Metrology and Sensing

Lecture 7: Wavefront sensors
2017-11-30
Herbert Gross
### Preliminary Schedule

<table>
<thead>
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<th>No</th>
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<td>1</td>
<td>19.10.</td>
<td>Introduction</td>
<td>Introduction, optical measurements, shape measurements, errors, definition of the meter, sampling theorem</td>
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<td>2</td>
<td>26.10.</td>
<td>Wave optics</td>
<td>Basics, polarization, wave aberrations, PSF, OTF</td>
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<td>Fringe projection</td>
<td>Moire principle, illumination coding, fringe projection, deflectometry</td>
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<td>Interferometry I</td>
<td>Introduction, interference, types of interferometers, miscellaneous</td>
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<td>Interferometry II</td>
<td>Examples, interferogram interpretation, fringe evaluation methods</td>
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<td>Hartmann-Shack WFS, Hartmann method, miscellaneous methods</td>
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<td>Tactile measurement, photogrammetry, triangulation, time of flight, Scheimpflug setup</td>
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<td>Speckle methods</td>
<td>Spatial and temporal coherence, speckle, properties, speckle metrology</td>
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<td>11</td>
<td>11.01.</td>
<td>Measurement of basic system properties</td>
<td>Basic properties, knife edge, slit scan, MTF measurement</td>
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<td>12</td>
<td>18.01.</td>
<td>Phase retrieval</td>
<td>Introduction, algorithms, practical aspects, accuracy</td>
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<td>13</td>
<td>25.01.</td>
<td>Metrology of aspheres and freeforms</td>
<td>Aspheres, null lens tests, CGH method, freeforms, metrology of freeforms</td>
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<td>14</td>
<td>01.02.</td>
<td>OCT</td>
<td>Principle of OCT, tissue optics, Fourier domain OCT, miscellaneous</td>
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<td>15</td>
<td>08.02.</td>
<td>Confocal sensors</td>
<td>Principle, resolution and PSF, microscopy, chromatical confocal method</td>
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Content

- Hartmann-Shack WFS
- Hartmann method
- Miscellaneous methods
Hartmann-Shack Wavefront Sensor

- Basic principle

Ref: S. Merx
- Lenslet array divides the wavefront into subapertures
- Every lenslet generates a single spot in the focal plane
- The averaged local tilt produces a transverse offset of the spot center
- Integration of the derivative matrix delivers the wave front $W(x,y)$
- Typical setup for component testing

- Lenslet array

![Diagram of a typical setup for component testing with labeled components: fiber illumination, collimator, beam-splitter, detector, lenslet array, telescope for adjustment of the diameter, test surface.]
Spot Pattern of a HS - WFS

- Aberrations produce a distorted spot pattern
- Calibration of the setup for intrinsic residual errors
- Problem: correspondence of the spots to the subapertures

a) spherical aberration
b) coma
c) trefoil aberration
Array Signal

- **Lenslet array ideal signal**

- **Real signal:**
  1. discretization
  2. quantization
  3. noise
- Dynamic range: ratio of spot diameter to size of sub-aperture
- Averaging of wavefront slope inside sub-aperture
1. Simple setup

2. With telescope and relay lens
General Setup of a HS - WFS

- Generalized setup:
  - adaptation of diameter
- Wavefront is scaled

\[
W_{sens} = |\Gamma| \cdot W_{pup}, \quad D_{sens} = \frac{D_{pup}}{|\Gamma|}
\]

- Relay lens
  - Adaptation of sensor size
Problem in practice: exact determination of the spot centroid:
- noise
- discretization
- quantization
- broadening by partial coherence
- broadening by local curvature
- error by centroid affecting coma
- error by partly illuminated pixels
Parametrization of a HS-WFS

Layout parametrization:

- **Fresnel number**
  \[
  N_F = \frac{D_{meas}^2}{4\lambda f N_{sub}^2 \eta^2} = \frac{D_{sub}^2}{4\lambda f}
  \]

- **Fill factor**
  \[
  \eta = \frac{D_{meas}}{D_{array}}
  \]

- **Spot size**
  \[
  \frac{D_{spot}}{D_{sub}} = \frac{1}{2N_F}
  \]

- **Spot size**
  \[
  D_{spot} = \frac{2\lambda f}{D_{sub}} = \frac{D_{sub}}{2N_F} = \frac{2\lambda f N_{sub} \eta}{D_{meas}}
  \]

- **Accuracy:**
  \[
  \theta_{\min} = \frac{k \cdot P \cdot m_{rel}}{f \cdot \Gamma} \quad \theta_{\max} = h \cdot \frac{D_{sub}}{2f \cdot \Gamma^2}
  \]
Parametrization of a HS-WFS

- Wavefront detectability

\[ W_{\text{min}} = \theta_{\text{min}} \cdot D_{\text{array}} \cdot N_{\text{sub}} \]

- Dynamic range

\[ R = \frac{\theta_{\text{max}}}{\theta_{\text{min}}} = \frac{h \cdot D_{\text{sub}}}{2kP \cdot m_{\text{rel}} \cdot \Gamma} \]

\[ R = \frac{\theta_{\text{max}}}{\theta_{\text{min}}} = \frac{D_{\text{sub}} - D_{\text{spot}}}{2k \cdot P \cdot m_{\text{rel}}} \cdot h = \frac{D_{\text{sub}} \cdot h}{k \cdot P \cdot \Gamma \cdot m_{\text{rel}}} \cdot \left( N_f - \frac{1}{2} \right) \leq 4000 \]
The wavefront is averaged over the area of the subaperture.

The Fresnel number determines the relation between:
- spot size and size of array cell/subaperture

Resolution:
- p: pixel size, N: number of subapertures

Relation between spots and corresponding subaperture:
- determines the dynamic range

Measurement of spot location with sub-pixel accuracy

Problems with partially illuminated subapertures

Nearly no problems with:
- spectral bandwidth
- coherence
- polarization

\[ N_F = \frac{\varnothing_{sub}^2}{4\lambda \cdot f} \]

\[ W_{\text{min}} = \frac{4p\lambda \cdot N}{\varnothing_{\text{mess}}} \cdot N_f \]

\[ \Delta x = -\frac{f}{n} \frac{\partial W}{\partial x} \]
Errors in the HS - Wavefrontsensor

- Tilted sensor plane
- Rotated sensor in the azimuth
- Scattering of focal lengths of the lenslets
- Average of slope inside the subaperture area
- Errors in the wavefront reconstruction algorithms
- Coma of lenses
- Wrong focal length due to dispersion for different wavelength
- Sensor plane not exactly matched with focal plane
- Partly illuminated lenslets
- Electronical noise
- Zernike errors due to bad known normalization radius / edge of pupil
- Geometrical distortions of the array
- Truncation of spot by the corresponding subaperture / cross talk
- Discrete finite number of pixels
- Quantization of signal on the detector
- Theoretical largest curvature: $R = f$
- Real size of point spread function:
- Larger curvature: cross talk generates errors

\[ R_{\text{min}} = \frac{f}{1 - \frac{1}{2N_F}} \]
HS-WFS : Discretization Errors

- Signal errors due to finite pixel size discretization of the point spread function
  N: number of pixels per sub-aperture
Averaging Error by Subapertures

- Averaging of the wavefront over the finite size of the subaperture
- Number of subapertures (linear): \( n_s \)
  - Index of the Zernikes: \( n \)
- Error decreases with growing \( n_s \)
- Error larger for higher order Zernikes \( n \)
- Larger errors of Zernike polynomials (radial order n) for small number of sub-apertures (ns)
- Larger gradients of high order Zernikes suffers strongly from averaging
- PV value less sensitive as rms value
- Larger ns more accurate and stable
- Determination of wavefront of a microscopic lens
- Number ns of subapertures (linear): 16, 32, 64, 100
- Calculated:
  1. gradient of wavefront
  2. reconstructed wavefront
  3. errors Zernikes
- Errors due to averaging and shifted center of the subaperture
Fresnel Number and Crosstalk

- Relative size of the spot in a HS WFS: determined by Fresnel number
- Small NF: large PSF, crosstalk of neighbouring apertures
- Larger error of centroid calculation for subapertures at the edge

\[
\frac{D_{\text{spot}}}{D_{\text{sub}}} = \frac{2\lambda \cdot f}{D_{\text{sub}}^2} = \frac{1}{2N_F}
\]
Partly illuminated sub-aperture: change of centroid and error of signal

Wrong signal for constant phase plateaus
- Example
  Change of point spread function due to partly illumination
## Comparison HS-Sensor - Interferometer

<table>
<thead>
<tr>
<th>Feature</th>
<th>HS-WFS</th>
<th>PSI-interf.</th>
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<tbody>
<tr>
<td>1  Complexity of the setup</td>
<td>+</td>
<td></td>
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<td>2  Cost of the equipment, additional high-quality components</td>
<td>+</td>
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<td>3  Spatial resolution</td>
<td></td>
<td>+</td>
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<tr>
<td>4  Robust measurement under environmental conditions</td>
<td>+</td>
<td></td>
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<tr>
<td>5  Noise due to image processing and pixelated sensor</td>
<td>+</td>
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<td>6  Perturbation by coherent scattering and straylight</td>
<td>+</td>
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<td>7  Algorithm for reconstructing the wavefront</td>
<td>+</td>
<td></td>
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<tr>
<td>8  Test systems with central obscuration</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>9  Speed of measurement</td>
<td>+</td>
<td></td>
</tr>
<tr>
<td>10 Absolute accuracy</td>
<td></td>
<td>+</td>
</tr>
<tr>
<td>11 Measurement possible independently of wavelength, polarization and coherence</td>
<td>+</td>
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Real Measurement of a HS-WFS

- Problem in practice: definition of the boundary
HS-WFS

- Real spot pattern:
  - broadening of spots
  - real boundary definition
  - signal to noise ratio
  - separation of spots
  - correspondence of spots to subapertures
HS-WFS

- Finding the boundary
- Special problem for determining Zernike polynomials
HS-WFS

- Assignment of spots
  - dynamic range limitation
  - integrability can solve the problem
  - practical help: shift arrows

- Pitfalls:
  - large broadening
  - overlapp of spots
  - missing spots
  - clear assignment of spots to sub-aperture
Measurement of the Human Eye by HS-WFS

- More or less motivating product advertisements
Hartmann Method

- Similar to Hastmann Shack Method with simple hole mask and two measuring planes
- Measurement of spot center position as geometrical transverse aberrations
- Problems: broadening by diffraction

\[ s'_y = s'_1 + (s'_2 - s'_1) \cdot \frac{y_1}{y_1 + y_2} \]
Hartmann Method

- Schematic drawing of transverse aberrations

- Distance of planes limited: overlap of spots

- Coherent coupling of sub-aperture fields, interference induces errors of centroid
Hartmann Method: Pinhole Array Geometry

- Possible geometry of the pinholes:
  - number of pinholes,
  - size of holes
  - distance / geometry

- Parameters determine the accuracy
Hartmann Method Properties

- z-positions critical for large spots diameters
- No dependence on spectral range and polarization
- Coherence is critical, interference for overlapping pinhole images
- Apodization not critical
- Averaging gives stable data evaluation
Hartmann Method

- Real pinhole pattern with signal
- Problems with cross talk and threshold
Separated spots in case of diffraction

\[ d_{s}^{(gesamt)} = \frac{d_{2}}{d_{1}} \cdot d_{s} + (d_{1} + d_{2}) \cdot \frac{D_{obj}}{f} + (d_{1} + d_{2}) \cdot \frac{2.44 \cdot \lambda}{d_{s}} \]
- Apodized beam: centroid rays pass through the perfect image point
- A centroid error is eliminated

- Reconstruction of the transverse aberrations delivers the wave aberration

\[ W(x, y) = -\frac{1}{R} \int_{0}^{x} \Delta x' dx \]
Hartmann Sensor

- Small power transmission

- Problem: diffraction spreading of light pencils
Hartmann Sensor

- Problem: diffraction spreading of light pencils
Pyramidal Wavefront Sensor

- Pyramidal components splits the wavefront
- Signal evaluation analog to 4-quadrant detector

\[
s_c = \frac{E_1 - E_2 + E_3 - E_4}{E_1 + E_2 + E_3 + E_4}
\]
Knife Edge Method

- Moving a knife edge perpendicular through the beam cross section

- Relationship between power transmission and intensity: Abel transform for circular symmetry

\[ P(x) = 2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{I(r) r \, dr}{\sqrt{r^2 - x^2}} \, d\xi \]

- Example: geometrical spot with spherical aberration

- Before caustic

- Zone rays below

- Near paraxial focus
Indirect Wavefront Sensing

- Foucault knife edge method

Ref: R. Kowarschik / J. Wyant
Zonal Aberration

- Eyepiece with strong zonal pupil aberration

- Illumination for decentered pupil: dark zones due to vignetting
  Kidney beam effect
Indirect Wavefront Sensing

- Foucault knife edge method

Ref: R. Kowarschik / J. Wyant
- Method very similar to moving knife edge
- Integration of slit length must be inverted:
  - inverse Radon transform
  - corresponds to tomographic methods
General Filter Techniques

- Generalized concept: filtering the wave
- Realizations:
  1. Foucualt knife edge
  2. slit
  3. Toepler schlieren method
  4. Ronchi test
  5. wire test
  6. Lyot test ($\lambda/4$ wire)
General Filter Techniques

- Knife edge filter for defocussing
- Changing intensity distribution as a function of the filter position
Ronchi Method

- Measurement of surfaces by fringe deformation
- Grating creates reference: fringe of 1st order after Ronchi grating
- Evaluation of the lateral aberrations of the wavefront by

\[
\frac{\partial W}{\partial x_p} = -\frac{\Delta x}{R}, \quad \frac{\partial W}{\partial y_p} = -\frac{\Delta y}{R}
\]

- Explanation geometrical or wave-optical
Ronchi Method

- Setup
- Problem: superposition of perturbing higher orders
Ronchi Method

- Ronchi pattern of low order aberrations
- Complex evaluation of patterns
Setup:

- First slit selects one cross section
- Lens images the slit
- $45^\circ$ rotated toroidal lens inverts space coordinate/angle
Cylindrical lens under 45°

- Cylindrical lens under 45°:
  Focal lengths \( f_x = +f_T \) and \( f_y = -f_T \)
- Rotation by 45°
- Angle deviation \( u \) is transformed into spatial \( v \) offset
- Imaging of the slit: angle aberration \( u \) gives transverse aberration \( y \)
Phase Space Analyzer in Case of Sph Aberration

- Image of a slit for a system with spherical aberration
- Image distance \( z \) is varied
- Cubic angle aberration more clearly seen in defocussed planes
- Not trivial: adjustment of proper slit width
Imaging of the slit

Special case

Slit coordinates:
angle $v$ creates deviation in $x'$ perpendicular to the slit

Special case:
thin slit and toroidal lens under 45°

Defocussing $c_4$: slit rotates

\[
\frac{1}{a} + \frac{1}{b} = \frac{1}{f}
\]

\[
d = f_L
\]

\[
x' = \left[1 - \frac{b}{f_L}\right] \cdot x + \left[\frac{f_L}{f_T} \cdot (b - f_L)\right] \cdot v
\]

\[
y' = \left[1 - \frac{b}{f_L}\right] \cdot y + \left[\frac{f_L}{f_T} \cdot (b - f_L)\right] \cdot u
\]

\[
x' = \left[1 - \frac{b}{f_L}\right] \cdot x \quad y' = \left[\frac{f_L}{f_T} \cdot (b - f_L)\right] \cdot u
\]

\[
y' = -\frac{f_L^2}{f_T} \cdot \left(1 - \frac{b}{f_L}\right) \cdot \frac{4\lambda \cdot x \cdot c_4}{r_{AP}^2}
\]
- **Application:** Measurement of microscopic lenses
- **Three colored points in intermediate image plane**
- **Measurement of:**
  - axial color
  - defocussing
  - spherical aberration

![Phase Space Analyser Diagram](image-url)