<table>
<thead>
<tr>
<th>No</th>
<th>Date</th>
<th>Subject</th>
<th>Ref</th>
<th>Detailed Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>05.04.</td>
<td>Wave optics</td>
<td>G</td>
<td>Complex fields, wave equation, k-vectors, interference, light propagation, interferometry</td>
</tr>
<tr>
<td>2</td>
<td>12.04.</td>
<td>Diffraction</td>
<td>B</td>
<td>Slit, grating, diffraction integral, diffraction in optical systems, point spread function, aberrations</td>
</tr>
<tr>
<td>3</td>
<td>19.04.</td>
<td>Fourier optics</td>
<td>B</td>
<td>Plane wave expansion, resolution, image formation, transfer function, phase imaging</td>
</tr>
<tr>
<td>4</td>
<td>26.04.</td>
<td>Quality criteria and resolution</td>
<td>B</td>
<td>Rayleigh and Marechal criteria, Strehl ratio, coherence effects, two-point resolution, criteria, contrast, axial resolution, CTF</td>
</tr>
<tr>
<td>5</td>
<td>03.05.</td>
<td>Polarization</td>
<td>G</td>
<td>Introduction, Jones formalism, Fresnel formulas, birefringence, components</td>
</tr>
<tr>
<td>6</td>
<td>10.05.</td>
<td>Photon optics</td>
<td>D</td>
<td>Energy, momentum, time-energy uncertainty, photon statistics, fluorescence, Jablonski diagram, lifetime, quantum yield, FRET</td>
</tr>
<tr>
<td>7</td>
<td>17.05.</td>
<td>Coherence</td>
<td>G</td>
<td>Temporal and spatial coherence, Young setup, propagation of coherence, speckle, OCT-principle</td>
</tr>
<tr>
<td>8</td>
<td>24.05.</td>
<td>Laser</td>
<td>B</td>
<td>Atomic transitions, principle, resonators, modes, laser types, Q-switch, pulses, power</td>
</tr>
<tr>
<td>9</td>
<td>31.05.</td>
<td>Gaussian beams</td>
<td>D</td>
<td>Basic description, propagation through optical systems, aberrations</td>
</tr>
<tr>
<td>10</td>
<td>07.06.</td>
<td>Generalized beams</td>
<td>D</td>
<td>Laguerre-Gaussian beams, phase singularities, Bessel beams, Airy beams, applications in superresolution microscopy</td>
</tr>
<tr>
<td>11</td>
<td>14.06.</td>
<td>PSF engineering</td>
<td>G</td>
<td>Apodization, superresolution, extended depth of focus, particle trapping, confocal PSF</td>
</tr>
<tr>
<td>12</td>
<td>21.06.</td>
<td>Nonlinear optics</td>
<td>D</td>
<td>Basics of nonlinear optics, optical susceptibility, 2nd and 3rd order effects, CARS microscopy, 2 photon imaging</td>
</tr>
<tr>
<td>13</td>
<td>28.06.</td>
<td>Scattering</td>
<td>G</td>
<td>Introduction, surface scattering in systems, volume scattering models, calculation schemes, tissue models, Mie Scattering</td>
</tr>
<tr>
<td>14</td>
<td>05.07.</td>
<td>Miscellaneous</td>
<td>G</td>
<td>Coatings, diffractive optics, fibers</td>
</tr>
</tbody>
</table>

D = Dienerowitz  
B = Böhme  
G = Gross
Contents

- Introduction
- Fresnel formulas
- Jones calculus
- Further descriptions
- Birefringence
- Components
- Applications
Scalar / vectorial Optics

- **Scalar:**
  Helmholtz equation

\[
\left( \Delta + k_o^2 n \right) E(\vec{r}) = 0
\]

- **Vectorial:**
  Maxwell equations

\[
\vec{k} \times \vec{H} = \omega \vec{D} - i \cdot \vec{j} \\
\vec{k} \times \vec{E} = \omega \vec{B} \\
\vec{k} \cdot \vec{D} = -i \cdot \rho \\
\vec{k} \cdot \vec{B} = 0
\]

\[
\vec{D} = \varepsilon_0 \varepsilon_r \vec{E} + \vec{P} \\
\vec{B} = \mu_0 \mu_r \vec{H} + \vec{M} \\
\vec{k} \cdot \vec{J} = \omega \rho \\
\vec{J} = \sigma \vec{E}
\]
Basic Forms of Polarisation

1. Linear components in phase

2. Circular phase difference of 90° between components

3. Elliptical arbitrary but constant phase difference
Basic Notations of Polarization

- Description of electromagnetic fields:
  - Maxwell equations
  - Vectorial nature of field strength
- Decomposition of the field into components
- Propagation plane wave:
  - Field vector rotates
  - Projection components are oscillating sinusoidal

\[ \vec{E} = A_x \cos \omega t \vec{e}_x + A_y \cos(\omega t - \delta) \vec{e}_y \]
Polarization Ellipse

- Elimination of the time dependence:
  Ellipse of the vector $E$

- Different states of polarization:
  - sense of rotation
  - shape of ellipse

\[
\frac{E_x^2}{A_x^2} + \frac{E_y^2}{A_y^2} - \frac{2E_x E_y \cos \delta}{A_x A_y} = \sin^2 \delta
\]
Polarization Ellipse

- Representation of the state of polarization by an ellipse
- Field components
  \[ E_x = A_x \cdot (\cos \theta \cdot \cos \delta_x + \sin \theta \cdot \sin \delta_x) \]
  \[ E_y = A_y \cdot (\cos \theta \cdot \cos \delta_y + \sin \theta \cdot \sin \delta_y) \]
- Axes of ellipse: a, b
- Rotation angle of the field \( \theta \)
  \[ \tan 2\theta = \frac{2A_x A_y}{A_x^2 - A_y^2} \cos \delta \]
- Angle of eccentricity
  \[ \tan \varepsilon = \frac{b}{a} \]
Circular Polarization

- Spiral curve of field vector
- Superposition of left and right handed circular polarized light: resulting linear polarization

Ref: Manset
Circular polarized Light

- Rotation of plane of polarization with t / z
- Phase angle 90°
- Generation by λ/4 plate out of linear polarized light
• Schematical illustration of the ray refraction (reflection at an interface)
• The cases of s- and p-polarization must be distinguished

a) s-polarization

b) p-polarization
Fresnel Formulas

- **Electrical transverse polarization**
  TE, s- or σ-polarization, E perpendicular to incidence plane

- **Magnetical transverse polarization**
  TM, p- or p-polarization, E in incidence plane

- **Boundary condition of Maxwell equations**
  at a dielectric interface:
  continuous tangential component of E-field

  \[ \varepsilon_1 \cdot E_{1n} = \varepsilon_2 \cdot E_{2n} \]

  \[ E_{1t} = E_{2t} \]

- **Amplitude coefficients for reflected field**
  transmitted field

  \[ r_{TE} = \left. \frac{E_r}{E_e} \right|_{TE} \]

  \[ r_{TM} = \left. \frac{E_r}{E_e} \right|_{TM} \]

  \[ t_{TE} = \left. \frac{E_t}{E_e} \right|_{TE} = r_{TE} + 1 \]

  \[ t_{TM} = \frac{n}{n'} \cdot (r_{TM} + 1) \]

- **Reflectivity and transmission of light power**

  \[ R = \frac{P_r}{P_e} = \left| r \right|^2 \]

  \[ T = \frac{P_t}{P_e} = \left( \frac{n'}{n \cdot \cos i'} \right) \cdot \left| t \right|^2 \]
Fresnel Formulas

- Coefficients of amplitude for reflected rays, s and p

\[ r_{E_\perp} = \frac{-\sin(i - i')}{\sin(i + i')} = \frac{n \cdot \cos i - \sqrt{n'^2 - n^2 \cdot \sin^2 i}}{n \cdot \cos i + \sqrt{n'^2 - n^2 \cdot \sin^2 i}} = \frac{n \cdot \cos i - n' \cdot \cos i'}{n \cdot \cos i + n' \cdot \cos i'} = \frac{k_{ez} - k_{tz}}{k_{ez} + k_{tz}} \]

\[ r_{E_\parallel} = \frac{\tan(i - i')}{\tan(i + i')} = \frac{n'^2 \cdot \cos i - n \cdot \sqrt{n'^2 - n^2 \cdot \sin^2 i}}{n'^2 \cdot \cos i + n \cdot \sqrt{n'^2 - n^2 \cdot \sin^2 i}} = \frac{n' \cdot \cos i - n \cdot \cos i'}{n' \cdot \cos i + n \cdot \cos i'} = \frac{n^2 \cdot k_{ez} - n^2 \cdot k_{tz}}{n^2 \cdot k_{ez} + n^2 \cdot k_{tz}} \]

- Coefficients of amplitude for transmitted rays, s and p

\[ t_{E_\perp} = \frac{2n \cdot \cos i}{n \cdot \cos i + n' \cdot \cos i'} = \frac{2n \cdot \cos i}{n \cdot \cos i + \sqrt{n'^2 - n^2 \cdot \sin^2 i}} = \frac{2n \cdot \cos i}{n \cdot \cos i + n' \cdot \cos i'} = \frac{2k_{ez}}{k_{ez} + k_{tz}} \]

\[ t_{E_\parallel} = \frac{2n \cdot \cos i}{n' \cdot \cos i + n \cdot \cos i'} = \frac{2n' \cdot n \cdot \cos i}{n'^2 \cdot \cos i + n \cdot \sqrt{n'^2 - n^2 \cdot \sin^2 i}} = \frac{2n \cdot \cos i}{n' \cdot \cos i + n \cdot \cos i'} = \frac{2n^2 \cdot k_{ez}}{n^2 \cdot k_{ez} + n^2 \cdot k_{tz}} \]
Fresnel Formulas

- Typical behavior of the Fresnel amplitude coefficients as a function of the incidence angle for a fixed combination of refractive indices

- $i = 0$
  Transmission independent on polarization
  Reflected $p$-rays without phase jump
  Reflected $s$-rays with phase jump of $\pi$ (corresponds to $r<0$)

- $i = 90^\circ$
  No transmission possible
  Reflected light independent on polarization

- Brewster angle: completely $s$-polarized reflected light
Fresnel Formulas: Energy vs. Intensity

Fresnel formulas, different representations:

1. Amplitude coefficients, with sign
2. Intensity coefficients: no additivity due to area projection
3. Power coefficients: additivity due to energy preservation
- Decomposition of the field strength $E$ into two components in $x/y$ or $s/p$

$$\vec{E}_0 = \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} A_x e^{-i\delta_x} \\ A_y e^{-i\delta_y} \end{pmatrix}$$

- Relative phase angle between components

$$\delta = \delta_y - \delta_x$$

- Polarization ellipse

$$\left(\frac{E_x}{A_x}\right)^2 + \left(\frac{E_y}{A_y}\right)^2 - 2 \frac{E_x E_y}{A_x A_y} \cos \delta = \sin^2 \delta$$

- Linear polarized light

$$\vec{E}_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \vec{E}_0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \vec{E}_0 = \begin{pmatrix} \cos \alpha \\ \sin \alpha \end{pmatrix}$$

- Circular polarized light

$$\vec{E}_{rz} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ i \end{pmatrix} \quad \vec{E}_{iz} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$$
- Jones representation of full polarized field: decomposition into 2 components
- Cascading of system components: Product of matrices
- In principle 3 types of components influencing polarization:
  1. Change of amplitude polarizer, analyzer
  2. Change of phase retarder
  3. Rotation of field components rotator

\[
\tilde{E} = J \cdot \tilde{E}_o, \quad \begin{pmatrix} E_p \\ E_s \end{pmatrix} = \begin{pmatrix} J_{ss} & J_{sp} \\ J_{ps} & J_{pp} \end{pmatrix} \begin{pmatrix} E_{op} \\ E_{os} \end{pmatrix}
\]

\[
\tilde{E}_n = J_n \cdot J_{n-1} \cdots J_2 \cdot J_1 \cdot \tilde{E}_1
\]

\[
J_{\text{trans}} = \begin{pmatrix} 1-t_s & 0 \\ 0 & 1-t_p \end{pmatrix}
\]

\[
J_{\text{ret}} = \begin{pmatrix} e^{-i\frac{\delta}{2}} & 0 \\ 0 & e^{i\frac{\delta}{2}} \end{pmatrix}
\]

\[
J_{\text{rot}}(\varphi) = \begin{pmatrix} \cos \varphi & \sin \varphi \\ -\sin \varphi & \cos \varphi \end{pmatrix}
\]
Jones Matrices

- Rotated component
  \[ J(\varphi) = D(-\varphi) J(0) D(\varphi) \]

- Rotation matrix
  \[ D(\varphi) = \begin{pmatrix} \cos \varphi & \sin \varphi \\ -\sin \varphi & \cos \varphi \end{pmatrix} \]

- Intensity
  \[ I_2 = \bar{E}_1 J^* J \bar{E}_1 \]

- Three types of components to change the polarization:
  1. amplitude: polarizer / analyzer
  2. phase: retarder
  3. orientation: rotator

- Propagation:
  1. free space
    \[ J_{PRO} = e^{-i \frac{2\pi}{\lambda} \Delta_{OPL}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \]
  2. dielectric interface
    \[ J_{TRA} = \begin{pmatrix} t_s & 0 \\ 0 & t_p \end{pmatrix} \]
    \[ J_{REF} = \begin{pmatrix} r_s & 0 \\ 0 & r_p \end{pmatrix} \]
  3. mirror
    \[ J_{SP} = r \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \]
Birefringence: Uniaxial Crystal

- Birefringence: index of refraction depends on field orientation
- Uniaxial crystal:
  - ordinary index $n_o$ perpendicular to crystal axis
  - extra-ordinary index $n_e$ along crystal axis
- Difference of indices
  \[ \Delta n = n_e - n_o \]

- Jones matrix
  \[
  J_{neo} = \begin{pmatrix}
  e^{-i\frac{\Delta n\cdot z}{\lambda}} & 0 \\
  0 & e^{i\frac{\Delta n\cdot z}{\lambda}}
  \end{pmatrix}
  \]

\[
J_{neo}(\alpha, \phi) = \begin{pmatrix}
\sin^2 \phi + \cos^2 \phi e^{i\alpha} & \sin \phi \cos \phi \left(e^{i\alpha} - 1\right) \\
\sin \phi \cos \phi \left(e^{i\alpha} - 1\right) & \cos^2 \phi + \sin^2 \phi e^{i\alpha}
\end{pmatrix}
\]

- Relative phase angle
  \[
  \alpha = \frac{2\pi z}{\lambda} (n_e - n_o)
  \]
## Descriptions of Polarization

<table>
<thead>
<tr>
<th></th>
<th>Parameter</th>
<th>Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Polarization ellipse</td>
<td>Ellipticity $\varepsilon$, orientation $\theta$</td>
</tr>
<tr>
<td>2</td>
<td>Complex parameter</td>
<td>Parameter $\zeta$</td>
</tr>
<tr>
<td>3</td>
<td>Jones vectors</td>
<td>Components of $E$</td>
</tr>
<tr>
<td>4</td>
<td>Stokes vectors</td>
<td>Stokes parameter $S_0 \ldots S_4$</td>
</tr>
<tr>
<td>5</td>
<td>Poincare sphere</td>
<td>Points on or inside the Poincare sphere</td>
</tr>
<tr>
<td>6</td>
<td>Coherence matrix</td>
<td>$2 \times 2$ - matrix $C$</td>
</tr>
</tbody>
</table>
Stokes Vector

- **Description of polarization from the energetic point of view**

- $S_0$ total intensity
  
  \[ S_0 = I(0,0) + I(90,0) \]
  
  \[ S_0 = E_x^2 + E_y^2 \]

- $S_1$ Difference of intensity in x-y linear
  
  \[ S_1 = I(0,0) - I(90,0) \]
  
  \[ S_1 = E_x^2 - E_y^2 \]

- $S_2$ Difference of intensity linear under 45° / 135°
  
  \[ S_2 = I(45,0) - I(135,0) \]
  
  \[ S_2 = 2E_x E_y \cdot \cos \delta \]

- $S_3$ Difference of circular components
  
  \[ S_3 = I(45,90) - I(45, -90) \]
  
  \[ S_3 = 2E_x E_y \cdot \sin \delta \]
Stokes Vector

- Description of a polarization state with Stokes parameter
  Interpretation: Components of the field on the Poincare sphere
- Also partial polarization is taken into account
  
  \[ S_0^2 \geq S_1^2 + S_2^2 + S_3^2 \]
- Relation
  Unequal sign: partial polarization

- Stokes vector 4x1

- Propagation:
  Müller matrix M

\[
\begin{pmatrix}
S_0 \\
S_1 \\
S_2 \\
S_3
\end{pmatrix}
= 
\begin{pmatrix}
m_{00} & m_{01} & m_{02} & m_{03} \\
m_{10} & m_{11} & m_{12} & m_{13} \\
m_{20} & m_{21} & m_{22} & m_{23} \\
m_{30} & m_{31} & m_{32} & m_{33}
\end{pmatrix}
\cdot
\begin{pmatrix}
S_0 \\
S_1 \\
S_2 \\
S_3
\end{pmatrix}
\]
Examples of Stokes Vectors

- Linear horizontal / vertical
  \[ \vec{S} = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad \vec{\bar{S}} = \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} \]

- Linear 45°
  \[ \vec{S} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix} \]

- Circular clockwise / counter-clockwise
  \[ \vec{S} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix} \quad \vec{\bar{S}} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix} \]
Partial Polarization

- Partial polarized light: degree of polarization
  
  \[ p = 0: \text{un-polarized} \]
  
  \[ p = 1: \text{fully polarized} \]
  
  \[ 0 < p < 1: \text{partial polarized} \]

- Determination of Stokes parameter:

  \[ p = \frac{I_{pol}}{I_{ges}} \]

  \[ p = \frac{\sqrt{S_1^2 + S_2^2 + S_3^2}}{S_0} \]

- Unpolarized light

  \[ S_0 = (1 - p)S_0 + pS_0 = S_u + S_p \]

  \[ S_1 = S_2 = S_3 = 0 \]

- Fully polarized light

  \[ S_0^2 = S_1^2 + S_2^2 + S_3^2 \]
Every point on a unit sphere describes one state of polarization.

In spherical coordinates:

\[
\mathbf{r} = \frac{1}{\sqrt{x^2 + y^2 + z^2}}, \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \cos 2\varepsilon \cdot \sin 2\theta \\ \cos 2\varepsilon \cdot \cos 2\theta \\ \sin 2\varepsilon \end{pmatrix}
\]

Points on the z-axis: circular polarized light.

Meridian line: linear polarization.

Points inside partial polarization.
Partial Polarization on the Poincare Sphere

- Fully polarized: point
- Unpolarized: full surface
- Partial polarized: probability distribution, points inside
- Stokes parameter $S_1, S_2, S_3$: Components along axis directions
- Radius of sphere, length of vector: $S_0$
- Projection into meridional plane: angle $2\theta$ of polarization ellipse
- Projection into meridian plane: eccentricity angle $2\varepsilon$
Polarization of a donat mode in the focal region:
1. In focal plane
2. In defocused plane

Ref: F. Wyrowski
- Jones matrix changes Jones vector
  \[ \tilde{E}' = J \cdot \tilde{E} \]

- Müller matrix changes Stokes vector
  \[ \tilde{S}' = M \cdot \tilde{S} \]

- Change of coherence matrix
  \[ C' = J \cdot C \cdot J^+ \]

- Procedure for real systems:
  1. Raytracing
  2. Definition of initial polarization
  3. Jones vector or coherence matrix local on each ray
  4. transport of ray and vector changes at all surfaces
  5. 3D-effects of Fresnel equations on the field components
  6. Coatings need a special treatment
  7. Problems: ray splitting in case of birefringence
Polarization Raytrace

- 3D calculus in global coordinates according to Chipman
- Ray trace defines local coordinate system

\[
\vec{x}_{L,1} = \frac{\vec{s}_{in} \times \vec{s}'_1}{\vec{s}_{in} \times \vec{s}'_1}, \quad \vec{y}_{L,j} = \vec{s}_{in} \times \vec{x}_{L,1}, \quad \vec{z}_{L,1} = \vec{s}_{in}
\]

\[
\vec{x}'_{L,1} = \vec{x}_{L,1}, \quad \vec{y}'_{L,1} = \vec{s}_{in} \times \vec{x}_{L,1}, \quad \vec{z}'_{L,1} = \vec{s}'_1
\]

- Transform between local and global coordinates

\[
T^{-1}_{1,in} = \begin{pmatrix}
    x_{L,x1} & x_{L,y1} & x_{L,z1} \\
    y_{L,x1} & y_{L,y1} & y_{L,z1} \\
    s_{x, in} & s_{y, in} & s_{z, in}
\end{pmatrix}, \quad T_{1, out} = \begin{pmatrix}
    x_{L,x1} & y'_{L,x1} & s'_{1x} \\
    x_{L,y1} & y'_{L,y1} & s'_{1y} \\
    x_{L,z1} & y'_{L,z1} & s'_{1z}
\end{pmatrix}
\]
Polarization Raytrace

- Embedded local 2x2 Jones matrix

\[
J_{1,\text{refr}} = \begin{pmatrix}
  j_{11} & j_{12} & 0 \\
  j_{21} & j_{22} & 0 \\
  0 & 0 & 1
\end{pmatrix}
\]

- Matrices of refracting surface and reflection

\[
J_r = \begin{pmatrix}
  t_p & 0 & 0 \\
  0 & t_s & 0 \\
  0 & 0 & 1
\end{pmatrix}, \quad J_r = \begin{pmatrix}
  r_p & 0 & 0 \\
  0 & r_s & 0 \\
  0 & 0 & 1
\end{pmatrix}
\]

- Field propagation

\[
\bar{E}_j = P_j \cdot \bar{E}_{j-1}
\]

\[
\begin{pmatrix}
  E_{x,j} \\
  E_{y,j} \\
  E_{z,j}
\end{pmatrix} = \begin{pmatrix}
  p_{xx} & p_{yx} & p_{zx} \\
  p_{xy} & p_{yy} & p_{zy} \\
  p_{xz} & p_{yz} & p_{zz}
\end{pmatrix} \begin{pmatrix}
  E_{x,j-1} \\
  E_{y,j-1} \\
  E_{z,j-1}
\end{pmatrix}
\]

- Cascading of operator matrices

\[
P_{\text{total}} = P_M \cdot P_{M-1} \cdots P_2 \cdot P_1
\]

- Transfer properties
  1. Physical changes
  2. Geometrical bending effects

\[
P_1 = T_{1,\text{out}} \cdot J_{1,\text{refr}} \cdot T_{1,\text{in}}^{-1}
\]

\[
Q_1 = T_{1,\text{out}} \cdot J_{1,\text{bend}} \cdot T_{1,\text{in}}^{-1}
\]
Diattenuation and Retardation

- Change of field strength: calculation with polarization matrix, transmission $T$
  
- Diattenuation

- Eigenvalues of Jones matrix

- Retardation: phase difference of complex eigenvalues

- To be taken into account:
  1. physical retardance due to refractive index: $P$
  2. geometrical retardance due to geometrical ray bending: $Q$

- Retardation matrix

\[
T = \frac{|P \cdot \vec{E}|^2}{|\vec{E}|^2} = \frac{\vec{E}^* \cdot P^T \cdot P \cdot \vec{E}}{\vec{E}^* \cdot \vec{E}}
\]

\[
D = \frac{T_{\text{max}} - T_{\text{min}}}{T_{\text{max}} + T_{\text{min}}}
\]

\[
J_{\text{ret}} \cdot \vec{w}_{1/2} = \Lambda_{1/2} \cdot \vec{w}_{1/2} = e^{i\Phi_{1/2}} \cdot \vec{w}_{1/2}
\]

\[
\delta = \text{arg}(\Lambda_1) - \text{arg}(\Lambda_2)
\]

\[
R = Q^{-1}_{\text{total}} \cdot P_{\text{total}}
\]
- Müller matrix visualization
  Interpretation not trivial

- Retardation and diattenuation map across the pupil

- Polarization Zernike pupil aberration according to M. Totzeck (complicated)
Anisotropic Media

- Different types of anisotropic media

Ref: Saleh / Teich
- Classical geometry of birefringent refraction refers on interface plane
- Grandfathers method:
  Calculation iterative due to non-linear equations in prism coordinates
- History: formulas according to Muchel / Schöppe
  Only plane setup considered, crystal axis in plane of incidence
Ray Splitting in Case of Birefringence

- Incident plane wave
- Different refractive indices for polarization direction
- Important: relative orientation against crystal axis
- Arbitrary incidence: splitting of rays
  - ordinary ray: perpendicular to optical axis
  - extra ordinary ray: in plane of incidence
Birefringence

- Different directions of E and D
- Ray splitting not identical to wave splitting
Anisotropic Media

- Relationship between electric field and displacement vector
  \[ \vec{D} = \varepsilon_0 \varepsilon_r \cdot \vec{E} \]

- Linear relationship of tensor-equation
  \[ D_j = \sum_l \varepsilon_{jl} E_l + \sum_{l,m} \gamma_{jlm} \nabla_m E_l + \sum_{l,m,q} \alpha_{jlmq} \nabla_m \nabla_q E_l + \]

- First term, coefficient \( \varepsilon \): local direction, birefringence
- Second term, coefficient \( \gamma \): gradient of field, third order, optical activity, polarization rotated
- Third term, \( \alpha \): forth order, intrinsic birefringence, spatial dispersion of birefringence

- General:
  Due to the tensor properties of the coefficients, all these effects are anisotropic

- The field \( \vec{E} \) and the displacement \( \vec{D} \) are no longer aligned
  \[ \vec{D} = -n^2 \vec{s} \times (\vec{s} \times \vec{E}) = -n^2 [\vec{s} \cdot (\vec{s} \cdot \vec{E}) - \vec{E}] \]
Vanishing solution determinante of the wave equation

\[ \frac{k_x^2}{c^2 - c_x^2} + \frac{k_y^2}{c^2 - c_y^2} + \frac{k_z^2}{c^2 - c_z^2} = 0 \]

Value of speed of light depending on the ray direction (phase velocity)

Alternative: axis 1/n
Index or Normal Ellipsoid

- Inverse matrix of the dielectric tensor: index or normal ellipsoid

\[
n_i = \sqrt{\varepsilon_i} \quad \varepsilon_r^{-1} = \begin{pmatrix}
    1 & 0 & 0 \\
    0 & \frac{1}{\varepsilon_x} & 0 \\
    0 & 0 & \frac{1}{\varepsilon_z}
\end{pmatrix}
\]

\[
\frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = \text{const}
\]

- Gives the refractive index as a function of the orientation
- Also possible: ellipsoid of k-values

\[
\frac{k_x^2 n_x^2}{n^2 - n_x^2} + \frac{k_y^2 n_y^2}{n^2 - n_y^2} + \frac{k_z^2 n_z^2}{n^2 - n_z^2} = 0
\]
- Special case uniaxial crystal: ellipsoid rotational symmetry
  - \( n_0 \) ordinary direction valid for two directions
  - \( n_e \) extra ordinary valid for only one direction
- Two cases:
  - \( n_e > n_0 \): positive (prolate, cigar)
  - \( n_e < n_0 \): negative (oblate, disc)
- Arbitrary orientation \( \theta \): intersection points

![Diagram of Index Ellipsoid for Uniaxial Crystals](image-url)
- Incident plane wave
- Osculating tangential plane at the ordinary index-sphere: defines normal to o-ray direction
- Osculating tangential plane at the index o/e-index ellipsoid: normal to e-direction
Classical uniaxial media used in polarization components:

1. Quartz, positive birefringent, small difference
2. Calcite, negative birefringent, larger difference

<table>
<thead>
<tr>
<th>material</th>
<th>sign</th>
<th>n_o</th>
<th>n_eo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calcite</td>
<td>negative</td>
<td>1.6584</td>
<td>1.4864</td>
</tr>
<tr>
<td>Quartz, SiO₂</td>
<td>positive</td>
<td>1.5443</td>
<td>1.5534</td>
</tr>
</tbody>
</table>
Refraction with Birefringence

- Optical symmetry axis of crystal material breaks symmetry
- Split of rays depends on the axis orientation
- Split of field into two orthogonal polarisation components
- Energy propagation (ray, Poynting) in general not perpendicular to wavefront

![Diagram showing refraction with birefringence](image-url)
Polarizer

- Polarizer with attenuation $c_{s/p}$
- Rotated polarizer
- Polarizer in y-direction

\[
J_{\text{LIN}} = \begin{pmatrix}
1 - c_s & 0 \\
0 & 1 - c_p
\end{pmatrix}
\]

\[
J_p(\phi) = \begin{pmatrix}
\cos^2 \phi & \sin \phi \cos \phi \\
\sin \phi \cos \phi & \sin^2 \phi
\end{pmatrix}
\]

\[
J_p(0) = \begin{pmatrix}
0 & 0 \\
0 & 1
\end{pmatrix}
\]
Pair Polarizer-Analyzer

- Polarizer and analyzer with rotation angle $\theta$
- Law of Malus:
  
  Energy transmission

\[ I(\theta) = I_o \cdot \cos^2 \theta \]
- Crossed pair of polarizer - analyzer plates
- No sample: transmission zero
- Polarized sample between the plates:
  - spatial resolved rotation of polarization
  - analysis of stress and strain
Retarder

- Phase difference $\delta$ between field components
- Retarder plate with rotation angle $\varphi$

- Special value:
  $\lambda / 4$ - plate generates circular polarized light
  1. fast axis $y$

  \[ J_{V}^{}(0, \pi / 2) = \begin{pmatrix} 1 & 0 \\ 0 & -i \end{pmatrix} \]

  2. fast axis $45^\circ$

  \[ J_{V}^{}(\pi / 4, \pi / 2) = \frac{i-1}{2} \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix} \]

\[
J_{RET} = \begin{pmatrix} e^{-\frac{i\delta}{2}} & 0 \\ 0 & e^{\frac{i\delta}{2}} \end{pmatrix}
\]

\[
J_{V}^{}(\delta, \varphi) = \begin{pmatrix} \cos^2 \varphi + \sin^2 \varphi e^{i\delta} & \sin \varphi \cos \varphi \left(1 - e^{-i\delta}\right) \\ \sin \varphi \cos \varphi \left(1 - e^{-i\delta}\right) & \sin^2 \varphi + \cos^2 \varphi e^{i\delta} \end{pmatrix}
\]
- Rotate the plane of polarization
- Realization with magnetic field: Faraday effect
  \[ \beta = |\vec{B}| \cdot L \cdot V \]
- Verdet constant V

\[ J_{\text{ROT}} = \begin{pmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{pmatrix} \]
Liquid Crystals

- **Types of LC media:**
  - nematic: long molecules, oriented in one direction
  - smectic: long molecules, oriented in one direction in several layers, different types A, B, C
  - discotic: plate-shapes molecules, oriented in a plane
  - cholesteric: skrew-shaped molecules

- **Functional principle:**
  - interaction creates domaines
  - orientation of the domaines by voltage
  - anisotropic behavior, birefringence
  - strong influence of temperature

- **Parameter to describe the order / entropy:**
  \[ S = \frac{1}{2} \cdot \langle 3 \cdot \cos^2 \theta - 1 \rangle \]

- **Application:**
  - displays
  - spatial light modulator
Orientation of the molecules in an external field (voltage)
Types of Liquid Crystals

- Nematic
- Smectic A
- Smectic C
Types of Liquid Crystals

**smectic C**

**cholesteric**
- Transmission changes due to polarization
- Black-White switch possible
- Reflective or refractive
Microscopic Contrast

- bright field
  - epi illumination
  - trans illumination
- polarization
- fluorescence

Ref: M. Kempe
Differential Interference Contrast

- Point spread function

\[ E_{psfdic}(x, y) = (1 - R) \cdot e^{-i\cdot\theta} \cdot E_{psf}(x - \Delta x, y) \]

\[ - R \cdot e^{+i\cdot\theta} \cdot E_{psf}(x + \Delta x, y) \]

- Parameter:
  1. Shear distance
  2. Phase offset
  3. Splitting ratio
Differential Interference Contrast

- Differential splitting:
  Large contrast at phase gradients
Differential Interference Contrast

- Lateral shift: preferred direction
- Visibility depends on orientation of details

**shift 45°**

**x-shift (0°)**

**y-shift (90°)**
DIC Phase Imaging

- Orientation of prisms and shift size determines the anisotropic image formation

Phase object  x shear  x-y shear  y shear

\[ \varphi_0 = 0 \]  \[ \varphi_0 = \pi/8 \]  \[ \varphi_0 = \pi/4 \]

Ref.: M. Kempe