Physical Optics

Lecture 13: Scattering
2017-06-28
Herbert Gross
<table>
<thead>
<tr>
<th>No</th>
<th>Date</th>
<th>Subject</th>
<th>Ref</th>
<th>Detailed Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>05.04.</td>
<td>Wave optics</td>
<td>G</td>
<td>Complex fields, wave equation, k-vectors, interference, light propagation, interferometry</td>
</tr>
<tr>
<td>2</td>
<td>12.04.</td>
<td>Diffraction</td>
<td>B</td>
<td>Slit, grating, diffraction integral, diffraction in optical systems, point spread function, aberrations</td>
</tr>
<tr>
<td>3</td>
<td>19.04.</td>
<td>Fourier optics</td>
<td>B</td>
<td>Plane wave expansion, resolution, image formation, transfer function, phase imaging</td>
</tr>
<tr>
<td>4</td>
<td>26.04.</td>
<td>Quality criteria and resolution</td>
<td>B</td>
<td>Rayleigh and Marechal criteria, Strehl ratio, coherence effects, two-point resolution, criteria, contrast, axial resolution, CTF</td>
</tr>
<tr>
<td>5</td>
<td>03.05.</td>
<td>Polarization</td>
<td>G</td>
<td>Introduction, Jones formalism, Fresnel formulas, birefringence, components</td>
</tr>
<tr>
<td>6</td>
<td>10.05.</td>
<td>Photon optics</td>
<td>D</td>
<td>Energy, momentum, time-energy uncertainty, photon statistics, fluorescence, Jablonski diagram, lifetime, quantum yield, FRET</td>
</tr>
<tr>
<td>7</td>
<td>17.05.</td>
<td>Coherence</td>
<td>G</td>
<td>Temporal and spatial coherence, Young setup, propagation of coherence, speckle, OCT-principle</td>
</tr>
<tr>
<td>8</td>
<td>24.05.</td>
<td>Laser</td>
<td>B</td>
<td>Atomic transitions, principle, resonators, modes, laser types, Q-switch, pulses, power</td>
</tr>
<tr>
<td>9</td>
<td>31.05.</td>
<td>Gaussian beams</td>
<td>D</td>
<td>Basic description, propagation through optical systems, aberrations</td>
</tr>
<tr>
<td>10</td>
<td>07.06.</td>
<td>Generalized beams</td>
<td>D</td>
<td>Laguerre-Gaussian beams, phase singularities, Bessel beams, Airy beams, applications in superresolution microscopy</td>
</tr>
<tr>
<td>11</td>
<td>14.06.</td>
<td>PSF engineering</td>
<td>G</td>
<td>Apodization, superresolution, extended depth of focus, particle trapping, confocal PSF</td>
</tr>
<tr>
<td>12</td>
<td>21.06.</td>
<td>Nonlinear optics</td>
<td>D</td>
<td>Basics of nonlinear optics, optical susceptibility, 2nd and 3rd order effects, CARS microscopy, 2 photon imaging</td>
</tr>
<tr>
<td>13</td>
<td>28.06.</td>
<td>Scattering</td>
<td>D</td>
<td>Introduction, surface scattering in systems, volume scattering models, calculation schemes, tissue models, Mie Scattering</td>
</tr>
<tr>
<td>14</td>
<td>05.07.</td>
<td>Miscellaneous</td>
<td>G</td>
<td>Coatings, diffractive optics, fibers</td>
</tr>
</tbody>
</table>

D = Dienerowitz          B = Böhme          G = Gross
Scattering

- Introduction
- Surface scattering in systems
- Volume scattering models
- Calculation schemes
- Tissue models
Definition of Scattering

• Physical reasons for scattering:
  - Interaction of light with matter, excitation of atomic vibration level dipols
  - Resonant scattering possible, in case of re-emission $\lambda$-shift possible
  - Direction of light is changed in complicated way, polarization-dependent

• Phenomenological description (macroscopic averaged statistics)
  1. Surface scattering:
     1.1 Diffraction at regular structures and boundaries:
         gratings, edges (deterministic: scattering ?)
     1.2 Extended area with statistical distributed micro structures
     1.3 Single micros structure: contamination, imperfections
  2. Volume scattering:
     2.1 Inhomogeneity of refractive index, striae, atmospheric turbulence
     2.2 Ensemble of single scattering centers (inclusions, bubble)

Therefore more general definition:
  - Interaction of light with small scale structures
  - Small scale structures usually statistically distributed (exception: edge, grating)
  - No absorption, wavelength preserved
  - Propagation of light can not be described by simple means (refraction/reflection)
1. Surface scattering
   1.1 Edge diffraction
   1.2 Scattering at topological small structures of a surface
       Continuous transition in macroscopic dimension: ripple due to manufacturing,
       micro roughness, diffraction due to phase differences
   1.3 Scattering at defects (contamination, micro defects), phase and amplitude

2. Scattering at single particles:
   2.1 Rayleigh scattering, d << \lambda
   2.2 Rayleigh-Debye scattering, d < \lambda
   2.3 Mie scattering, spherical particles d > \lambda

3. Volume scattering
   3.1 Scattering at inhomogeneities of the refractive index,
       e.g. atmospheric turbulence, striae
   3.2 Scattering at crystal boundaries (e.g. ceramics)
   3.3 Scattering at statistical distributed dense particles
       e.g. biological tissue
Aspects of Scattering

- Geometry regular - statistical distributed
- Single - multi scattering
- Density of scatterers low - high, independence, saturation, change of illumination
- Near - far field
- Scaling, size of scatterers vs. wavelength, micro - macro
- Coherence, scattering vs. re-emission
- Polarization dependence
- Discret scatterers vs. continuous n-variations
- Absorption
- Diffraction vs. geometrical approach
- Steady state vs time dependence
- Wavelength dispersion of material parameters
- Finite volume size - boundary conditions
Approximations in Scattering Models

- Geometry simplified
- Boundaries simplified, mostly at infinity
- Isotropic scattering characteristic
- Perfect statistics of distributed particles
- Multiple scattering neglected
- Discretization of volume
- Angle dependence of phase function simplified
- Scattering centers independent
- Scatterers point like objects
- Spatially varying material parameters ignored
- Field assumed to be scalar
- Decoherence effects neglected
- Absorption neglected
- Interaction of scatterers neglected
- $\lambda$-dispersion of material data neglected
Different surface geometries:
Every micro-structure generates a specific straylight distribution

Ref.: J. Stover, p.10
Phenomenology of Surface Scattering

- Scattering at rough surfaces:
  statistical distribution of light scattering in the angle domain

- Angle indicatrix of scattering:
  - peak around the specular angle
  - decay of larger angle distributions depends on surface treatment
- Autocorrelation function of a rough surface

\[ C(\Delta x) = \langle h(x) \cdot h(x + \Delta x) \rangle = \frac{1}{L} \cdot \int h(x) \cdot h(x + \Delta x) \, dx \]

- Correlation length \( \tau_c \): Decay of the correlation function, statistical length scale

- Value at difference zero

\[ C(0) = \sigma_{\text{rms}}^2 \]

- Special case of a Gaussian distribution

\[ C(x) = \sigma_{\text{rms}}^2 \cdot e^{-\frac{1}{2} \left( \frac{x}{\tau_c} \right)^2} \]
Surface Characterization

\[ h(x) \]

\[ \Delta x \]

\[ C(\Delta x) \]

\[ A(k) \]

\[ PSD(k) \]

\[ \langle h_1 h_2 \rangle \]

\[ C(\Delta x) = \frac{1}{L} \cdot \int h(x) \cdot h(x + \Delta x) \, dx \]

\[ A(k) = \int_0^L h(x, y) \cdot e^{-ikx} \, dx \]

\[ PSD(k) = \frac{1}{L} \cdot \left| \int h(x) \cdot e^{ikx} \, dx \right|^2 \]
PSD of a Surface

- Fourier transform of a surface spectral amplitude density
  \[ A(k) = \int_0^L h(x, y) \cdot e^{-ikx} \, dx \]

- PSD power spectral density relative power of frequency components
  \[ F_{PSD}(v_x, v_y) = \frac{1}{A} \left| \frac{1}{2\pi} \iint h(x, y) \cdot e^{2\pi i(x \cdot v_x + y \cdot v_y)} \, dx \, dy \right|^2 \]

- Areae under PSD-curve

- Meaningful range of frequencies

- Polished surfaces are similar and have fractal structure, PSD has slope 1.5 ... 2.5

- Relation to auto-correlation function of the surface
  \[ F_{PSD}(v) = \hat{F}[C(x)] = \frac{1}{\pi} \cdot \int_0^\infty C(x) \cdot \cos(xv) \, dx \]
PSD Ranges

- Typical impact of spatial frequency ranges on PSF
- Low frequencies: loss of resolution classical Zernike range
- High frequencies: Loss of contrast statistical
- Large angle scattering
- Mif spatial frequencies: complicated, often structured fals light distributions
BSDF of a Surface

- Description of scattering characteristic of a surface: BSDF (bidirectional scattering distribution function)

- Straylight power into the solid angle $d\Omega$ from the area element $dA$ relative to the incident power $P_i$

  $$F_{BSDF} = \frac{dL_s}{dP_i} = \frac{dP_s}{\cos \theta \cdot dP_i \cdot d\Omega}$$

- The BSDF works as the angle response function

  $$P(\theta, \varphi) = \int F_{BSDF} (\theta_i, \varphi_i, \theta, \varphi) \cdot P(\theta_i, \varphi) \cdot \cos \theta_i d\Omega_i$$

- Special cases: formulation as convolution integral
BSDF of a Surface

- 3D description of a surface

- Large angles:
  consideration of the cosines
  \[
  \alpha = \sin \theta_{s} \cdot \sin \varphi_{s} \\
  \beta = \sin \theta_{s} \cdot \cos \varphi_{s} - \sin \theta_{\text{spec}}
  \]

- Example distribution
Model Functions of Surfaces

- Exponential correlation decay
  PSD is Lorentzian function
  \[ C(x) = \sigma_{\text{rms}}^2 \cdot e^{-\frac{x}{\tau_c}} \]
  \[ F_{\text{PSD}}(s) = \frac{1}{\pi} \cdot \frac{\sigma_{\text{rms}}^2 \cdot \tau_c}{1 + (s \cdot \tau_c)^2} \]

- Gaussian correlation
  \[ C(x) = \sigma_{\text{rms}}^2 \cdot e^{-\frac{1}{2} \left( \frac{x}{\tau_c} \right)^2} \]
  \[ F_{\text{PSD}}(s) = \frac{\tau_c \cdot \sigma_{\text{rms}}^2}{\sqrt{4\pi}} \cdot e^{-\left( \frac{s\tau_c}{2} \right)^2} \]

- Fractal surface with Hausdorff parameter D
  \[ F_{\text{PSD}}(s) = \frac{\Gamma\left( \frac{n+1}{2} \right)}{2\Gamma\left( \frac{1}{2} \right) \cdot \Gamma\left( \frac{n}{2} \right)} \cdot \frac{K_n}{s^{n+1}} \]

- K correlation model parameter B, s
  \[ F_{\text{PSD}}(s) = \frac{A}{\left[ 1 + (s \cdot B)^2 \right]^{C/2}} \]
BSDF Model of Harvey-Shack

- Empirical model function of BSDF

- Notations:
  - Sine of scattering angle
  - Slope parameter $m$
  - Glance angle
  - Reference and pivot angle: $\beta_{ref}$
  - BSDF value at reference: $a$

- Simple isotropic scalar model

Mathematical expression:

$$F_{BSDF}(\beta) = a \cdot \left( \frac{|\beta - \beta_{spec}|}{\beta_{ref}} \right)^m$$

$$\beta = \sin \theta_s$$

$$\beta_{spec} = \sin \theta_{spec}$$
Roughness of Optical Surfaces

Roughness of optical surfaces, Dependence of treatment technology

- Grinding
- Polishing
- Computer controlled polishing
- Diamond turning
- Plasma etching
- Ductile manufacturing
- Ion beam finishing
- Magneto-rheological treatment

Rms roughness [nm]

Material removal [qmm/s]

10^-6 10^-4 10^-2 10^0 10^+2
- Maximum BRDF at angle of reflection
- Larger BRDF for skew incidence

**BRDF of Black Lacquer**

Ref.: A. Bodemann
Particles on Optical Surfaces

- Model of Mie scattering at particle contamination

Ref: B. Görtz / Linos
- Cleaned surface

![Graph showing size of particles in μm and number of particles for cleaned surface and dark field microscopic image.](image)

Ref: B. Götz / Linos
Sources of Stray Light

1. Direct imaging ray path

2. Direct reflected ray (zero order false light)
   - Reflected from optical surface
   - Reflected by mechanical parts

3. Scattered light
   - Scattered on surface micro structure
     - Optical surface
     - Mechanical surface
   - Scattered on particles
   - Volume scattering

4. Diffraction
   - Apertures and baffles

Ref: B. Goerz
Straylight and Ghost Images

- Different reasons
- Various distributions
Scattering of Light

- Scattering of light in diffuse media like frog

Ref: W. Osten
Photometrical calculation of the transfer of energy density

\[ dP = L \cdot \cos \theta \cdot dA \cdot d\Omega \]

Integration of the solid angle by raytrace

in the system model

\[ P = E_s \cdot g \cdot T \cdot F_{BRDF} \cdot \Omega \]

g : geometry factor
surface response : BSDF
T : transmission
Decomposition of the system into different ray paths

Properties:
- extrem large computational effort
- important sampling guarantees quantitative results for large dynamic ranges
- mechanical data necessary and important
  - often complicated geometry and not compatible with optical modelling
- surface behavior (BRDF) necessary with large accuracy
Model Options

- 3 major approaches
- Analytical vs. numerical solutions

[Diagram with options]

- Rigorous Maxwell solutions
  - Analytical
  - Numerical
  - T-matrix
  - PSTD
  - FDTD
  - GLMT

- Radiation transport equation
  - Analytical
  - Numerical
  - Monte Carlo
  - FD-grid-based
  - SH expansion
  - Finite elements
  - Features: polarization (PMC), electric field (EMC), particles fixed, time resolved

- Diffusion equation
  - Analytical
  - Numerical
  - FD-grid based
  - FE
  - Layered
  - Bricks
  - Cylinder
Model Validity Ranges

- Typical tissue features
- Model validity ranges

- cell membranes
- macromolecule aggregates, stiations in collagen fibrils
- mitochondria
- cells

- Typical scale size $\lambda = d$

- single: Rayleigh
- single: Mie
- volume: Maxwell
- volume: RTE
The Volume Dilemma

- Problem:
  - Exact solutions of scattering: Maxwell equations
  - Volume sampling requires large memory
  - Realistic simulations: small volumes (2 μm³)
  - Real sample volumes cannot be calculated directly

- Approach:
  - Calculation of response function of microscopic scattering particles with Maxwell equations
  - Empirical approximation of scattering phase function p(θ,φ)
  - Solution of transport theory with approximated scattering function

Ref: A. Kienle
Model Validity Ranges

- Simple view: diagram volume vs. density

\[ n(x,y,z) g, m_s, m_a \]
Approximations and assumptions:
1. low density, no interaction of scatter events
2. no absorption
3. statistical distribution of many isolated small scatter centers

Approach: description with BSDF function
\[ F_{BSDF} = \frac{1}{4\pi \cdot \cos \theta_i \cos \theta_s} \cdot C_s \cdot p(\theta) \cdot \rho_s \]
Analytical solutions:
- Spherical particles
  1. generalized Lorentz-Mie theory, near and far field
  2. multi sphere configurations
  3. layered structures
- Spheroids
- Cylinders
  1. single cylinders, with oblique incidence, near and far field
  2. stacked cylinders
  3. multi cylinder configurations, perpendicular incidence

Numerical solutions in time domain:
- Arbitrary geometries
- Finite difference time domain method (FDTD), only small volumes (2μm³), Δx = λ/20
- Pseudospectral method (PSTD), Δx = λ/4

Stationary solutions:
- Discrete dipole approximation for arbitrary geometries
- T-matrix method

Ref: A. Kienle
Rigorous Scattering at Sphere

- Maxwell solution in the nearfield

\[ \lambda = 600 \text{ nm} \]

\[ r = 1 \mu \text{m} \]

\[ n_{\text{out}}=1.33 / n_{\text{in}}=1.59 \]

\[ n_{\text{out}}=1.59 / n_{\text{in}}=1.33 \]

\[ n_{\text{out}}=1.33 / n_{\text{in}}=1.59 + 5 \text{i} \]

\[ r = 2 \mu \text{m} \]

Ref: J. Schäfer
Multiple Scattering

- Change of scattering cross section due to shadowing effects
- Phase function depends on neighboring particle

Ref: J. Schäfer
Aggregation to Extended Samples

- Larger aggregates of simple single scatter particles
- Can be treated rigorous for moderate numbers/volumes

Ref: S. Tseng
Rayleigh-Scattering

- Scattering at particles much smaller than the wavelength
  \[ d << \lambda \]

- Scattering efficiency decreases with growing wavelength
  \[ Q_s = \frac{128\pi^4 a^4}{3\lambda^4} \left| \frac{n_s^2 - n^2}{n_s^2 + 2n^2} \right| \]

- Angle characteristic depends on wavelength

- Phase function
  \[ p(\theta) = \frac{3}{16\pi} \cdot (1 + \cos^2 \theta) \]

- Example: blue color of the sky
Mie Scattering

- Result of Maxwell equations for spherical dielectric particles, valid for all scales
- Interesting for larger sizes $d \gg \lambda$

- Macroscopic interaction:
  - Interference of partial waves,
  - Complicated angle distribution
- Usually dominating: forward scattering
- Parameter: $n$, $n'$, $d$, $\lambda$, $(\alpha)$
- Example: small water droplets ($d=10 \, \mu m$)
- Limitation: interaction of neighboring particles

- Approximation of parameter $5\lambda < 2\pi \cdot n \cdot a < 50\lambda \quad n < n_s < 1.1n$

Cross section

$$\sigma = 3.28 \cdot \left( \frac{2\pi nd}{\lambda} \right)^{0.37} \left( \frac{n'}{n} - 1 \right)^{2.09}$$

Ref.: M. Möller
- Shape of the phase function due to Mie scattering:
  - growing complexity with radius of sphere
  - interference condition complicated
  with several points of stationary phase

Ref: J. Schäfer
Radiative transport equation: photon density model (gold standard for large volumes),
Purely energetic approach, no diffraction
Integration of PDE by raytracing or expansion in spherical harmonics
Options:
1. time, space and frequency domain
2. fluorescence
3. polarization
4. flexible incorporation of boundaries and surfaces, voxel based
Analytical solutions for special geometries:
1. several source geometries
2. space extended to infinity
3. Already some minor differences to Monte-Carlo approach due to assumptions
Not included features:
1. diffraction, no description of speckles, interference
2. no coherent back scattering
3. no dependencies of neighboring scatterers
Radiance Transport Equation

- Description of the light propagation with radiance transport equation for photon density balance:
  1. incoming photons
  2. outgoing photons
  3. absorption, extinction
  4. emission, source

\[
\frac{1}{c} \frac{\partial L(\vec{r}, t, \vec{s})}{\partial t} + \vec{s} \cdot \nabla L(\vec{r}, t, \vec{s}) + (\mu_a + \mu_s) \cdot L(\vec{r}, t, \vec{s}) = \mu_s \cdot \int L(\vec{r}, t, \vec{s}) \cdot p(\vec{s}, \vec{s}') d\omega + Q(\vec{r}, t, \vec{s})
\]

- Numerical solution approach:
  Expansion into spherical harmonics
Simulation of Diffuse Light Propagation

- Excitation of tissue by focussed laser illumination
- Propagation of fluorescence light in tissue by diffusion
Diffuse Light Propagation: General Model

- General radiation transfer equation (RTE, Boltzmann)

\[
\frac{1}{c} \frac{\partial L(\vec{r}, t, \vec{s})}{\partial t} + \vec{s} \cdot \nabla L(\vec{r}, t, \vec{s}) + (\mu_a + \mu_s) \cdot L(\vec{r}, t, \vec{s}) = \mu_s \cdot \int L(\vec{r}, t, \vec{s}') \cdot p(\vec{s}, \vec{s}') d\omega + Q(\vec{r}, t, \vec{s})
\]

- Local balance of photon numbers:

1. Incoming photons, divergence
2. Outgoing photons, scattering
3. Absorbed photons, extinction
4. Emitted photons, source

- Problem: direction dependence of scattering

- Approximative phase function \( p \) of the scattering process: Henyey-Greenstein

\[
p_{HG}(\vec{s}, \vec{s}') = \frac{1}{4\pi} \cdot \frac{1 - g^2}{\left[1 + g^2 - 2g\vec{s} \cdot \vec{s}'\right]^{3/2}}
\]
Severe approximations assumed:
- perfect isotropic scattering
- no wave optical effects
- description of light as photon density evolution

Time dependent or steady state

Analytical solutions for special geometries
1. Infinity
2. semi-infinity
3. bricks
4. Layered structures
5. cylinder
6. Spheres
Diffusion Equation

- Approximation:
  - scattering dominates gradient effects
  - scattering interaction is isotropic
- Radiance:
  \[ L(\vec{r},t,\vec{s}) = \frac{1}{4\pi} \left[ \Phi(\vec{r},t) + 3D \cdot \vec{s} \cdot \nabla \Phi(\vec{r},t) \right] \]
- Diffusion equation
  \[ \frac{1}{c} \frac{\partial \Phi(\vec{r},t)}{\partial t} = \nabla \left[ D \cdot \nabla \Phi(\vec{r},t) \right] - \mu_a \cdot \Phi(\vec{r},t) + S(\vec{r},t) \]
- Isotropic diffusion constant
  \[ D = \frac{1}{3 \left[ \mu_a + \mu_s \cdot (1 - g) \right]} \]
- Mean free path
  \[ L = \frac{1}{\mu_{eff}} = \frac{D}{\mu_a} = \frac{1}{\sqrt{3} \cdot \mu_a \cdot \left[ \mu_a + \mu_s \cdot (1 - g) \right]} \]
- Stationary and isotropic
  \[ \nabla^2 \Phi(\vec{r},t) - \frac{\mu_a}{D} \Phi(\vec{r},t) = -\frac{S(\vec{r},t)}{D} \]
Modelling Fluorescence

- Models:
  1. Maxwell theory simulation
  2. Monte Carlo calculation
  3. Diffusion theory
- Different approaches
- Under investigation

Ref: A. Kienle
Comparison of Methods

- Analytical solutions with Maxwell solver
  Multiple cylinder geometry

- RTE with Maxwell analytic
  Multiple spheres

Ref: A. Kienle
Comparison of Methods

- RTE with Maxwell analytic with polarization
  Multiple spheres

Ref: A. Kienle
Comparison of Methods

- RTE with Maxwell numeric
  Multiple spheres
Comparison of Methods

- Diffusion versus Monte Carlo method
  Spatial domain

- Diffusion versus Monte Carlo method
  Time domain

Ref: A. Kienle
Henyey-Greenstein Scattering Model

- Henyey-Greenstein model for human tissue
  Phase function

- Asymmetry parameter $g$:
  Relates forward / backward scattering
  $g = 0$ : isotropic
  $g = 1$ : only forward
  $g = -1$: only backward

- Rms value of angle spreading

$$\theta_{rms} = \sqrt{2(1 - g)}$$

- Typical for human tissue:
  $g = 0.7 \ldots 0.9$

$$p_{HG}(\theta, g) = \frac{1}{4\pi} \cdot \frac{1 - g^2}{(1 + g^2 - 2g \cos \theta)^{3/2}}$$

$$g = \langle \cos \theta \rangle = \int p(\theta) \cos \theta d(\cos \theta) = 2\pi \int_{0}^{\pi} p(\theta) \cos \theta \sin \theta d\theta$$
Henyey-Greenstein Model

- Simple model for phase function in the case of scattering in biological tissue:

- Values for human tissue

<table>
<thead>
<tr>
<th>Tissue</th>
<th>λ</th>
<th>μs [mm⁻¹]</th>
<th>μτ [mm⁻¹]</th>
<th>g</th>
</tr>
</thead>
<tbody>
<tr>
<td>Human dermis</td>
<td>635</td>
<td>0.18</td>
<td>24.4</td>
<td>0.775</td>
</tr>
<tr>
<td>Liver</td>
<td>635</td>
<td>0.23</td>
<td>31.3</td>
<td>0.68</td>
</tr>
<tr>
<td>Lung</td>
<td>635</td>
<td>0.081</td>
<td>32.4</td>
<td>0.75</td>
</tr>
<tr>
<td>Fundus, healthy</td>
<td>514</td>
<td>0.423</td>
<td>5.312</td>
<td>0.79</td>
</tr>
<tr>
<td>Fundus, diseased</td>
<td>514</td>
<td>0.689</td>
<td>13.43</td>
<td></td>
</tr>
<tr>
<td>Retinal pigment</td>
<td>800</td>
<td>14</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ciliar body</td>
<td>694</td>
<td>2.52</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Skin epidermis</td>
<td>800</td>
<td>4.0</td>
<td>42.0</td>
<td>0.852</td>
</tr>
</tbody>
</table>
Henyey-Greenstein Model

- Extended representation 1:
  superposition of two terms

\[ p_{DHG}(\theta, a, g_1, g_2) = a \cdot \frac{1}{4\pi} \cdot \frac{1 - g_1^2}{(1 + g_1^2 - 2g_1 \cos \theta)^{3/2}} + (1 - a) \cdot \frac{1}{4\pi} \cdot \frac{1 - g_2^2}{(1 + g_2^2 - 2g_2 \cos \theta)^{3/2}} \]

\[ = a \cdot p_{HG}(\theta, g_1) + (1 - a) \cdot p_{HG}(\theta, g_2) \]

- Extended representation 2:
  Two parameters
  More realistic

\[ p_{CS}(\theta, g) = \frac{3}{2} \cdot \frac{1 - g^2}{2 + g^2} \cdot \frac{1 + \cos^2 \theta}{(1 + g^2 - 2g \cos \theta)^{3/2}} \]
LSM-Simulation: Cell Model

- cell model after Starosta & Dunn
  (3D Computation of Focused Beam Propagation through Multiple Biological Cells, OE 17, 12455, 2009)
  - cell: ellipsoid with $n = 1.36$
  - nucleus: sphere with $n = 1.4$
  - 100 mitochondria: ellipsoids with $n = 1.4$
  - 4 fluorescence beads (zero Stokes-shifts)

Ref: S. Siegler
Bio-medical real sample examples

cancer cell

dentin

cell complex

blood vessel

muscle fibers

wood
Approaches of Biological Straylight Simulation

- Large scale / cells: macroscopic range
  Diffusion equation, isotropic
  Some analytical solutions, numerical with spherical harmonics expansion
  Parameters: effective $\mu'_s$, $\mu_a$, $n$

- Medium scale / cell fine structure: mesoscopic range
  transport theory, radiation propagation only numerical solutions, scalar anisotropic
  Preferred: Monte-Carlo raytrace
  Some analytical solutions
  Parameters: $\mu_s, \mu_a, n, p(\phi, \theta)$

- Fine scale: microscopic range
  Only small volumes, with polarization
  Maxwell equation solver, FDTD, PSTD, Some analytical solutions
  Parameter: complex index $n(r)$

- Correct scaling: feature size vs. wavelength, depends on application
Scattering in Tissue

- Scattering in biological tissue: Relevant for therapeutic spectral window $\lambda = 650 \text{ nm}...1.3 \mu\text{m}$

- Definition / typical numbers:
  - coefficient of absorption: $\mu_a \approx 0.01 ... 1 \text{ mm}^{-1}$
  - coefficient of scattering: $\mu_s \approx 10 ... 100 \text{ mm}^{-1}$
    dominating: scattering with forward direction
  - total attenuation of ballistic photons
    $\mu_t = \mu_a + \mu_s$
  - Albedo (scattering contribution on attenuation)
    $a = \mu_s / \mu_t \approx 0.99 ... 0.999$
  - mean free path of photons
    $s = 1 / \mu_s \approx 10 ... 100 \mu\text{m}$

Ref.: M. Möller
Modelling Light Scattering in Tissue: Backscattering

- Processes

- Simulation

Ref: A. Kienle