If an obscuration-free Schiefspiegler mirror system is considered, the problem of the astigmatism at the oblique mirrors is the major concern. This primary aberration can be corrected by different means:

1. Selecting an appropriate combination of mirror curvatures and incidence angles with spherical mirrors only to sum up to zero corresponding to the Coddington formula.
2. Making every mirror toric to avoid any astigmatism at every surface
3. Correcting the errors with the last mirror in total. This provokes larger intermediate aberrations and eventually higher order aberrations.

These three options are discussed and compared in this exercise.

a) Establish a 3-mirror system with the radii -700, -100, -300 mm of the mirrors. The air distances in between are both 200. Find the optimal image distance for a wavelength of 1 \( \mu \text{m} \) and an incoming collimated beam diameter of 50 mm. The ray bendings at the mirrors are 22°, -30° and 20°. Calculate the astigmatism of the system. Is there a significant contribution of higher order?

b) Now change the bending at the second mirror only and correct the astigmatism and defocus terms of Zernike (c4, c5) by allowing to change also the final image distance. Can the astigmatism be corrected? What are the main residual aberrations? Calculate the astigmatism also between the mirrors and indicate the compensation effect.

c) The second approach is a step by step correction of the astigmatism after every mirror by using a biconic shape with toric surface. What are the final residual aberrations?

d) Finally make the last mirror M3 astigmatic to compensate for the effects of the previous mirrors.

Solution:

a)
The astigmatism is -55 waves and is only primary.

\[
\begin{align*}
2 & 1 & 5.12168302 : 1 \\
2 & 2 & 0.00000000 : (p) \ast \cos (A) \\
2 & 3 & -3.61752902 : (p) \ast \sin (A) \\
2 & 4 & 5.09630545 : (2p^2 - 1) \\
2 & 5 & -54.65505506 : (p^{2}) \ast \cos (2A) \\
2 & 6 & 0.00000000 : (p^2) \ast \sin (2A) \\
2 & 7 & 0.00000000 : (3p^2 - 2) p \ast \cos \theta \\
2 & 8 & -1.51999788 : (3p^2 - 2) p \ast \sin \theta \\
2 & 9 & -0.02499298 : (6p^4 - 6p^2 + 1) \\
2 & 10 & 0.00000000 : (p^3) \ast \cos (3A) \\
2 & 11 & 0.67529873 : (p^3) \ast \sin (3A) \\
2 & 12 & 0.09837220 : (4p^2 - 3) \ast \cos (2A) \\
2 & 13 & 0.00000000 : (4p^2 - 3) \ast \sin (2A)
\end{align*}
\]

b) The astigmatism can be corrected.

By a universal plot, the feasible ranges of the bending can be seen.
The values $+23^\circ$ or $-23^\circ$ seems to make sense. Both solutions are possible. In any case we have the same correction for positive or negative bendings, because the cos is even. But the obscuration can be different. Here luckily both solutions work:

The astigmatism is
1. $-16.9\ \lambda$ after the M1
2. $+58.7\ \lambda$ after M2, Therefore M2 contributes $+75.6\ \lambda$
3. zero after M3, therefore M3 contributes $-58.7\ \lambda$

The largest residual aberration is coma with c8 = $-3.83\ \lambda$. The c11 = 0.7 $\lambda$ is the largest higher order aberration here.
The following figure shows the grid distortion on mirror M3 due to astigmatism and coma:

c) First we are looking for the mirror M1:
The radii are -712.567 and -686.6265 mm.
Secondly, the mirror M2 is made biconic with optimal shape.

Finally, the mirror M3 is corrected too. Here also z4 is required to be zero for a final focusing.
Now there is nearly no astigmatism between the mirrors. The residual aberration of coma is reduced to $-2.18 \lambda$ in comparison to the first scheme. All higher orders are neglectable, in particular also c11.
The result is better than those before, the residual coma is -2.07 \lambda. But as expected, the higher order (c11, c12) are larger here due to the induced aberrations.

### Exercise 3-2: Scheimpflug System with Freeforms

A Scheimpflug system is plane symmetric and has no symmetry in the meridional plane. Therefore a freeform can be used to improve the uniformity of the performance over the field of view.

a) Establish a Scheimpflug system with 3 lenses and the following fixed data:

- focal length \( f = 50 \) mm
- numerical aperture: \( NA = 0.05 \)
- field size 15° in every coordinate direction
- wavelength 589 nm
- object distance 40 mm, tilted by y-tangent = 0.7 (surface type: tilted)
- first lens \( L_1 \): SK4, thickness 2 mm (positive)
- stop 10 mm after first lens
- second lens BAF4, thickness 2 mm, 1 mm after stop (negative)
- third lens SK4, thickness 5 mm, 15 mm behind second lens (positive)

Optimize the image distance, the best image plane tilt and the lens bendings for the 5 field points. Is the performance diffraction limited? What is the change of uniformity for the selected field points?

b) Define the last surface to be a Zernike freeform surface and reoptimize the system. What is the result? Show the shape of the deviation from the sphere of the new surface.

**Solution:**

```plaintext
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**Solution:**

```
The system is not diffraction limited. The spot sizes are changing from 2.2 times the Airy size to the factor 3.5 times the Airy. Therefore the performance is changing by 60%.

b) After defining the last surface to be freeform, the system data looks as follows. As normalization radius, 12 mm are used for the Zernikes. All 36 Polynomials are used.

to get a reasonable layout, the image plane is moved in -y direction by 10 mm.
The performance of the spot is now diffraction limited at the selected field points. In the corner, the performance is more degraded.

Shape of the surface without the circular part:
We see a small anamorphic shape with some differences in the vertical direction at the boundary.
Exercise 3-3: Pair of Thick Meniscus Lenses

Thick meniscus lenses are useful to the correction of field flatness.

a) Establish for a collimated beam at 550 nm wavelength and with 10 mm diameter a symmetrical pair of thick meniscus lenses. The material should be BK7 and the thicknesses 10 mm respectively. The stop is located in the center of symmetry with 3 mm distance to both lenses, the field angle is 2°. Optimize the two radii of curvature to get a collimated beam out and to have a Petval radius of +50 mm.

b) Check the Petzval curvature with Zemax. What are the focal lengths of the lenses?

c) Now a stronger Petzval curvature with a radius of 25 mm is required. Calculate this system. What is now the dominating residual aberration in the system? How can this error be removed? What happens, if the spherical correction by an asphere is performed? Is the system now well corrected?

d) Now establish only one thick meniscus lens with a front radius of +10 mm. Select the rear radius to get a afocal setup. Generate an Universal plot to calculate the Petzval curvature as a function of the thickness in the range \( t = 0.5...25 \text{ mm} \).

Solution:

a) The system looks like the following:

<table>
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<tr>
<th>Surf: Type</th>
<th>Comment</th>
<th>Radius</th>
<th>Thickness</th>
<th>Glass</th>
<th>Semi-Diameter</th>
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<tr>
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<td>Infinity</td>
<td></td>
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</tr>
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<td>BK7</td>
<td>5.88557</td>
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<tr>
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<td>12.89230 V</td>
<td>10.00000</td>
<td></td>
<td>6.00000 U</td>
</tr>
<tr>
<td>3*</td>
<td>Standard</td>
<td>9.35880 V</td>
<td>3.00000</td>
<td></td>
<td>6.00000 U</td>
</tr>
<tr>
<td>STO</td>
<td>Standard</td>
<td>Infinity</td>
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<td></td>
<td>6.00000 U</td>
</tr>
<tr>
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<td>Standard</td>
<td>-9.35880 F</td>
<td>10.00000</td>
<td>BK7</td>
<td>6.00000 U</td>
</tr>
<tr>
<td>6</td>
<td>Standard</td>
<td>-12.89230 F</td>
<td>10.00000</td>
<td></td>
<td>5.42710</td>
</tr>
<tr>
<td>INA</td>
<td>Standard</td>
<td>Infinity</td>
<td>-</td>
<td></td>
<td>5.82257</td>
</tr>
</tbody>
</table>
b) The Seidel list gives the Petzval curvature, which is exactly 50 mm. The focal lengths are near to -2000 mm.

c) If the 25 mm Petzval radius is required, the system is strongly curved.

From the Seidel diagram and the Zernike coefficients it can be seen, that spherical aberration is the dominating error.
The spherical aberration can be corrected by making one of the surfaces aspherical. If the last surface is selected, we get the following result by selecting an afocal angle criterion.
Due to the effect of the asphere, the spherical aberration is corrected, but now coma is strongly growing. Astigmatism is also not small and not well corrected too.

d) The system looks like the following figures, the solve guarantees the parallel marginal ray path.

For the universal plot, the operand PETC as the Petzval curvature is set in the merit function. The universal plots shows a grows of the curvature with the thickness as follows: