Metrology and Sensing

Lecture 11: Measurement of basic system properties
2019-01-08
Herbert Gross
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Content

- Basic system properties
- Knife edge method
- Slit scan method
- MTF measurement
- Deviation in measurement eyepiece
- Achievable accuracy depends on focal length
- Accuracy of a few arc seconds possible
Alignment Telescope

- Measurement of small lateral displacements
  \[ x' = m \cdot x \]
- Zero measurement with target cross and crossing hairs

![Diagram of Alignment Telescope]

- Test object
- Objective
- Target
- Eyepiece

- Reference
- Deviation
- Y
- Y'
Autocollimation Principle

- Spherical test surface:
  - incoming and outgoing wavefront spherical
  - concentric waves around center of curvature: autocollimation

- Aspherical test surface
- Collimated incident light
- Calibrated collimator with focal length $f_c$ and test chart with size $y$
- Selection of sharp image plane
- Analysis of image size

\[ f' = -f'_c \cdot \frac{y'}{y} \]
- Setup with distance object-image $L > 4f$
- Known location of the principal plane $P$ of the system
  - distance $d_P$ between principal planes
- Selection of two system locations with sharp image
- Relative axial shift $D$ between the two setups

\[
f = \frac{L - d_H}{4} - \frac{D^2}{4(L - d_H)}
\]
Measurement of Focal Length with Focometer

- Telecentric movable measurement microscope with offset $y$: Abbe focometer
- Focusing of two different test charts with sizes $y_1$ and $y_2$
- Determination of the focal length by

\[
\tan u = \frac{y}{f} = \frac{y_2 - y_1}{e}
\]
- Setup with fiber and plane mirror for autocollimation
- Change of distance between test lens and fiber
- Analysis of the recoupled power into the fiber (confocal) gives the focal point
Measurement of Focal Length with Focimeter

- Afocal setup with sharp image plane
- Measurement of long focal lengths
- Insertion of test system in collimated light segment and refocussing
- Applying the lens makers formula

\[
\frac{1}{f} = \frac{1}{f_2} - \frac{x}{f_2^2}
\]
- Setup with collimator and two Ronchi rulings
- System under test is inserted
- Grating period $d$ and azimuthal angle $\theta$ between the gratings
- Moire pattern is rotated by angle $\alpha$, if test lens acts as focusing element
- Radius of curvature $R$ or focal length

$$R = \frac{d}{\theta \cdot \tan \alpha}$$
• Setup with Ronchi grating in collimated light gives a series of Talbot images
• The Talbot planes are imaged by the system under test
• Analysis of the image plane by lens formula gives the desired focal length
• By use of several planes, the position of the principal plane can be eliminated
• A second Ronchi grating can be used to find the accurate image planes
Criteria for best focus:

1. Paraxial centre of curvature for the paraxial spherical wave of an on axis object point.
2. Maximum of the Strehl ratio
3. Smallest rms-value of the wave aberration
4. Highest contrast of the modulation of an object feature of given spatial frequency
5. Highest value of the slope of an edge
6. Highest value of the entropy of the detected digital image

Requirements for focus detection procedure

1. Steep curve dependency to get high accuracy
2. Robust definition to deliver a large dynamic range
3. Suppression of side lobe effects to guarantee an unambiguous solution
4. High frequency pre-filtering to be noise insensitive
Determination of Best Focus

- Blur of defocussed plane

- Minimum of image entropy ($w_j$ is intensity in pixel No $j$)

$$E = - \sum_j w_j \cdot \log_2 w_j$$

- Maximum of image contrast
Determination of Best Focus

- Phase analysis by Zernike coefficient $c_4$

$$c_4 = \frac{-1}{4n\lambda} \cdot \Delta z \cdot NA^2$$

Measurement with two Ronchi gratings
Determination of Best Focus

- Measurement by image analysis:

  1. Maximum gradient of edges

     \[ g = |\nabla I(x, y)| = \sqrt{\left(\frac{\partial I}{\partial x}\right)^2 + \left(\frac{\partial I}{\partial y}\right)^2} \]

  2. Power of gradients

     \[ G = \iint |\nabla I(x, y)|^2 \, dx \, dy \]

  3. Laplacian

     \[ L = \iint |\nabla^2 I(x, y)|^2 \, dx \, dy \]
Measurement of Principal Planes

- Measurement for systems in air via the nodal planes
- Imaging of a test pattern with a collimator onto a detector
- Invariant lateral image location for rotated system around the nodal point
- Critical: vignetting effects for large angles
Measurement of Principal Planes

- Setup of the test lens with different object locations: axial shift \( D \)
- Analysis of the lens imaging formula

\[
\frac{1}{a_j + \Delta} + \frac{1}{a'_j - \Delta} = \frac{1}{f}
\]

- Minimizing the error of several measurements \( j \)

\[
\delta = f \cdot (a_j + a'_j) + \Delta \cdot (a_j - a'_j) - a_j a'_j + \Delta^2
\]
Measurement of Pupil Size

- Setup with collimating auxiliary lens

\[ \tan u = \frac{D}{2f} \]

- Determination with measuring microscope (dynameter)
- Setup with Ronchi grating
- Measurement of the lateral shift of higher diffraction orders at distance z

- High-NA in microscopy: $NA > 1$
  - Measurement of total internal reflection of fluorescence light

**Diagram:**
- Test system
- Ronchi grating
- Image plane

**Graphs:**
- Fluorescence intensity
- Pupil illumination

**Formulas:**
- $D_{Exp}$
- $\theta$
- $f$
- $z$
- $\Delta x$
- $\alpha$
- $D$
- $r_{max}$
- $r_{TIR}$
- $f_{TIR}$
Measurement of Telecentricity

- Measurement of object sided telecentricity errors by lateral shift of image location during defocussing

- High accuracy measurement by interferometry and measurement of Zernike coefficients $c_{2/3}$. 
- Measurement of reflexes at lens vertex points
- Analysis of confocal signal in autocollimation
- Avoiding spherical aberration induced errors by ring illumination
• Measurement of tilt errors (plane or spherical surface) in autocollimation
• Projection of the cross
• Observation of lateral shift in Fourier plane \[ x = f \cdot 2\varphi \]
- Projection of test marker
- Autocollimation of sharp image, focal point coincides with center of curvature of surface with radius $r$
- Rotation of test system: tilt of surface induces a lateral shift of the image
- Problems with inner surfaces

$$v = \beta \cdot v_M = 2 \cdot \beta \cdot r \cdot \kappa$$
- Thin collimated beam through lens
- Focussing of the beam onto detector
- Measurement of wedge angle by lateral shift $v$
- Tilt angle of lens not detectable
- Not feasible for very short focal lengths

$$\phi = (n - 1) \cdot \theta = (n - 1) \cdot (\alpha_1 - \alpha_2) = \frac{v}{f}$$
Wedge Angle of a Thin Lens

Thin lens with wedge error

\[ \vartheta = -\frac{v}{(n-1) \cdot f} \]

\[ \vartheta = \kappa_1 - \kappa_2 = \left( \frac{1}{r_1} - \frac{1}{r_2} \right) \cdot v \]
Tolerances: Centering Methods

- Mechanical

- Ray in transmission

- Rays in reflection

Ref.: M. Peschka
• Measurement of the centering of lens by inspection of the interferogram of front and rear surface
Two equivalent forms of description:
1. Three single surface tilt angles
2. Decenter v, overall tilt angle K, tilt of cemented inner surface $\kappa_c$, better correspondance to manufacturing steps
Centering in Bonding Process

- Centering carrier lens

- Adjusting second lens

Ref.: M. Peschka
Tolerances and Mechanical Interface

- Coupling optics – mechanics
  Interface glass – metal

- Mechanical design of mountings and housing

- Typical options:
  1. Filling cylinder with fixating screw
  2. Cementing, later centering
  3. Lace / bordering

- Critical:
  - Centering tolerances
  - reference surfaces
  - analysis of complete geometry (kinematic)
Mechanical Mounting Geometry

- Filling of lenses into mounting cylinder with spacers
- Accumulation of centering errors by transportation of reference
- Definition of lens positions by:
  1. mechanical play inside mounting
  2. fixating ring screw
  3. planarity of spacers
Testing of Prism Angles

- Measurement of $90^\circ$ angle in air
- Interferogram

\[ \alpha = \frac{m \cdot \lambda}{2D} \]
**Xray CT-Tomography**

- Determination of
  1. number of lenses
  2. thicknesses
  3. radii of curvature
  4. refractive indices (!?)

- Typical uncertainties of linear dimensions:
  0.050 mm ... 0.010 mm

- Errors are scaling with size

- Contrast for plastics poor

- Different absorption for glass types

- Cross section vs CT-projection image

Ref: B. Satzer
Xray CT-Tomography

- Various cross sections

Ref: B. Satzer
Reasons for reduced system transmission:
1. Absorption in the bulk material of the components
2. Scattering in the bulk materials by inclusions or finite scattering parameters
3. Absorption in the coatings of the surfaces
4. Partial reflection or transmission at the coatings at transmissive or reflective surfaces
5. Blocking of light via mechanical or diaphragm parts of the system due to vignetting
6. Scattering of light by local surface imperfections or non-perfect polished surfaces
7. Deflection of light by diffraction of the light at edges
8. Deflection of light in unwanted higher orders of diffractive elements

Usually strong dependency on:
1. field position
2. wavelength of light
3. used pupil location
4. polarization

Critical:
1. absolute values for test lens
2. influence of auxiliary components
3. change of vignetting and incidence angles
Measurement of Transmission

- Measurement of transmission:
  a) Calibration setup
  b) Measurement setup

- Reasons for measurement errors:
  1. Absorption in the component materials
  2. Absorption in the coatings
  3. Finite reflectivity of the coatings
  4. Vignetting of the aperture bundle for oblique chief rays
  5. Natural vignetting according for oblique chief rays and projection of tilted planes
  6. False light from surrounding light sources, which reach the image plane
  7. Scattering of light at components of the system mechanical design
  8. False light due to ghost images or narcissus in infrared systems

\[ T = \frac{P_{\text{in}}}{P_{\text{out}}} \]
Measurement of Ghost Images and Veiling Glare

- Measurement of unwanted light: 2 different approaches:
  1. object area black, surrounded by bright source
detection of irradiance in image region
  2. intensive isolated point light source in the object plane at different locations
detection of artificial distributions in the image area: glare spread function
- Testsample plate on prism
- Near grazing incidence, measurement of total internal reflection at interface
- Sharp shadow boundary of transmitted light beam
- Evaluation

\[
\sin \theta = n_2 \cdot \cos \varphi \\
n = n_2 \cdot \sin \varphi \\
n = \sqrt{n_2^2 - \sin^2 \theta}
\]
- Measurement of the refractive index of a liquid
- Thin film of test liquid between prisms, adjustment of total internal reflection
- Special setup with direct sight prisms, no color fringes

Abbe Refractometer

- Test liquid
- Measuring prism
- Amici prism without deviation
- Compensators
- Objective
- Eyepiece
- Telescope
- Image
- Standard not sufficient
- Appearance:
  like wood in transmission shadow image

### Definition of Striae

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<th>Schlierengrade</th>
<th>MIL-G-174B</th>
<th>DIN ISO 10110-4</th>
<th>SCHOTT (nur intern)</th>
<th>SCHOTT (Katalog)</th>
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<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>≥ 30 nm</td>
<td>1: ≤ 10%</td>
<td>Intensität mit Beachtung der Proben dicke (50 mm)</td>
<td></td>
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<tr>
<td></td>
<td>2: ≤ 5%</td>
<td></td>
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<tr>
<td></td>
<td>3: ≤ 2%</td>
<td></td>
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<td>4: ≤ 1%</td>
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*Note: Images of glass samples are also present.*
Measurement with Shadowgraphy

- Advantages:
  1. Fast, cheap
  2. Simple setup
  3. Small requirements on sample surface quality

- Disadvantage:
  1. No quantitative calibration until now
- Change of contrast for rotation in experiment
- Simulation of shadow image with data of interferograms

Rotation of Sample

**Shadow Image**

-3.3°  -1.5°  0°  1.2°  2.6°

**Interferogram**

**Shadow Image calculated out of interferogram data**
- Measurement of striae and index inhomogeneities at a plane plate
- Dark field method:
  - direct light blocked
  - deviated and scattered light reaches the camera
Ellipsometry for Polarization Measurement

- Measurement of material data by polarization
- Incident linear polarized light
- Reflected light elliptical polarized
- Compensation of ellipticity, quantitative determination of null-test parameter
- **Wavefront**
  - PV-, rms-value, fractional pupil area
  - Rayleigh-, Marechal criteria for diffraction limit

- **Point spread function**
  - Strehl ratio, width, second moment, area equivalent, correlation, power in the bucket
In principle, the complete information on the optical systems performance can be recovered from measurements of:
1. Point spread function
2. Line spread function
3. Edge spread function

The ESF and the LSF covers the spatial frequencies in the pupil only in one direction. The complete information requires the measurements of ESF / LSF in several angle orientations (azimuth)

Between the ESF and the LSF, a simple relationship allows a conversion

\[ I_{LSF}(x_i) = \frac{d I_{ESF}(x_i)}{dx_i} \]

There are three symmetry classes, which can be distinguished:
1. Circular symmetry
   The Abel transform allows a non-iterative calculation of the PSF from the LSF/ESF
2. Mirror symmetry with decoupling of x and y
   The calculation can be performed in two separated 1D-sections
3. General case without symmetry
   A complete tomographic reconstruction is necessary
Knife Edge Measurement

- Characteristic s-shaped curve for the energy transmission
- Definition of centroid by corresponding threshold values
Knife Edge Method

- Moving a knife edge perpendicular through the beam cross section
- Relationship between power transmission and intensity: Abel transform for circular symmetry
  \[
P(x) = 2\int_{\infty}^{\infty} \int_{\infty}^{\infty} \frac{I(r) r dr}{\sqrt{r^2 - \xi^2}} d\xi \]
- Example: geometrical spot with spherical aberration
- Method very similar to moving knife edge
- Integration of slit length must be inverted:
  - inverse Radon transform
  - corresponds to tomographic methods
Examples:
Gaussian profile

\[ I(x, y) = \frac{2P}{\pi \cdot w_x w_y} \cdot e^{-\frac{x^2}{w_x^2} - \frac{y^2}{w_y^2}} \]

Variation of the ratio between beam width \( W \) and slit width \( v \):

- \( w / v \) large: width of the slit can be neglected
- \( w / v \) small: strongly changed profile
- Measurement of an edge image
- Evaluating the derivative:
  Line spread function
- Fourier transform:
  optical transfer function

\[
I_{LSF}(x') = \frac{d I_{ESF}(x')}{d x'}
\]

\[
H_{OTF}(s) = \hat{F}[I_{LSF}(x')]
\]
- mathematical relationships

- Direct analysis of the frequency content

\[ H_{MTF}(\nu) = \left| \frac{A'(\nu)}{A(\nu)} \right| \]

- Problem: zero crossing points
  solution: filtering, windowed calculation
Setup:
Imaging of a grating

Possible realizations:
1. Density type grating, the sine wave is modelled by gray levels
2. Area type gratings, the sine wave is modelled by geometrical sine-shaped structures

Area coded sine grating:
MTF Measurement

- Definitions:
  1. Fourier transform of PSF
     \[ H_{OTF}(v_x, v_y) = \int \int I_{PSF}(x', y') \cdot e^{2\pi i (v_x x' + v_y y')} \, dx' \, dy' \]
  2. Autocorrelation of pupil function
     \[ H_{OTF}(v_x, v_y) = \frac{\int \int P(x_p + \frac{\lambda f' v_x}{2}, y_p + \frac{\lambda f' v_y}{2}) \cdot P^*(x_p - \frac{\lambda f' v_x}{2}, y_p - \frac{\lambda f' v_y}{2}) \, dx_p \, dy_p}{\int \int |P(x_p, y_p)|^2 \, dx_p \, dy_p} \]

- Overview: classification
  1. Imaging of special test structures and analyzing the corresponding image contrast behavior
     1.1 If the structures are sine grating structures, a single frequency response is determined
     1.2 If the structures have a large frequency content like points, lines, edges or bar patterns, a careful analysis of the higher frequency components and calculation the OTF from the measurement data must be performed
  2. Direct measuring of the autocorrelation function of the optical system pupil corresponding to the Duffieux-integral formulation of the transfer function
  3. Measurement of the point spread function and digital calculation of the transfer function by performing the Fourier transform.
MTF Measurement

- Conditions for measuring the incoherent transfer function:

1. An object is illuminated by incoherent light.
2. The object acquires, through its structures, all relevant spatial frequencies that have to be measured.
3. The object is imaged by the test system.
4. Spatial resolution is provided for the detection of the image intensity. As a rule this is achieved by an adjustable slit located in front of the detector. Alternatively, the slit can be fixed and scanning is accomplished by the imaged grating or object structure.
5. The contrast is derived from the intensity distribution and analysed as a function of the spatial frequency

- Possible test structures of the object:
  1. Point object
  2. Edge object
  3. Line object or slits
  4. Bar pattern
  5. Random transparencies
  6. Sine gratings with one or several periods
  7. Special test charts like the Siemens star
MTF-Measurement by Imaging Gratings

- Realization by a rotating radial grating
  Spatial frequency depends on azimuthal angle

\[ \text{slit} \]

\[ q \]

\[ \frac{1}{s} \]

\[ a \]

\[ g \]

\[ \theta \]

\[ \text{rotation for spatial scanning} \]

\[ \text{rotation for the angle of the slit / grating : spatial frequency} \]
MTF Measurement by Pupil Autocorrelation

- **Basis**: Duffieux integral

\[
H_{OTF}(v_x, v_y) = \frac{\int \int P(x_p + \frac{\lambda f'v_x}{2}, y_p + \frac{\lambda f'v_y}{2}) \cdot P^*(x_p - \frac{\lambda f'v_x}{2}, y_p - \frac{\lambda f'v_y}{2}) dx_p dy_p}{\int \int |P(x_p, y_p)|^2 dx_p dy_p}
\]

Diagram showing the setup with labels for:
- Laser source
- Focusing lens
- Stop
- Lens under test
- Beam splitter
- Roof mirror
- Variable phase
- Variable shear
- Lens diffuser
- Sensor
Typical shortcomings of MTF measurements:

1. Mechanical tolerances of the movable parts of the setup like line scan, rotatable edges and alignment errors

2. Application of precise correction factors for finite size slits

3. Truncation errors of the finite lengths structures of the object

4. Calibration of the spatial frequency variable, in particular for finite fields of view with projection changes of lengths and pattern widths

5. Poorly known residual aberrations of auxiliary optical components

6. Use of incorrect spectral and coherence constraints of the illumination

7. Shortcomings of sensor performance

8. Perturbing glare and stray light