Metrology and Sensing

Lecture 11: Measurement of basic system properties
2017-01-03
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Content

- Basic system properties
- Knife edge method
- Slit scan method
- MTF measurement
Measurement of Focal Length with Collimator

- Collimated incident light
- Calibrated collimator with focal length $f_c$ and test chart with size $y$
- Selection of sharp image plane
- Analysis of image size

\[ f' = - f'_c \cdot \frac{y'}{y} \]
Measurement of Focal Length According to Gauss

- Setup with distance object-image $L > 4f$
- Known location of the principal plane $P$ of the system distance $d_P$ between principal planes
- Selection of two system locations with sharp image
- Relative axial shift $D$ between the two setups

\[
f = \frac{L - d_H}{4} - \frac{D^2}{4(L - d_H)}\]
Telecentric movable measurement microscope with offset $y$: Abbe focometer

- Focusing of two different test charts with sizes $y_1$ and $y_2$
- Determination of the focal length by

$$\tan u = \frac{y}{f} = \frac{y_2 - y_1}{e}$$
Measurement of Focal Length by Confocal Setup

- Setup with fiber and plane mirror for autocollimation
- Change of distance between test lens and fiber
- Analysis of the recoupled power into the fiber (confocal) gives the focal point
Measurement of Focal Length with Collimator

- Collimated incident light
- Calibrated collimator with focal length $f_c$ and test chart with size $y$
- Selection of sharp image plane
- Analysis of image size

$$f' = -f'_c \cdot \frac{y'}{y}$$
- Setup with distance object-image $L > 4f$
- Known location of the principal plane $P$ of the system
distance $d_P$ between principal planes
- Selection of two system locations with sharp image
- Relative axial shift $D$ between the two setups

\[ f = \frac{L - d_H}{4} - \frac{D^2}{4(L - d_H)} \]
Measurement of Focal Length with Abbe-Focometer

- Telecentric movable measurement microscope with offset $y$
- Focusing of two different test charts with sizes $y_1$ and $y_2$
- Determination of the focal length by

\[
\tan u = \frac{y}{f} = \frac{y_2 - y_1}{e}
\]
- Setup with fiber and plane mirror for autocollimation
- Change of distance between test lens and fiber
- Analysis of the recoupled power into the fiber (confocal) gives the focal point

**Measurement of Focal Length by Confocal Setup**
Measurement of Focal Length with Focimeter

- Afocal setup with sharp image plane
- Measurement of long focal lengths
- Insertion of test system in collimated light segment and refocussing
- Applying the lens makers formula

\[
\frac{1}{f} = \frac{1}{f_2} - \frac{x}{f_2^2}
\]
- Setup with collimator and two Ronchi rulings
- System under test is inserted
- Grating period $d$ and azimuthal angle $\theta$ between the gratings
- Moire pattern is rotated by angle $\alpha$, if test lens acts as focusing element
- Radius of curvature $R$ or focal length

$$R = \frac{d}{\theta \cdot \tan \alpha}$$
- Setup with Ronchi grating in collimated light gives a series of Talbot images
- The Talbot planes are imaged by the system under test
- Analysis of the image plane by lens formula gives the desired focal length
- By use of several planes, the position of the principal plane can be eliminated
- A second Ronchi grating can be used to find the accurate image planes
Determination of Best Focus

- Criteria for best focus:

1. Paraxial centre of curvature for the paraxial spherical wave of an on axis object point.
2. Maximum of the Strehl ratio
3. Smallest rms-value of the wave aberration
4. Highest contrast of the modulation of an object feature of given spatial frequency
5. Highest value of the slope of an edge
6. Highest value of the entropy of the detected digital image

- Requirements for focus detection procedure

1. Steep curve dependency to get high accuracy
2. Robust definition to deliver a large dynamic range
3. Suppression of side lobe effects to guarantee an unambiguous solution
4. High frequency pre-filtering to be noise insensitive
Determination of Best Focus

- **Blur of defocussed plane**

- **Minimum of image entropy**
  \[ E = - \sum_j w_j \cdot \log_2 w_j \]

- **Maximum of image contrast**
Phase analysis by Zernike coefficient $c_4$

$$c_4 = -\frac{1}{4n\lambda} \cdot \Delta z \cdot NA^2$$

Measurement with two Ronchi gratings
Measurement by image analysis:

1. Maximum gradient of edges
   \[ g = |\nabla I(x, y)| = \sqrt{\left( \frac{\partial I}{\partial x} \right)^2 + \left( \frac{\partial I}{\partial y} \right)^2} \]

2. Power of gradients
   \[ G = \iint |\nabla I(x, y)|^2 \, dx \, dy \]

3. Laplacian
   \[ L = \iint |\nabla^2 I(x, y)|^2 \, dx \, dy \]
Measurement of Principal Planes

- Measurement for systems in air via the nodal planes
- Imaging of a test pattern with a collimator onto a detector
- Invariant lateral image location for rotated system around the nodal point
- Critical: vignetting effects for large angles
Measurement of Principal Planes

- Setup of the test lens with different object locations: axial shift $D$
- Analysis of the lens imaging formula

\[
\frac{1}{a_j + \Delta} + \frac{1}{a'_j - \Delta} = \frac{1}{f}
\]

- Minimizing the error of several measurements $j$

\[
\delta = f \cdot (a_j + a'_j) + \Delta \cdot (a_j - a'_j) - a_j a'_j + \Delta^2
\]
Measurement of Pupil Size

- Setup with collimating auxiliary lens
  \[ \tan u = \frac{D}{2f} \]

- Determination with measuring microscope (dynameter)
Measurement of Pupil Size

- Setup with Ronchi grating
- Measurement of the lateral shift of higher diffraction orders at distance $z$

- High-NA in microscopy: $NA > 1$
  Measurement of total internal reflection of fluorescence light
- Measurement of object sided telecentricity errors by lateral shift of image location during defocussing

- High accuracy measurement by interferometry and measurement of Zernike coefficients $c_{2/3}$. 
Measurement of Lens Position

- Measurement of reflexes at lens vertex points
- Analysis of confocal signal in autocollimation
- Avoiding spherical aberration induced errors by ring illumination

![Diagram of lens measurement system](image)
- Measurement of tilt errors (plane or spherical surface) in autocollimation
- Projection of the cross
- Observation of lateral shift in Fourier plane $x = f \cdot 2\varphi$
- Projection of test marker
- Autocollimation of sharp image, focal point coincides with center of curvature of surface with radius $r$
- Rotation of test system: tilt of surface induces a lateral shift of the image
- Problems with inner surfaces

$$v = \beta \cdot v_M = 2 \cdot \beta \cdot r \cdot \kappa$$
- Thin collimated beam through lens
- Focussing of the beam onto detector
- Measurement of wedge angle by lateral shift $v$
- Tilt angle of lens not detectable
- Not feasible for very short focal lengths

$$\varphi = (n-1) \cdot \theta = (n-1) \cdot (\alpha_1 - \alpha_2) = \frac{v}{f}$$
Reasons for reduced system transmission:
1. Absorption in the bulk material of the components
2. Scattering in the bulk materials by inclusions or finite scattering parameters
3. Absorption in the coatings of the surfaces
4. Partial reflection or transmission at the coatings at transmissive or reflective surfaces
5. Blocking of light via mechanical or diaphragm parts of the system due to vignetting
6. Scattering of light by local surface imperfections or non-perfect polished surfaces
7. Deflection of light by diffraction of the light at edges
8. Deflection of light in unwanted higher orders of diffractive elements

Usually strong dependency on:
1. field position
2. wavelength of light
3. used pupil location
4. polarization

Critical:
1. absolute values for test lens
2. influence of auxiliary components
3. change of vignetting and incidence angles
**Measurement of Transmission**

- **Measurement of transmission:**
  
  a) Calibration setup
  
  ![Calibration setup diagram]

  b) Measurement setup
  
  ![Measurement setup diagram]

  \[
  T = \frac{P_{in}}{P_{out}}
  \]

- **Reasons for measurement errors:**
  1. Absorption in the component materials
  2. Absorption in the coatings
  3. Finite reflectivity of the coatings
  4. Vignetting of the aperture bundle for oblique chief rays
  5. Natural vignetting according for oblique chief rays and projection of tilted planes
  6. False light from surrounding light sources, which reach the image plane
  7. Scattering of light at components of the system mechanical design
  8. False light due to ghost images or narcissus in infrared systems
Measurement of unwanted light: 2 different approaches:

1. object area black, surrounded by bright source
detection of irradiance in image region

2. intensive isolated point light source in the object plane at different locations
detection of artificial distributions in the image area: glare spread function
- Measurement of the refractive index of a liquid
- Thin film of test liquid between prisms, adjustment of total internal reflection
- Special setup with direct sight prisms, no color fringes
Physical Image Quality Criteria

- Wavefront
  PV-, rms-value, fractional pupil area
  Rayleigh-, Marechal criteria for diffraction limit

- Point spread function
  Strehl ratio, width, second moment, area equivalent, correlation, power in the bucket

\[ \text{SR} / D_s \]
\[ \text{STDEV} \]
\[ \text{LIB} \]
\[ \text{EW} \]
\[ \text{SM} \]
\[ \text{FWHM} \]
\[ \text{CW} \]
\[ \text{Ref} \]
\[ \text{WEA} \]
\[ P=50\% \]
In principle, the complete information on the optical systems performance can be recovered from measurements of:

1. Point spread function
2. Line spread function
3. Edge spread function

The ESF and the LSF covers the spatial frequencies in the pupil only in one direction. The complete information requires the measurements of ESF / LSF in several angle orientations (azimuth).

Between the ESF and the LSF, a simple relationship allows a conversion

\[ I_{LSF}(x_i) = \frac{d I_{ESF}(x_i)}{dx_i} \]

There are three symmetry classes, which can be distinguished:

1. Circular symmetry
   The Abel transform allows a non-iterative calculation of the PSF from the LSF/ESF
2. Mirror symmetry with decoupling of x and y
   The calculation can be performed in two separated 1D-sections
3. General case without symmetry
   A complete tomographic reconstruction is necessary
- Characteristic s-shaped curve for the energy transmission
- Definition of centroid by corresponding threshold values
Knife Edge Method

- Moving a knife edge perpendicular through the beam cross section

- Relationship between power transmission and intensity: Abel transform for circular symmetry

\[ P(x) = 2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{I(r) r dr}{\sqrt{r^2 - \xi^2}} d\xi \]

- Example: geometrical spot with spherical aberration
- Method very similar to moving knife edge
- Integration of slit length must be inverted:
  - inverse Radon transform
  - corresponds to tomographic methods
**Slit-Scan-Method**

- Examples:
  - Gaussian profile

- Variation of the ratio between beam width $W$ and slit width $v$:
  - $w / v$ large: width of the slit can be neglected
  - $w / v$ small: strongly changed profile

$$I(x, y) = \frac{2P}{\pi \cdot w_x \cdot w_y} \cdot e^{-\frac{2x^2}{w_x^2} - \frac{2y^2}{w_y^2}}$$

$$I_{ssc}(x) = \frac{1}{2} I_o \cdot \left[ \text{erf}\left(\sqrt{2} \frac{x + v}{w_x}\right) + \text{erf}\left(\sqrt{2} \frac{-x + v}{w_x}\right) \right]$$
- Measurement of an edge image
- Evaluating the derivative: Line spread function
- Fourier transform: optical transfer function

\[ I_{LSF}(x') = \frac{d I_{ESF}(x')}{d x'} \]

\[ H_{OTF}(s) = \hat{F}[I_{LSF}(x')] \]
MTF-Measurement by Imaging Gratings

- Setup:
  Imaging of a grating

- Possible realizations:
  1. Density type grating, the sine wave is modelled by gray levels
  2. Area type gratings, the sine wave is modelled by geometrical sine-shaped structures

- Area coded sine grating:
MTF Measurement

- Definitions:
  1. Fourier transform of PSF
     \[ H_{OTF}(v_x, v_y) = \int \int I_{PSF}(x', y') e^{2\pi i (v_x x' + v_y y')} \, dx' \, dy' \]
  2. Autocorrelation of pupil function
     \[ H_{OTF}(v_x, v_y) = \frac{\int \int P(x_p + \frac{\lambda f' v_x}{2}, y_p + \frac{\lambda f' v_y}{2}) \, P^*(x_p - \frac{\lambda f' v_x}{2}, y_p - \frac{\lambda f' v_y}{2}) \, dx_p \, dy_p}{\int \int |P(x_p, y_p)|^2 \, dx_p \, dy_p} \]

- Overview: classification
  1. Imaging of special test structures and analyzing the corresponding image contrast behavior
     1.1 If the structures are sine grating structures, a single frequency response is determined
     1.2 If the structures have a large frequency content like points, lines, edges or bar patterns, a careful analysis of the higher frequency components and calculation the OTF from the measurement data must be performed
  2. Direct measuring of the autocorrelation function of the optical system pupil corresponding to the Duffieux-integral formulation of the transfer function
  3. Measurement of the point spread function and digital calculation of the transfer function by performing the Fourier transform.
Conditions for measuring the incoherent transfer function:

1. An object is illuminated by incoherent light.
2. The object acquires, through its structures, all relevant spatial frequencies that have to be measured.
3. The object is imaged by the test system.
4. Spatial resolution is provided for the detection of the image intensity. As a rule this is achieved by an adjustable slit located in front of the detector. Alternatively, the slit can be fixed and scanning is accomplished by the imaged grating or object structure.
5. The contrast is derived from the intensity distribution and analysed as a function of the spatial frequency

Possible test structures of the object:

1. Point object
2. Edge object
3. Line object or slits
4. Bar pattern
5. Random transparencies
6. Sine gratings with one or several periods
7. Special test charts like the Siemens star
MTF-Measurement by Edge Image Analysis

- Mathematical relationships

\[ I_{LSF}(x') = \frac{d I_{ESF}(x')}{d x'} \]

\[ H_{OTF}(\nu) = \hat{F}[I_{LSF}(x')] \]

- Direct analysis of the frequency content

\[ H_{MTF}(\nu) = \left| \frac{A'(\nu)}{A(\nu)} \right| \]

- Problem: zero crossing points
  Solution: filtering, windowed calculation
MTF-Measurement by Imaging Gratings

- Setup:
  Imaging of a grating

- Possible realizations:
  1. Density type grating, the sine wave is modelled by gray levels
  2. Area type gratings, the sine wave is modelled by geometrical sine-shaped structures

- Area coded sine grating:
MTF-Measurement by Imaging Gratings

- Realization by a rotating radial grating
  Spatial frequency depends on azimuthal angle
MTF Measurement by Pupil Autocorrelation

- Basis: Duffieux integral

\[ H_{OTF}(v_x, v_y) = \frac{\int \int P(x_p + \frac{\lambda f' v_x}{2}, y_p + \frac{\lambda f' v_y}{2}) \cdot P^*(x_p - \frac{\lambda f' v_x}{2}, y_p - \frac{\lambda f' v_y}{2}) \, dx_p \, dy_p}{\int \int |P(x_p, y_p)|^2 \, dx_p \, dy_p} \]
Sources of Errors in MTF Measurements

Typical shortcomings of MTF measurements:

1. Mechanical tolerances of the movable parts of the setup like line scan, rotatable edges and alignment errors

2. Application of precise correction factors for finite size slits

3. Truncation errors of the finite lengths structures of the object

4. Calibration of the spatial frequency variable, in particular for finite fields of view with projection changes of lengths and pattern widths

5. Poorly known residual aberrations of auxiliary optical components

6. Use of incorrect spectral and coherence constraints of the illumination

7. Shortcomings of sensor performance

8. Perturbing glare and stray light