Lecture 6: Optimization I
2014-06-05
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## Preliminary Schedule

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1. Principles of nonlinear optimization
2. Optimization in optical design
3. Global optimization methods
4. Sensitivity of variables in optical systems
5. Systematic methods and optimization process
6. Optimization in Zemax
Basic Idea of Optimization

- Topology of the merit function in 2 dimensions
- Iterative down climbing in the topology
Mathematical description of the problem:

- \( n \) variable parameters
- \( m \) target values
- Jacobi system matrix of derivatives,
  Influence of a parameter change on the
  various target values,
  sensitivity function
- Scalar merit function
- Gradient vector of topology
- Hesse matrix of 2nd derivatives

\[
\begin{align*}
\mathbf{x} \\
\mathbf{f}(\mathbf{x}) \\
J_{i,j} &= \frac{\partial f_i}{\partial x_j} \\
F(\mathbf{x}) &= \sum_{i=1}^{m} w_i \cdot \left[y_i - f(\mathbf{x})\right]^2 \\
g_j &= \frac{\partial F}{\partial x_j} \\
H_{j,k} &= \frac{\partial^2 F}{\partial x_j \partial x_k}
\end{align*}
\]
Optimization Principle for 2 Degrees of Freedom

- Aberration depends on two parameters
- Linearization of sensitivity, Jacobian matrix
  Independent variation of parameters
- Vectorial nature of changes:
  Size and direction of change
- Vectorial decomposition of an ideal step of improvement,
  linear interpolation
- Due to non-linearity:
  change of Jacobian matrix,
  next iteration gives better result
Nonlinear Optimization

- Linearized environment around working point
  Taylor expansion of the target function
  \[ \tilde{f} = \tilde{f}_0 + J \cdot \tilde{x} \]

- Quadratic approximation of the merit function

- Solution by linear Algebra
  system matrix \( A \)
  cases depending on the numbers of \( n \) / \( m \)

\[
A^+ = \begin{cases} 
A^{-1} & \text{if } m = n \\
\left( A^T A \right)^{-1} \cdot A^T & \text{if } m > n \ (\text{under determined}) \\
A^T \cdot \left( A A^T \right)^{-1} & \text{if } m < n \ (\text{over determined}) 
\end{cases}
\]

- Iterative numerical solution:
  Strategy: optimization of
  - direction of improvement step
  - size of improvement step
Derivative vector in merit function topology:
- Necessary for gradient-based methods
- Numerical calculation by finite differences
- Possibilities and accuracy

\[ g_{jk} = \frac{\partial f_j(\bar{x})}{\partial x_k} = \nabla_{x_k} f_j(\bar{x}) \]

\[ g_{jk} = \frac{f_j^{\text{right}} - f_j}{\Delta x_k} \]
Effect of Constraints on Optimization

Effect of constraints

- Initial point
- Global minimum
- Local minimum
- $x_1 < 0$
- Path without constraint
- Path with constraint
- Constraint $x_1 < 0$
Boundary Conditions and Constraints

- Types of constraints
  1. Equation, rigid coupling, pick up
  2. One-sided limitation, inequality
  3. Double-sided limitation, interval

- Numerical realizations:
  1. Lagrange multiplier
  2. Penalty function
  3. Barrier function
  4. Regular variable, soft-constraint

\[ F(x) \quad \text{permitted domain} \]
\[ F_0(x) \quad p \text{ small} \]
\[ p \text{ large} \]
\[ x_{\text{max}} \quad x_{\text{min}} \]
Local working optimization algorithms
Principle of searching the local minimum

- **Optimization Minimum Search**
- **Gauss-Newton method** with compromise
- **Steepest descent**
- **Nearly ideal iteration path**
- **Quadratic approximation** around the starting point

The diagram illustrates the topology of the merit function showing the nearly ideal iteration path, steepest descent, starting point, method with compromise, and quadratic approximation around the starting point.
Optimization: Convergence

- Adaptation of direction and length of steps: rate of convergence
- Gradient method: slow due to zig-zag
Optimization and Starting Point

- The initial starting point determines the final result.
- Only the next located solution without hill-climbing is found.
- Method of local minima escape: temporarily added term to overcome local minimum

\[ \Delta F_{esc}(\vec{x}) = \Delta F_0 \cdot e^{-\beta(F(\vec{x}) - F_{0})^2} \]

- Optimization and adaptation of escape parameters
Goal of optimization:
Find the system layout which meets the required performance targets according of the specification

Formulation of performance criteria must be done for:
- Apertur rays
- Field points
- Wavelengths
- Optional several zoom or scan positions

Selection of performance criteria depends on the application:
- Ray aberrations
- Spot diameter
- Wavefornt description by Zernike coefficients, rms value
- Strehl ratio, Point spread function
- Contrast values for selected spatial frequencies
- Uniformity of illumination

Usual scenario:
Number of requirements and targets quite larger than degrees od freedom,
Therefore only solution with compromise possible
Optimization in Optical Design

- **Merit function:**
  Weighted sum of deviations from target values

- **Formulation of target values:**
  1. fixed numbers
  2. one-sided interval (e.g. maximum value)
  3. interval

- **Problems:**
  1. linear dependence of variables
  2. internal contradiction of requirements
  3. initial value far off from final solution

- **Types of constraints:**
  1. exact condition (hard requirements)
  2. soft constraints: weighted target

- **Finding initial system setup:**
  1. modification of similar known solution
  2. Literature and patents
  3. Intuition and experience

\[
\Phi = \sum_{j=1,m} g_j \cdot (f_j^{ist} - f_j^{soll})^2
\]
Characterization and description of the system delivers free variable parameters of the system:

- Radii
- Thickness of lenses, air distances
- Tilt and decenter
- Free diameter of components
- Material parameter, refractive indices and dispersion
- Aspherical coefficients
- Parameter of diffractive components
- Coefficients of gradient media

General experience:
- Radii as parameter very effective
- Benefit of thickness and distances only weak
- Material parameter can only be changes discrete
Constraints in the optimization of optical systems:

1. Discrete standardized radii (tools, metrology)
2. Total track
3. Discrete choice of glasses
4. Edge thickness of lenses (handling)
5. Center thickness of lenses (stability)
6. Coupling of distances (zoom systems, forced symmetry, ...)
7. Focal length, magnification, working distance
8. Image location, pupil location
9. Avoiding ghost images (no concentric surfaces)
10. Use of given components (vendor catalog, availability, costs)
Constraints on thickness values and distances

- Maximum center thickness of air: $MXCA$
- Minimum center thickness of air: $MNCA$
- Minimum edge thickness of air: $MNEA$
- Maximum center thickness of glass: $MXCG$
- Minimum center thickness of glass: $MNCG$
- Minimum edge thickness of glass: $MNEG$
Lack of Constraints in Optimization

Illustration of not usefull results due to non-sufficient constraints

- Negative edge thickness
- Negative air distance
- Lens thickness too large
- Lens stability too small
- Air space too small
- Typical in optics:
  Twisted valleys in the topology

- Selection of local minima
- Typical merit function of an achromate
- Three solutions, only two are useful
Global Optimization

- No unique solution
- Constraints not sufficient
  fixed: unwanted lens shapes
- Many local minima with nearly the same performance
- Saddel points in the merit function topology
- Systematic search of adjacent local minima is possible
- Exploration of the complete network of local minima via saddelpoints
Saddel Point Method

- Example Double Gauss lens of system network with saddelpoints
- Special problem in glass optimization: finite area of definition with discrete parameters $n$, $\nu$
- Restricted permitted area as one possible constraint
- Model glass with continuous values of $n$, $\nu$ in a pre-phase of glass selection, freezing to the next adjacent glass
Merit Function in Zemax

- Default merit function
- Criterion
- Ray sampling (high NA, aspheres,...)
- Boundary values on thickness of center and edge for glass / air
- Special options
  - Add individual operands
  - Editor: settings, weight, target actual value relative contribution to sum of squares
  - Several wavelengths, field points, aperture points, configurations: many requirements
  - Sorted result: merit function listing
If the number of field points, wavelengths or configurations is changed:
the merit function must be updated explicitly
Help function in Zemax: many operands

**Optimization Operand Definitions**

ZEMAX supports optimization operands which are used to define the merit function. Each operand may be assigned a weight which indicates the relative importance of that operand, as well as a target, which is the desired value for that operand.

**First-order optical properties:**
AMAG, ENPP, EFFL, EFLX, EFLY, EPDI, EXPD, EXPP, ISEN, LINV, OBSN, PIMH, PMAG, Powe, Powp, Powr, SFNO, TFNO, WENO

**Aberrations:**
ABCD, ANAC, ANAR, ANAX, ANAY, ANCX, ANCY, ASTII, AXCL, BIOC, BIOD, RSER, COMA, DIXM, DI SA, DISG, DISG, DIST, FGGS, FCCT, FCUR, LACI, LONA, OPDC, OPDM, OPDX, QSCD, PEC, PETZ, RSCE, RSCH, RSRE, RSRE, RWCF, RWCH, RWRE, RWRH, SPCH, SPHA, TRAC, TRAD, TRAE, TRAI, TRAR, TRAX, TRAY, TROX, TRC, ZERN

**MTF data:**
GMTA, GMTS, GMTT, MSWA, MSWS, MSWT, MTFA, MTFS, MTFT, MTHA, MTHS, MTHT

**PSF/Strehl Ratio Data:**
STRH

**Encircled energy:**
DENC, DENE, FRP, GENC, GENC, XENC, XENF

**Constraints on lens data:**
CCTG, COLT, COVA, CTGT, CILT, CVTA, CVTG, CVVL, CVVA, DMGT, DMLT, DMVA, ETGT, ETLT, ETVA, FTTG, FTLT, MCA, MCNG, MNCT, MNCV, MNEA, MNEG, MNF, MNP, MXCA, MXCG, MXCT, MXCV, MXFA, MXGE, MXET, MXPD, MNSD, MXSD, TTGT, TTHI, TILT, TVSA, XNEA, XNET, XNEG, XNEA, XNEA, XNEA, ZTH

**Constraints on lens properties:**
CVOS, MNDT, MXDT, SAGX, SAGY, SSSAG, STHI, TMAS, TOTL, VOLU, NORMX, NORY, NORZ, NORD

**Constraints on parameter data:**
PMGT, PMLT, PMVA
Merit Function in Zemax

- Classical definition of the merit function in Zemax:

\[ MF^2 = \frac{\sum W_i(V_i - T_i)^2}{\sum W_i} \]

- Special merit function options: individual operands can be composed:
  - sum, diff, prod, divi,... of lines, which have a zero weight itself
  - mathematical functions sin, sqrt, max ....
  - less than, larger than (one-sided intervals as targets)

- Negative weights:
  requirement is considered as a Lagrange multiplier and is fulfilled exact

- Optimization operands with derivatives:
  building a system insensitive for small changes (wide tolerances)

- Further possibilities for user-defined operands:
  construction with macro language (ZPLM)

- General outline:
  - use simple operands in a rough optimization phase
  - use more complex, application-related operands in the final fine-tuning phase
Defining variables: indicated by V in lens data editor
toggle: CNTR z or right mouse click

Auxiliary command: remove all variables, all radii variable, all distances variable

If the initial value of a variable is quite bad and a ray failure occurs, the optimization can not run and the merit function not be computed
Variable Glass in Zemax

- Modell glass: characterized by index, Abbe number and relative dispersion
- Individual choice of variables
- Glass moves in Glass map
- Restriction of useful area in glass map is desirable (RGLA = regular glass area)
- Re-substitution of real glass: next neighbor in n-n-diagram
- Choice of allowed glass catalogs can be controlled in General-menu
- Other possibility to reset real glasses: direct substitution
Methods Available in Zemax

- General optimization methods
  - local
  - global
- Easy-one-dimensional optimizations
  - focus
  - adjustment
  - slider, for visual control
- Special aspects:
  - solves
  - aspheres
  - glass substitutes
Methods Available in Zemax

- Classical local derivative:
  - DLS optimization (Marquardt)
  - orthogonal descent

- Hammer:
  - Algorithm not known
  - Useful after convergence
  - needs long time
  - must be explicitly stopped

- Global:
  - global search, followed by local optimization
  - Save of best systems
  - must be explicitly stopped
Conventional DLS-Optimization in Zemax

- Optimization window:
  - Choice of number of steps / cycles
- Automatic update of all windows possible for every cycle (run time slows down)
- After run: change of merit function is seen
- Changes only in higher decimals: stagnation
- Window must be closed (exit) explicitly
Visual optimization

- Menu Tools / Design / Visual optimization
- Change of variable quantities by slider and instantaneous change of all windows
- 'Optimization' under visual control of the consequences