Entanglement
EPR Paradox 1935

Completeness:
every element of reality must be represented in a complete physical theory

Element of reality:
value can be predicted with certainty (exists independently of measurement)

Locality:
Speed of light is the limit. It is possible to separate physical systems so that they do not influence one another

EPR showed (with their definitions of completeness, elements of reality, and locality)
→ quantum mechanics is not a complete theory
EPR Paradox for polarization states

Heisenberg uncertainty:
\[ \Delta \hat{\sigma}_1 \cdot \Delta \hat{\sigma}_3 \geq 1 \]

If we know result of a measurement along H/V
\[ \hat{\sigma}_3 |H\rangle = |H\rangle \rightarrow \langle \hat{\sigma}_3 \rangle = \langle H|\hat{\sigma}_3|H\rangle = 1 \]

Then we know nothing about result of a measurement along D/A
\[ \hat{\sigma}_2 |H\rangle = |V\rangle \rightarrow \langle \hat{\sigma}_2 \rangle = \langle H|\hat{\sigma}_3|H\rangle = \langle H|V\rangle = 0 \]

QM is incomplete
(the result of the D/A measurement is Pre-determined, we just don’t know it..)

QM is complete
But the properties associated with complementary observables \( \hat{\sigma}_2 \) and \( \hat{\sigma}_3 \) can’t have simultaneous reality

* This is a modified version of the EPR argument according to Bohm
EPR Paradox for polarization states

Entangled photons A and B:

\[ |\psi^-\rangle = \frac{1}{\sqrt{2}} (|H\rangle_A |V\rangle_B - |V\rangle_A |H\rangle_B) \]

\[ = \frac{1}{\sqrt{2}} (|D\rangle_A |A\rangle_B - |A\rangle_A |D\rangle_B) \]

Prefect anti-correlation; if A and B perform the same polarization measurement, they get opposite results:

\[ \langle \psi^- | \hat{\sigma}^A |_{\bar{n}} \hat{\sigma}^B |_{\bar{n'}} |\psi^-\rangle = -1 \]

A measures \( \hat{\sigma}_1 \) and gets result \( \sigma_1 = \pm 1 \)

A knows result for B due to anti-correlation \( \sigma_1 = \mp 1 \)

A can predict with certainty \( \sigma_1 \) of Bob's photon, which may be far away. This can't affect his reality (locality assumption).

\( \implies \sigma_1 \) of Bob's photon must be an element of reality (according to EPR def.)

What if A measures \( \hat{\sigma}_2 \) instead?

A can predict with certainty \( \sigma_2 \) of Bob's photon

\( \implies \sigma_2 \) of Bob's photon must also be an element of reality
EPR Paradox for polarization states

**QM is incomplete**
(the result of the D/A measurement is
Pre-determined, we just don’t know it..)

**QM is complete**
But the properties associated with complementary observables $\hat{O}_2$ and $\hat{O}_3$
can’t have simultaneous reality

Can we “complete” the quantum theory by adding some hidden variables?

Nope. The quantum correlation of entangled systems is stronger than any correlation local realistic theory would permit...
Bell’s thought experiment

\[ P(A|\lambda, a_i) \]

\[ A = 1 \]

\[ A = -1 \]

\[ P(B|\lambda, b_i) \]

\[ B = 1 \]

\[ B = -1 \]

\[ [A(\lambda, a_1) + A(\lambda, a_2)]B(\lambda, b_1) + [A(\lambda, a_1) - A(\lambda, a_2)]B(\lambda, b_2) = \pm 2 \]

\[ P(A, B|a, b) = \sum_{\lambda} P(\lambda) P(A|a, \lambda) P(B|b, \lambda) \]

\[ S_{\text{classical}} \]

\[ |\langle A(a')B(b) \rangle + \langle A(a)B(b) \rangle + \langle A(a')B(b') \rangle + \langle A(a)B(b') \rangle| \leq 2 \]
Bell Inequality

\[ S_{QM} = |E_{QM}(\alpha_1, \beta_1) + E_{QM}(\alpha_1, \beta_2) + E_{QM}(\alpha_2, \beta_1) - E_{QM}(\alpha_2, \beta_2)| \]

Quantum Mechanical Calculation:

\[ E_{QM}(\alpha, \beta) = P_{11} + P_{-1-1} - P_{1-1} - P_{-11} = -\cos 2(\alpha - \beta) \]

\[ S^{max}_{QM} = 2\sqrt{2} > 2 \]

\[ \alpha_1 = 0^\circ, \alpha_2 = 45^\circ, \beta_1 = -22.5^\circ, \beta_2 = 22.5^\circ, \]

QM in experimental disagreement with Bell’s assumption of local hidden variables!!!
Loopholes: Fair Sampling

Real experiments: loss

Draw 3 Balls

Loopholes: Fair Sampling

Real experiments: loss

Draw 3 Balls

→ All Balls in the bucket are green?

Closing Fair Sampling loophole

Real experiments: loss

\[ E_{QM}(\alpha, \beta) = P_{11} + P_{-1-1} - P_{1-1} - P_{-11} = -\eta^2 \cos 2(\alpha - \beta) \]

\[ \eta^2 S_{QM}^{max} = \eta^2 2\sqrt{2} > 2 \quad \eta > 84\% \]
Loopholes: Locality

The very first experiments were done with fixed polarizer settings so that it could not be guaranteed that

\[ P_A(A|\lambda, a, b, B) = P_A(A|\lambda, a) \]

\[ P_B(A|\lambda, a, b, B) = P_B(B|\lambda, b) \]

This loophole was closed by guaranteeing both outcome and setting independence by appropriate space-like separation in the sense of special relativity.

first addressed by A. Aspect et al. and fully closed by G. Weihs et al.
Locality

Space-like separate the setting choice and outcome on one side from the measurement on the other side.
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Space-like separate the setting choice and outcome on one side from the measurement on the other side
Addressing Freedom-of-Choice

Space-like separate setting choice from emission event
Loophole-free Bell test

(a) Alice \quad \sim 29 \text{m} \quad \text{Source} \quad \sim 29 \text{m} \quad \text{Bob}

(b) Source

(c) Measurement Station (Alice/Bob)
The Source
Pump Polarization V
Pump Polarization H
Pump Polarization H

\[ |\psi\rangle = \frac{1}{\sqrt{1 + r^2}} (|VH\rangle + r|HV\rangle) \]

The measurement stations

The measurement stations
Results close all significant loopholes*
The freedom-of-choice loophole

Handsteiner et al. (2017): "Cosmic Bell Test - Measurement Settings from Milky Way Stars

This experiment pushes back to at least \(\sim 7.8\) Gyr ago the most recent time by which any local-realist influences could have exploited the "freedom-of-choice" loophole to engineer the observed Bell violation.
E91 Protocol

\[ |\Psi\rangle_{AB} = \frac{1}{\sqrt{2}} \left( |1_A, 1_B\rangle + |0_A, 0_B\rangle \right) \]

3 Measurement settings each. After detecting photons A and B group the results into 2 groups of detection events:

- photons measured using the same basis by Alice and Bob \(\rightarrow\) sifted key
- photons measured using the Bell basis \(\rightarrow\) Calculate Bell parameter \(\rightarrow\) Security

Modified BB84 with entangled photons

Encoding and sifting steps identical to BB84 with single photons
- photons measured using the same basis by Alice and Bob → sifted key
- photons measured using the Bell basis → Calculate Bell parameter → Security