Optical Design with Zemax for PhD - Basics

Lecture 3: Properties of optical systems I
2013-05-23
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1. Aspheres
2. Gratings
3. Diffractive elements
4. Special surfaces
Aspherical Surface Types

- Conic section
  Special case spherical

- Cone

- Toroidal surface with
  radii $R_x$ and $R_y$ in the two
  section planes

- Generalized onic section without
  circular symmetry

- Roof surface

\[
z = c \left( \frac{x^2 + y^2}{1 + \sqrt{1 - (1 + \kappa) c^2 (x^2 + y^2)}} \right)
\]

\[
z = \frac{\sqrt{x^2 + y^2}}{\theta}
\]

\[
z = R_y - \sqrt{\left( R_y - R_x + \sqrt{R_x^2 - x^2} \right)^2 - y^2}
\]

\[
z = \frac{c_x x^2 + c_y y^2}{1 + \sqrt{1 - (1 + \kappa_x) c_x^2 x^2 - (1 + \kappa_y) c_y^2 y^2}}
\]

\[
z = |y| \cdot \tan \theta
\]
Conic Sections

- Explicite surface equation, resolved to $z$
  
  Parameters: curvature $c = 1 / R$
  conic parameter $\kappa$

- Influence of $\kappa$ on the surface shape

\[
z = \frac{c(x^2 + y^2)}{1 + \sqrt{1 - (1 + \kappa)c^2(x^2 + y^2)}}
\]

<table>
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<tr>
<th>Parameter</th>
<th>Surface shape</th>
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<td>$\kappa = -1$</td>
<td>paraboloid</td>
</tr>
<tr>
<td>$\kappa &lt; -1$</td>
<td>hyperboloid</td>
</tr>
<tr>
<td>$\kappa = 0$</td>
<td>sphere</td>
</tr>
<tr>
<td>$\kappa &gt; 0$</td>
<td>oblate ellipsoid (disc)</td>
</tr>
<tr>
<td>$0 &gt; \kappa &gt; -1$</td>
<td>prolate ellipsoid (cigar )</td>
</tr>
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- Relations with axis lengths $a, b$ of conic sections

\[
\kappa = \left(\frac{a}{b}\right)^2 - 1 \\
c = \frac{b}{a^2} \\
b = \frac{1}{c(1 + \kappa)} \\
a = \frac{1}{c\sqrt{|1 + \kappa|}}
\]
Aspherical Shape of Conic Sections

- Conic aspherical surface
- Variation of the conical parameter $\kappa$

$$z = \frac{cy^2}{1 + \sqrt{1 - (1 + \kappa) \cdot c^2 y^2}}$$
Parabolic Mirror

Equation
\[ z = \frac{cy^2}{1 + \sqrt{1 - (1 + \kappa)y^2c^2}} \]

\( c \): curvature \( 1/R_s \)
\( \kappa \): eccentricity \( ( = -1 ) \)

Radii of curvature:
\[ R_{\text{tan}} = R_s \cdot \sqrt{1 + \left( \frac{y}{R_s} \right)^2} \]
\[ R_{\text{tan}} = R_s \cdot \left[ 1 + \left( \frac{y}{R_s} \right)^2 \right]^{\frac{3}{2}} \]
**Simple Asphere – Parabolic Mirror**

- **Equation**
  
  \[ z = \frac{y^2}{2R_s} \]

- Radius of curvature in vertex: \( R_s \)
- Perfect imaging on axis for object at infinity
- Strong coma aberration for finite field angles
- Applications:
  1. Astronomical telescopes
  2. Collector in illumination systems

**Diagram:**

- **axis \( w = 0^\circ \)**
- **field \( w = 2^\circ \)**
- **field \( w = 4^\circ \)**
Simple Asphere – Elliptical Mirror

- Equation
- Radius of curvature $r$ in vertex, curvature $c$
- Eccentricity $\kappa$
- Two different shapes: oblate / prolate
- Perfect imaging on axis for finite object and image location
- Different magnifications depending on used part of the mirror
- Applications:
  Illumination systems

$$z = \frac{cy^2}{1 + \sqrt{1 - (1 + \kappa)y^2c^2}}$$
Equation

\[ z = \frac{cy^2}{1 + \sqrt{1 - (1 + \kappa)y^2c^2}} \]

\( c \): curvature \( 1/R \)

\( \kappa \): Eccentricity

Ellipsoid Mirror

Ellipsoid

F'

F

a

b

oblate vertex radius \( R_{SO} \)

prolate vertex radius \( R_{sp} \)
Aspheres - Geometry

- Reference: deviation from sphere
- Deviation $\Delta z$ along axis
- Better conditions: normal deviation $\Delta r_s$
Asphere: Perfect Imaging on Axis

- Perfect stigmatic imaging on axis:
  Hyperoloid rear surface

\[
\left( \frac{z + \frac{s}{n+1}}{s} \right)^2 - \frac{r^2}{s^2 \cdot \left( \frac{n-1}{n+1} \right)} = 1
\]

- Strong decrease of performance for finite field size:
  dominant coma
- Alternative:
  ellipsoidal surface on front surface
- Improvement by higher orders
- Generation of high gradients
Aspheres: Correction of Higher Order

- Correction at discrete sampling
- Large deviations between sampling points
- Larger oscillations for higher orders
- Better description: slope, defines ray bending

Residual spherical transverse aberrations

Corrected points with $y' = 0$

Paraxial range

$y' = c \frac{dz_A}{dy}$

Perfect correcting surface

Points with maximal angle error

Corrected points residual angle deviation

Real asphere with oscillations
Mechanisms of light deviation and ray bending

- Refraction
- Reflection
- Diffraction according to the grating equation
- Scattering (non-deterministic)

\[
n \cdot \sin \theta = n' \cdot \sin \theta' \\
\theta = -\theta' \\
g \cdot (\sin \theta - \sin \theta_o) = m \cdot \lambda
\]
Grating Diffraction

- Maximum intensity: constructive interference of the contributions of all periods

- Grating equation

\[ g \cdot (\sin \theta - \sin \theta_o) = m \cdot \lambda \]
Grating Equation

- Intensity of grating diffraction pattern (scalar approximation $g >> \lambda$)
- Product of slit-diffraction and interference function
- Maxima of pattern: coincidence of peaks of both functions: grating equation
  \[ g \cdot (\sin \theta - \sin \theta_o) = m \cdot \lambda \]
- Angle spread of an order decreases with growing number of periods $N$
- Oblique phase gradient:
  - relative shift of both functions
  - selection of peaks/order
  - basic principle of blazing

\[ I = N^2 \cdot g^2 \cdot \left[ \frac{\sin \left( \frac{\pi ug}{\lambda} \right)}{\frac{\pi ug}{\lambda}} \cdot \frac{\sin \left( \frac{N\pi ug}{\lambda} \right)}{N \cdot \sin \left( \frac{\pi ug}{\lambda} \right)} \right]^2 \]
### Ideal Diffraction Grating

- Ideal diffraction grating: monochromatic incident collimated beam is decomposed into discrete sharp diffraction orders
- Constructive interference of the contributions of all periodic cells
- Only two orders for sinusoidal

**Equation:**

\[ g = \frac{1}{s} \]

**Diagram:**

- Incident collimated light
- Diffraction orders: +4, +3, +2, +1, 0, -1, -2, -3, -4
- Grating constant: \( g = \frac{1}{s} \)
Real diffraction grating:
1. Finite number of periods
2. Finite width of diffraction orders
- Original lens height profile \( h(x) \)
- Wrapping of the lens profile: \( h_{\text{red}}(x) \) Reduction on maximal height \( h_{2\pi} \)
- Digitalization of the reduced profile: \( h_{q}(x) \)
- Surface with grating structure:
  new ray direction follows the grating equation
- Local approximation in the case of space-varying
  grating width
  \[ \bar{s}' = \frac{n}{n'} \cdot \bar{s} + \frac{m\lambda g}{n'd} \cdot \hat{g} + \gamma \cdot \bar{e} \]
- Raytrace only into one desired diffraction order
- Notations:
  - g : unit vector perpendicular to grooves
  - d : local grating width
  - m : diffraction order
  - e : unit normal vector of surface
- Applications:
  - diffractive elements
  - line gratings
  - holographic components
Diffractive Optics:

- Local micro-structured surface
- Location of ray bending: macroscopic carrier surface
- Direction of ray bending: local grating micro-structure
Surface properties and settings

- Setting of surface properties

![Surface properties and settings diagram](image-url)
Important Surface Types

- Special surface types
- Data in Lens Data Editor or in Extra Data Editor
- Gradient media are described as 'special surfaces'
- Diffractive / micro structured surfaces described by simple ray tracing model in one order
Important Surface Types

- Standard spherical and conic sections
- Even asphere classical asphere
- Paraxial ideal lens
- Paraxial XY ideal toric lens
- Coordinate break change of coordinate system
- Diffraction grating line grating
- Gradient 1 gradient medium
- Toroidal cylindrical lens
- Zernike Fringe sag surface as superposition of Zernike functions
- Extended polynomial generalized asphere
- Black Box Lens hidden system, from vendors
- ABCD paraxial segment
Raytracing in GRIN media

- Ray: in general curved line
- Numerical solution of Eikonal equation
- Step-based Runge-Kutta algorithm
  - 4th order expansion, adaptive step width
- Large computational times necessary for high accuracy

\[
\frac{d^2 \vec{r}}{dt^2} = n \cdot \nabla n = \vec{D} = \begin{pmatrix}
  n \frac{\partial n}{\partial x} \\
  n \frac{\partial n}{\partial y} \\
  n \frac{\partial n}{\partial z}
\end{pmatrix}
\]
Description of GRIN media

- Analytical description of grin media by Taylor expansions of the function \( n(x,y,z) \)
- Separation of coordinates
  \[ n = n_{o,\lambda} + c_1 h + c_2 h^2 + c_3 h^4 + c_4 h^6 + c_5 h^8 + c_6 z + c_7 z^2 + c_8 z^3 + c_9 z^4 \]
  \[ + c_{10} x + c_{11} x^2 + c_{12} x^3 + c_{13} y + c_{14} y^2 + c_{15} y^3 \]
- Circular symmetry, nested expansion with mixed terms
  \[ n = n_{o,\lambda} + c_1 h^2 + c_2 h^4 + c_3 h^6 + c_4 h^8 + z\left(c_5 + c_6 h^2 + c_7 h^4 + c_8 h^6 + c_9 h^8\right) \]
  \[ + z^2\left(c_{10} + c_{11} h^2 + c_{12} h^4 + c_{13} h^6 + c_{14} h^8\right) + z^3\left(c_{15} + c_{16} h^2 + c_{17} h^4 + c_{18} h^6 + c_{19} h^8\right) \]
- Circular symmetry only radial
  \[ n = n_{o,\lambda} \sqrt{1 + c_2 \left( c_1 h \right)^2 + c_3 \left( c_1 h \right)^4 + c_4 \left( c_1 h \right)^6 + c_5 \left( c_1 h \right)^8 + c_6 \left( c_1 h \right)^{10}} \]
- Only axial gradients
  \[ n = n_{o,\lambda} \sqrt{1 + c_2 \left( c_1 z \right)^2 + c_3 \left( c_1 z \right)^4 + c_4 \left( c_1 z \right)^6 + c_5 \left( c_1 z \right)^8} \]
- Circular symmetry, separated, wavelength dependent
  \[ n = n_{o,\lambda} + c_{1,\lambda} h^2 + c_{2,\lambda} h^4 + c_{3,\lambda} h^6 + c_{4,\lambda} h^8 + c_{5,\lambda} z + c_{6,\lambda} z^2 + c_{7,\lambda} z^3 \]
Gradient Lens Types

- Curved ray path in inhomogeneous media
- Different types of profiles

- Radial gradient rod lens
- Axial gradient rod lens
- Radial and axial gradient rod lens
- Radial gradient lens
- Axial gradient lens
- Radial and axial gradient lens
Collecting Radial Selfoc Lens

- Thick Wood lens with parabolic index profile
- Principal planes at 1/3 and 2/3 of thickness

\[ n(r) = n_0 - n_2 \cdot r^2 \]

- \( n_2 > 0 \): collecting lens
- \( n_2 < 0 \): negative lens
Gradient Lenses

- Refocusing in parabolic profile
- Helical ray path in 3 dimensions
Gradient Lenses

- Types of lenses with parabolic profile

\[ n(r) = n_0 - n_2 \cdot r^2 \]

\[ = n_0 \cdot \left(1 - n_r \cdot r^2\right) \]

\[ = n_0 \cdot \left(1 - \frac{1}{2} A \cdot r^2\right) \]

- Pitch length

\[ p = 2\pi \cdot \sqrt{\frac{n_0}{2n_2}} = \frac{2\pi}{\sqrt{2n_r}} \]