Lens Design II

Lecture 10: Higher order aberrations
2019-12-18
Herbert Gross
<table>
<thead>
<tr>
<th>Week</th>
<th>Date</th>
<th>Topic</th>
<th>Subtopics</th>
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<tr>
<td>1</td>
<td>16.10</td>
<td>Aberrations and optimization</td>
<td>Repetition</td>
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<tr>
<td>2</td>
<td>23.10</td>
<td>Structural modifications</td>
<td>Zero operands, lens splitting, lens addition, lens removal, material selection</td>
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<td>3</td>
<td>30.10</td>
<td>Aspheres</td>
<td>Correction with aspheres, Forbes approach, optimal location of aspheres, several aspheres</td>
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<td>4</td>
<td>06.11</td>
<td>Freeforms</td>
<td>Freeform surfaces, general aspects, surface description, quality assessment, initial systems</td>
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<td>13.11</td>
<td>Field flattening</td>
<td>Astigmatism and field curvature, thick meniscus, plus-minus pairs, field lenses</td>
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<td>6</td>
<td>20.11</td>
<td>Chromatical correction I</td>
<td>Achromatization, axial versus transversal, glass selection rules, burried surfaces</td>
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<td>7</td>
<td>27.11</td>
<td>Chromatical correction II</td>
<td>Secondary spectrum, apochromatic correction, aplanatic achromates, spherochromatism</td>
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<tr>
<td>8</td>
<td>04.12</td>
<td>Special correction topics I</td>
<td>Symmetry, wide field systems, stop position, vignetting</td>
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<td>9</td>
<td>11.12</td>
<td>Special correction topics II</td>
<td>Telecentricity, monocentric systems, anamorphicotic lenses, Scheimpflug systems</td>
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<td>10</td>
<td>18.12</td>
<td>Higher order aberrations</td>
<td>High NA systems, broken achromates, induced aberrations</td>
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<td>11</td>
<td>08.01</td>
<td>Further topics</td>
<td>Sensitivity, scan systems, eyepieces</td>
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<td>12</td>
<td>15.01</td>
<td>Mirror systems</td>
<td>special aspects, double passes, catadioptric systems</td>
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<td>13</td>
<td>22.01</td>
<td>Zoom systems</td>
<td>Mechanical compensation, optical compensation</td>
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<td>14</td>
<td>29.01</td>
<td>Diffractive elements</td>
<td>Color correction, ray equivalent model, straylight, third order aberrations, manufacturing</td>
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<td>15</td>
<td>05.02</td>
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</table>
1. Higher orders
2. Higher orders in an achromate
3. Aplanatic-concentric surfaces
4. Merte surface
5. Aspheres
6. High-NA systems
7. Induced aberrations
Paraxial approximation

- Taylor expansion of the sin-function
- Definition of allowed error $10^{-4}$
- Deviation of the various approximations:
  - linear: $5^\circ$
  - cubic: $24^\circ$
  - 5th order: $54.2^\circ$
Contribution of various orders of the sin-expansion

\[ \sin i = i - \frac{i^3}{6} + \frac{i^5}{120} - \frac{i^7}{5040} + \frac{i^9}{362880} - \ldots \]
Higher Order Aberrations

- Relevance of higher order expansion terms
- Nearly perfect geometrical imaging possible in the special cases:
  1. small aperture
  2. small field

![Diagram showing the relationship between field size, aperture size, and order of aberrations.](image-url)
Higher Order Aberrations

- Expansion approach for aberrations: cartesian product of invariants of rotational symmetry
  \[
  u = \frac{x^2 + y^2}{2}, \quad v = x \cdot x_p + y \cdot y_p, \quad w = \frac{x_p^2 + y_p^2}{2}
  \]

- Third order aberrations
  exponent sum 4
  \[
  W_{sph} = S \cdot (x_p^2 + y_p^2)^2
  \]
  \[
  W_{coma} = C \cdot y \cdot y_p \cdot (x_p^2 + y_p^2)
  \]
  \[
  W_{ast} = A \cdot y^2 \cdot y_p^2
  \]

- Fifth order aberrations
  exponent sum 6
  \[
  W_{sph} = S_{sph zone} \cdot (x_p^2 + y_p^2)^3
  \]
  \[
  W_{coma} = C_{linear} \cdot y \cdot y_p \cdot (x_p^2 + y_p^2)^2
  \]
  \[
  W_{sph} = S_{sph skew1} \cdot y^2 \cdot (x_p^2 + y_p^2)^2
  \]
  \[
  W_{sph} = S_{sph skew2} \cdot y^2 \cdot y_p^2 \cdot (x_p^2 + y_p^2)
  \]
  \[
  W_{coma} = C_{ell coma1} \cdot y^3 \cdot y_p \cdot (x_p^2 + y_p^2)^6
  \]
  \[
  W_{ptz} = C \cdot y^2 \cdot (x_p^2 + y_p^2)
  \]
  \[
  W_{dist} = D \cdot y^3 \cdot y_p
  \]
  \[
  W_{ast} = A_{ast} \cdot y^4 \cdot y_p^2
  \]
  \[
  W_{ptz} = C_{petz} \cdot y^4 \cdot (x_p^2 + y_p^2)
  \]
  \[
  W_{dist} = D_{dist} \cdot y^5 \cdot y_p
  \]
  \[
  W_{sph} = P \cdot y^6
  \]
## 5th Order Aberrations

<table>
<thead>
<tr>
<th>No</th>
<th>Field power</th>
<th>Pupil power</th>
<th>Azimuthal power</th>
<th>Term</th>
<th>Name</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>6</td>
<td>0</td>
<td>$W_{060} r_p^6$</td>
<td>Secondary spherical aberration</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>5</td>
<td>1</td>
<td>$W_{151} y' r_p^5 \cos \theta$</td>
<td>Secondary coma</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>4</td>
<td>2</td>
<td>$W_{242} y'^2 r_p^4 \cos^2 \theta$</td>
<td>Secondary astigmatism, wing error</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>$W_{333} y^3 r_p^3 \cos^3 \theta$</td>
<td>Trefoil error, arrow error</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>4</td>
<td>0</td>
<td>$W_{240} y'^2 r_p^4$</td>
<td>Skew spherical aberration</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>$W_{331} y^3 r_p^3 \cos \theta$</td>
<td>Skew coma</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>$W_{422} y'^4 r_p^2 \cos^2 \theta$</td>
<td>Skew astigmatism</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>$W_{420} y'^4 r_p^2$</td>
<td>Secondary field curvature</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>$W_{511} y^5 r_p \cos \theta$</td>
<td>Secondary distortion</td>
</tr>
</tbody>
</table>
- Partial correction of residual spherical aberration by 5th order or 5th and 7th order
- Different (alternating) sign of coefficients
- Residual total error significantly smaller
- Large gradients at the edge
- 3rd and 5th order compensation: residual zonal error at \( \frac{1}{\sqrt{2}} = 0.707 \) of the pupil radius

a) 2 orders

b) 3 orders
Higher Order Aberrations

- Sine condition fulfilled: linear coma removed
- Remaining aberrations of higher order:
  1. Oblique tangential spherical
     \[ W = c_3 \cdot y^2 \cdot r_p^2 \cdot y_p^2 \]
     \[ \Delta x' = -\frac{1}{n'u'} c_3 \cdot y^2 \cdot r_p^3 \cdot \sin \varphi_p \cos^2 \varphi_p \]
     \[ \Delta y' = -\frac{1}{n'u'} c_3 \cdot y^2 \cdot r_p^3 \cdot \cos \varphi_p \left(1 + \cos^2 \varphi_p\right) \]
  2. Oblique sagittal spherical
     \[ W = c_2 \cdot y^2 \cdot r_p^4 \]
     \[ \Delta x' = -\frac{1}{n'u'} 4c_1 \cdot y^2 \cdot r_p^3 \cdot \sin \varphi_p \]
     \[ \Delta y' = -\frac{1}{n'u'} c_2 \cdot y^2 \cdot r_p^3 \cdot \cos \varphi_p \]
  3. Elliptical coma
     \[ W = c_4 \cdot y^3 \cdot r_p^2 \cdot y_p + c_5 \cdot y^3 \cdot y_p^3 \]
     \[ \Delta x' = -\frac{1}{n'u'} c_4 \cdot y^3 \cdot r_p^2 \cdot \sin 2\varphi_p \]
     \[ \Delta y' = -\frac{1}{n'u'} \left[ c_4 \cdot y^3 \cdot r_p^2 \left(2 + \cos 2\varphi_p\right) + 3c_5 \cdot y^3 \cdot r_p^2 \cdot \cos^2 \varphi_p \right] \]
Higher Order Aberrations: Achromate, Aspheres

- Splitted achromate

- Aspherical surface

Ref: H. Zügge
Relaxed System

- Example: achromate with cemented/splitted setup
- Equivalent performance
- Inner surfaces of splitted version more sensitive

a) Cemented achromate \( f=100 \text{ mm} \), \( \text{NA}=0.1 \)

b) Splitted achromate \( f=100 \text{ mm} \), \( \text{NA}=0.1 \)

Ref: H. Zügge
- Spherical aberration vanishes for all orders

- Aplanatic lens at high NA:
  effective real NA is higher than paraxial

- Further possible aplanatic lenses
  1. less practical importance
  2. used in microscopic objective front lens

\[
\sin u_{\text{real'}} = 0.894 \\
\sin u_{\text{ideal'}} = 0.707
\]
- Large aperture: start with corrected marginal ray
- Large field: start with corrected chief ray
- Small difference in refractive index
- Growing higher order contributions
Higher Order Aberrations: Merte Surface

- Merte surface:
  - low index step
  - strong bending
  - mainly higher aberrations generated

From: H. Zügge
Microscope Objective Lens Types

- Medium magnification system
  40x0.65

- High NA system 100x0.9
  without field flattening

- High NA system 100x0.9
  with flat field

- Large-working distance
  objective lens 40x0.65
Microscope Objective Lens

- Examples of large-working-distance objective lenses
- Aplanatic-concentric shell-lenses in the front group
- Large diameter of the lens
- Microscope Lens
- Zernike spectrum shows large higher contributions

Ref: H. Zügge
- Aberration expansion: perturbation theory

- Linear independent contributions only in lowest correction order: Surface contributions of Seidel additive

- Higher order aberrations (5th order,...): nonlinear superposition
  - 3rd order generates different ray heights and angles at next surfaces
  - induces aberration of 5th order
  - together with intrinsic surface contribution: complete error

- Separation of intrinsic and induced aberrations: refraction at every surface in the system
Induced Aberrations

- Example spherical aberration
- Induced effects for higher orders
Structuring Color Aberrations

- Dependence on aperture and $\Delta n$-power

<table>
<thead>
<tr>
<th>longitudinal aberration</th>
<th>$\Delta n$ expansion</th>
<th>1st order</th>
<th>2nd order</th>
<th>3rd order</th>
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<tbody>
<tr>
<td>monochrom</td>
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<td></td>
<td></td>
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<tr>
<td>1=$\Delta n^0$</td>
<td>$\Delta n^1$</td>
<td>$\Delta n^2$</td>
<td>$\Delta n^3$</td>
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<tr>
<td>paraxial</td>
<td>$s^{(0)}$</td>
<td>CHL$^{(1)}$</td>
<td>CHL$^{(2)}$</td>
<td>CHL$^{(3)}$</td>
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<td>$r_p^0$</td>
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<td>primary Seidel</td>
<td>$Sph_{3rd}$</td>
<td>sphChr$^{(1)}_{3rd}$</td>
<td>sphChr$^{(2)}_{3rd}$</td>
<td>sphChr$^{(3)}_{3rd}$</td>
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<tr>
<td>$r_p^2$</td>
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<tr>
<td>secondary</td>
<td>$Sph_{5th}$</td>
<td>sphChr$^{(1)}_{5th}$</td>
<td>sphChr$^{(2)}_{5th}$</td>
<td>sphChr$^{(3)}_{5th}$</td>
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<td>$r_p^4$</td>
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<td>tertiary</td>
<td>$Sph_{7th}$</td>
<td>sphChr$^{(1)}_{7th}$</td>
<td>sphChr$^{(2)}_{7th}$</td>
<td>sphChr$^{(3)}_{7th}$</td>
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<tr>
<td>$r_p^6$</td>
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</tbody>
</table>
Surface No. j in the system:
- intermediate imaging with object, image, entrance and exit pupil
- separate calculations with ideal/real perturbed object point
- pupil distortion must be taken into account
Induced Aberrations

- Mathematical formulation:
  1. incoming aberrations form previous surface

\[ W_{\text{entr, } j}(\vec{r}_p) = \sum_{i=1}^{j-1} [W_i^{(3)}(\vec{r}_p) + W_i^{(5)}(\vec{r}_p)] \]

2. transfer into exit pupil surface \( j \)

\[ W_{\text{exit, } j}(\vec{r}_p) = \sum_{i=1}^{j-1} [W_i^{(3)}(\vec{r}_p + \delta \vec{r}_p) + W_i^{(5)}(\vec{r}_p)] + [W_j^{(3)}(\vec{r}_p) + W_j^{(5)}(\vec{r}_p)] \]

3. complete/total aberration

\[ W_{\text{compl, } j}(\vec{r}_p) = W_{\text{exit, } j}(\vec{r}_p) - W_{\text{entr, } j}(\vec{r}_p) \]

\[ = W_j^{(3)}(\vec{r}_p) + W_j^{(5)}(\vec{r}_p) + \sum_{i=1}^{j-1} W_i^{(3)}(\delta \vec{r}_p) \]

4. subtraction total/intrinsic: induced aberrations

\[ W_{\text{induc, } j}(\vec{r}_p) = \sum_{i=1}^{j-1} W_i^{(3)}(\delta \vec{r}_p) \]

- Interpretation:
  Induced aberration is generated by pupil distortion together with incoming perturbed 3rd order aberration

- Similar effects obtained for higher orders

- Usually induced aberrations are larger than intrinsic one
Example Gabor telescope
- a lens pre-corrects a spherical mirror to obtain vanishing spherical aberration
- due to the strong ray deviation at the plate, the ray heights at the mirror changes significantly
- as a result, the mirror has induced chromatical aberration, also the intrinsic part is zero by definition

Surface contributions and chromatic difference (Aldi, all orders)
Induced Axial Color Aberration

- **Dialyt achromate:**
  - Induced axial color at 2nd lens
  - Correction by distance $t$

Ref: A. Berner
Induced Aberrations in Endoscopes

- Several relay systems: accumulation of residual aberrations
- Growing induced aberrations