Physical Optics

Lecture 10: PSF Engineering
2020-01-09
Michael Kempe
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K = Kempe  
G = Gross
- Point Spread Function
  - At low Fresnel numbers
  - At high numerical aperture
- Apodization effects
- Applications:
  - Extended depth of focus
  - Stimulated emission depletion microscopy
  - Optical tweezers
Point Spread Function

Point-spread function: image of an infinitely small object point (size $\ll$ lambda)

Object $\rightarrow$ Optical System $\rightarrow$ Image

"Black Box"

Incoherent imaging:

$$I_{image}(x', y', z') = \int \int dx \, dy \, I_{PSF}(x', x, y', y, z', z) \cdot I_{obj}(x, y, z)$$

It also is the smallest spot to which light can be focussed for a given aperture $NA = n \cdot sin\alpha$
“Ideal“ Point Spread Function

Circular homogeneous illuminated aperture:

- Transverse intensity:
  Airy distribution
  Dimension: $D_{\text{Airy}}$
  normalized lateral coordinate:
  $$v = 2 \pi \frac{r}{\lambda \ NA}$$
  $$I(0,v) = \left[ \frac{2J_1(v)}{v} \right]^2 I_0$$
  $$D_{\text{Airy}} = \frac{1.22 \cdot \lambda}{NA}$$

- Axial intensity:
  sinc-function
  Dimension: Rayleigh unit $R_E$
  normalized axial coordinate
  $$u = 2 \pi \frac{z \ n}{\lambda \ NA^2}$$
  $$I(u,0) = \left[ \frac{\sin(u/4)}{u/4} \right]^2 I_0$$
  $$R_E = \frac{n \cdot \lambda}{NA^2}$$
Axial and Lateral Ideal Point Spread Function

- Comparison of both cross sections

Ref: R. Hambach
Low Fresnel Number Focussing

- Small Fresnel number $N_F$ (far field): geometrical focal point (center point of spherical wave) not identical with best focus (peak of intensity)
  - Optimal intensity is located towards optical system (pupil)
  - Focal caustic asymmetrical around the peak location
  - Explanation: the photometric distance law shows effects inside the depth of focus

\[ N_F = \frac{r_p^2}{\lambda \cdot z} \]
Low Fresnel Number Focussing

- Example: focussing by a micro lens

- Data:
  - radius of aperture: 0.1 mm
  - wavelength: 1 μm
  - focal length: 10 mm
  - Numerical aperture: NA = 0.01
  - Fresnel number: 1

- Results:
  - highest intensity in front of geometrical focus

Ref: R. Hambach
Low Fresnel Number Focussing

- System with small Fresnel number:
  Axial intensity distribution $I(z)$ is asymmetric

$$I(z) = I_0 \cdot \left(1 - \frac{u}{2\pi \cdot N_F}\right)^2 \cdot \left(\frac{\sin \frac{u}{4}}{\frac{u}{4}}\right)^2$$

- Example microscopic 100x/0.9 system:
  $a = 1\, \text{mm}, \, z = 100\, \text{mm}$:
  $N_F = 18$
High-NA Focusing

1. Transfer from entrance to exit pupil in high-NA: geometrical effect due to projection (photometry) $\rightarrow$ apodization

   \[ A(r') = A_0(r)\sqrt{\cos(u)} \]

2. Tilt of field vector components leads to:
   a) a $z$-component of the field
   b) a coupling between $x$- and $y$-components.
Vectorial Diffraction at High NA

Linear Polarization

Pupil

\[ \text{I}_{\text{ges}} \]

\[ \text{I}_x \]

\[ \text{I}_y \]

\[ \text{I}_z \]
Point Spread Function Modeling at High NA

- Comparison of PSF intensity profiles for different models as a function of defocussing

![Graph showing error of model vs NA for different models: paraxial, scalar high-NA, vectorial, low-NA, paraxial, scalar high-NA, vectorial.]

**NA = 0.98**

- Paraxial
- High-NA scalar
- High-NA vectorial
Apodization: Gaussian Illumination

- Gaussian beam pupil illumination

Ref: R. Hambach
Spherical Aberration

- Pupil phase error at large radius:
  - Axial asymmetrical distribution off axis
  - Peak moves axially towards lens

Ref: R. Hambach
Apodization: Annular Ring Pupil

- Ring illumination:
  - Generation of Bessel beams

Ref: R. Hambach

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Pupil intensity

Intensity $I(r)$ at focal point, $z=10$

Ref: Institute of Applied Physics

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R. Hambach
- Farfield of a ring pupil:
  - outer radius $a_a$
  - inner radius $a_i$
  - parameter $\varepsilon = \frac{a_i}{a_a} < 1$
- Ring structure increases with $\varepsilon$
- Depth of focus increases

\[
\Delta z = \frac{2\lambda}{n \sin^2 \theta \cdot (1 - \varepsilon^2)}
\]

- Application:
  - Telescope with central obscuration
- Intensity at focus

\[
I(x) = \frac{1}{(1 - \varepsilon^2)^2} \left[ \frac{2J_1(x)}{x} - \varepsilon^2 \cdot \frac{2J_1(\varepsilon x)}{\varepsilon x} \right]^2
\]
- Ring pupil illumination
- Enlarged depth of focus
- Lateral resolution constant due to large angle incidence
- Can not be understood geometrically
Apodisation of the pupil:
1. Homogeneous circle  \(\rightarrow\) Airy
2. Gaussian  \(\rightarrow\) Gaussian
3. Ring illumination  \(\rightarrow\) Bessel

PSF in focus:
different convergence to zero for larger radii

Encircled energy:
very different energy distribution

Definition of "compactness" of the central peak:
1. FWHM:
   Airy more compact as Gauss
   Bessel more compact as Airy
2. Energy 95%:
   Gauss more compact as Airy
   Bessel extremly worse
Application: Extended Depth of Focus

- Schematic drawing of the principal ray path in case of extended depth of focus
- Where is the energy going?
- What are the constraints and limitations?
Depth of Focus

- Depth of focus depends on numerical aperture

1. Large aperture: small depth of focus
2. Small aperture: large depth of focus

Ref: O. Bimber
Extended Depth of Focus

Approaches
1. PSF engineering (with/without digital postprocessing)
   - Phase pupil mask
   - Amplitude pupil mask
   - Complex Pupil mask
2. Plenoptic principle (with arrays, detection of intensity as a function of direction of incidence)
3. Time multiplexing of a fast variable system

Criteria for Engineering (application dependend)
- Axial gain factor of focal depth, uniformity over z (along axis)
- Strehl definition, transverse resolution, peak height of side lobes
- Contrast of image
- MTF-/OTF-requirements
- Energy transmission
• Ring shaped masks according to Toraldo:
  - discrete rings
  - absorbing rings or pure phase shifts
  - original setup: only 0 / \( \pi \) values of phase
  - special case:
    Fresnel zone plate

• Pure phase rings:
  amplitude of PSF

\[
E(r') = \sum_{j=1}^{n} e^{2\pi i \Phi_j} \left[ \varepsilon_j^2 \cdot \frac{2J_1(kr' \sin u' \varepsilon_j)}{kr' \sin u' \varepsilon_j} - \varepsilon_{j-1}^2 \cdot \frac{2J_1(kr' \sin u' \varepsilon_{j-1})}{kr' \sin u' \varepsilon_{j-1}} \right]
\]
Toraldo Phase Mask

- Here: pure phase mask

Ref: R. Hambach
EDF with Complex Toraldo Mask

- $I(r,z)$

- $I(x)$

- $I(z)$

$I(z)$ depth for 80%: 13 RE
Cubic Phase Plate (CCP)

- Phase Mask with cubic polynomial shape

\[ P(x) = \begin{cases} e^{i\alpha x^3} & \text{for } |x| \leq 1 \\ 0 & \text{otherwise} \end{cases} \]

- Effect of mask:
  - depth of focus enlarged
  - PSF broadened, but nearly constant
  - Deconvolution possible

- Problems:
  - variable PSF over field size
  - noise increased
  - image artefacts
Cubic Phase Mask: PSF and OTF

Conventional imaging

System with cubic phase mask

MTF eines beugungsbegrenzten optischen Systems

MTF von DeepView

Im Fokus
1 Schärferlinie defokussiert
2 Schärferlinien defokussiert

focus defocussed

focus defocussed
Cubic Phase Mask: PSF with Deconvolution

Psf traditional

primary Psf with phase mask

Psf with phase mask deconvolved

Defocus

primary

with mask

with mask deconvolved
Application: Stimulated Emission Depletion (STED) Microscopy

Principle

The fluorescence excitation achieved by a pump beam is selectively removed by stimulated emission with a co-aligned erase beam $\rightarrow$ nonlinear (saturated) de-excitation

Erase beam engineering by phase mask

STED Microscopy


The Nobel Prize in Chemistry 2014 was awarded jointly to Eric Betzig, Stefan W. Hell and William E. Moerner "for the development of super-resolved fluorescence microscopy."
Stimulated Emission Depletion Microscopy

STED imaging of the endoplasmic reticulum in a mammalian cell
Fluorophore: YFP

Application: Optical Tweezers

- **Optical tweezers:**
  optical trap by using gradient forces of a focused light beam in 3D for non-conducting particles

- The forces on the particles move it towards the region of higher intensity

- The change of optical momentum due to the refracted rays creates a net force

  momentum of a photon → force

  \[ p = h \cdot \frac{\lambda}{n} \]

  \[ F = m \cdot a = \frac{dp}{dt} \]

  Typical size: fN (10^{-15} Newton)

Nobel prize in physics in 2018: **Arthur Ashkin** "for the optical tweezers and their application to biological systems"

Ref: D. Simons
Trapping Forces

- Forces in a collimated beam

- Forces in a focussed beam
Optical Tweezers

- Basic setup for particle trapping
- Branches:
  1. laser illumination
  2. conventional illumination
  3. observation

Ref: D. Simons
Optical Tweezers

- Optical forces in inhomogeneous fields:
  - 3 different profiles:
    a) strong focussed, b) weak focussed, c) ring profile

1. profile \( I(x,y) \)

2. axial forces (weak)

3. lateral forces

Ref: D. Simons
Optical Tweezers: Applications

- Applications:
  - controlled movement of cells and particles
  - measurement of force in biological systems

Ref: S. Zwick

A motor molecule walks inside the light trap

1 The kinesin molecule attaches to a small sphere held by the optical tweezers.
2 Kinesin marches away along the cell skeleton. It pulls the sphere, making it possible to measure the kinesin’s stepwise motion.
3 Finally, the motor molecule can no longer withstand the force of the light trap and the sphere is forced back to the centre of the beam.

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