Lens Design I

Lecture 4: Properties of optical systems III
2019-05-02
Herbert Gross
<table>
<thead>
<tr>
<th></th>
<th>Date</th>
<th>Topic</th>
<th>Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.04.</td>
<td>Basics</td>
<td>Introduction, Zemax interface, menus, file handling, preferences, Editors, updates, windows, coordinates, System description, 3D geometry, aperture, field, wavelength</td>
</tr>
<tr>
<td>2</td>
<td>18.04.</td>
<td>Properties of optical systems I</td>
<td>Diameters, stop and pupil, vignetting, layouts, materials, glass catalogs, raytrace, ray fans and sampling, footprints</td>
</tr>
<tr>
<td>3</td>
<td>25.04.</td>
<td>Properties of optical systems II</td>
<td>Types of surfaces, cardinal elements, lens properties, Imaging, magnification, paraxial approximation and modelling, telecentricity, infinity object distance and afocal image, local/global coordinates</td>
</tr>
<tr>
<td>4</td>
<td>02.05.</td>
<td>Properties of optical systems III</td>
<td>Component reversal, system insertion, scaling of systems, aspheres, gratings and diffractive surfaces, gradient media, solves</td>
</tr>
<tr>
<td>5</td>
<td>09.05.</td>
<td>Advanced handling I</td>
<td>Miscellaneous, fold mirror, universal plot, slider, multiconfiguration, lens catalogs</td>
</tr>
<tr>
<td>6</td>
<td>16.05.</td>
<td>Aberrations I</td>
<td>Representation of geometrical aberrations, spot diagram, transverse aberration diagrams, aberration expansions, primary aberrations</td>
</tr>
<tr>
<td>7</td>
<td>23.05.</td>
<td>Aberrations II</td>
<td>Wave aberrations, Zernike polynomials, measurement of quality</td>
</tr>
<tr>
<td>8</td>
<td>06.06.</td>
<td>Aberrations III</td>
<td>Point spread function, optical transfer function</td>
</tr>
<tr>
<td>9</td>
<td>13.06.</td>
<td>Optimization I</td>
<td>Principles of nonlinear optimization, optimization in optical design, general process, optimization in Zemax</td>
</tr>
<tr>
<td>10</td>
<td>20.06.</td>
<td>Optimization II</td>
<td>Initial systems, special issues, sensitivity of variables in optical systems, global optimization methods</td>
</tr>
<tr>
<td>11</td>
<td>27.06.</td>
<td>Advanced handling II</td>
<td>System merging, ray aiming, moving stop, double pass, IO of data, stock lens matching</td>
</tr>
<tr>
<td>12</td>
<td>04.07.</td>
<td>Correction I</td>
<td>Symmetry principle, lens bending, correcting spherical aberration, coma, astigmatism, field curvature, chromatical correction</td>
</tr>
<tr>
<td>13</td>
<td>11.07.</td>
<td>Correction II</td>
<td>Field lenses, stop position influence, retrofocus and telephoto setup, aspheres and higher orders, freeform systems, miscellaneous</td>
</tr>
</tbody>
</table>
1. Solves
2. System changes
3. Aspheres
4. Gratings and diffractive surfaces
5. Gradient media
Modifications and System Setups

System changes:
- Tilt/Decenter Elements
- Reverse Elements
- Scale Lens
- Make Focal
- Add Fold Mirror
- Delete Double Pass
- Local to Global
- Global to Local
- Convert Semi-Diameters to Circular Apertures
- Convert Semi-Diameters to Floating Apertures
- Convert Semi-Diameters to Maximum Apertures
- Remove All Apertures
- Replace Vignetting With Apertures
- Value of the parameter depends on other requirements.
- Pickup of radius/thickness: linear dependence on other system parameters.
- Determined to have fixed:
  - Marginal ray height
  - Chief ray angle
  - Marginal ray normal
  - Chief ray normal
  - Aplanatic surface
  - Element power
  - Concentric surface
  - Concentric radius
  - F number
  - Marginal ray height
  - Chief ray height
  - Edge thickness
  - Optical path difference
  - Position
  - Compensator
  - Center of curvature
  - Pupil position

Solves
Solves

- Examples for solves:
  1. last radius forces given image aperture
  2. get symmetry of system parts
  3. multiple used system parts
  4. moving lenses with constant system length
  5. bending of a lens with constant focal length
  6. non-negative edge thickness of a lens
  7. bending angle of a mirror (i'=i)
  8. decenter/tilt of a component with return
- Open different menus with a click in the corresponding editor cell
- Solves can be chosen individually
- Individual data for every surface in this menu
Aspherical Correction

- Correction of spherical aberration by an asphere

Ref: A. Herkommer
Conic sections

- Explicite surface equation, resolved to \( z \)
  
  Parameters: curvature \( c = 1 / R \)
  
  conic parameter \( \kappa \)

- Influence of \( \kappa \) on the surface shape

\[
z = \frac{c(x^2 + y^2)}{1 + \sqrt{1 - (1 + \kappa)c^2(x^2 + y^2)}}
\]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Surface shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \kappa = -1 )</td>
<td>paraboloid</td>
</tr>
<tr>
<td>( \kappa &lt; -1 )</td>
<td>hyperboloid</td>
</tr>
<tr>
<td>( \kappa = 0 )</td>
<td>sphere</td>
</tr>
<tr>
<td>( \kappa &gt; 0 )</td>
<td>oblate ellipsoid (disc)</td>
</tr>
<tr>
<td>( 0 &gt; \kappa &gt; -1 )</td>
<td>prolate ellipsoid (cigar)</td>
</tr>
</tbody>
</table>

- Relations with axis lengths \( a, b \) of conic sections

\[
\kappa = \left( \frac{a}{b} \right)^2 - 1
\]

\[
c = \frac{b}{a^2}
\]

\[
b = \frac{1}{c(1 + \kappa)}
\]

\[
a = \frac{1}{c \sqrt{1 + \kappa}}
\]
Aspherical shape of conic sections

- Conic aspherical surface
- Variation of the conical parameter $\kappa$

\[
z = \frac{cy^2}{1 + \sqrt{1 - (1 + \kappa) \cdot c^2 y^2}}
\]
Parabolic mirror

Equation
\[ z = \frac{c y^2}{1 + \sqrt{1 - (1 + \kappa) y^2 c^2}} \]

\( c \) : curvature \( 1/\text{R}_s \)
\( \kappa \) : eccentricity \( ( = -1 ) \)

Radii of curvature:
\[ R_{\text{tan}} = \text{R}_s \cdot \sqrt{1 + \left( \frac{y}{\text{R}_s} \right)^2} \]
\[ R_{\text{tan}} = \text{R}_s \cdot \left[ 1 + \left( \frac{y}{\text{R}_s} \right)^2 \right]^{\frac{3}{2}} \]
Ellipsoid mirror

Equation

\[ z = \frac{cy^2}{1 + \sqrt{1 - (1 + \kappa)y^2c^2}} \]

c: curvature 1/R

\(\kappa\): Eccentricity
Sag of a surface

- Sag $z$ at height $y$ for a spherical surface:

$$z = r - \sqrt{r^2 - y^2}$$

- Paraxial approximation: quadratic term

$$z_p \approx \frac{y^2}{2r}$$
Polynomial Aspherical Surface
Standard rotational-symmetric description

- Basic form of a conic section superimposed by a Taylor expansion of $z$

$$z(r) = \frac{cr^2}{1+\sqrt{1-(1+\kappa)c^2r^2}} + \sum_{m=0}^{M} a_m r^{2m+4}$$

$r$ ... radial distance to optical axis

$$r = \sqrt{x^2 + y^2}$$

c  ... curvature

$k$  ... conic constant

$a_m$  ... aApherical coefficients

Ref: K. Uhlendorf
Surface properties and settings

- Setting of surface properties
Important Surface Types

- Special surface types
- Data in Lens Data Editor or in Extra Data Editor
- Gradient media are described as 'special surfaces'
- Diffractive / micro structured surfaces described by simple ray tracing model in one order

- Standard spherical and conic sections
- Even asphere classical asphere
- Paraxial ideal lens
- Paraxial XY ideal toric lens
- Coordinate break change of coordinate system
- Diffraction grating line grating
- Gradient 1 gradient medium
- Toroidal cylindrical lens
- Zernike Fringe sag surface as superposition of Zernike functions
- Extended polynomial generalized asphere
- Black Box Lens hidden system, from vendors
- ABCD paraxial segment
Surface Analysis in Zemax

- Analysis of surfaces
Surface Analysis in Zemax

- Analysis of surface sag

![Diagram of surface analysis in Zemax](image.png)
Surface Analysis in Zemax

- Analysis of surface curvature

![Surface Curvature](image1)

- ![Surface Curvature Cross Section](image2)
Surface Analysis in Zemax

- Analysis of freeform surfaces
Maximum intensity: constructive interference of the contributions of all periods

Grating equation

\[ g \cdot (\sin \theta - \sin \theta_o) = m \cdot \lambda \]
Ideal diffraction grating:
- Monochromatic incident collimated beam is decomposed into discrete sharp diffraction orders.
- Constructive interference of the contributions of all periodic cells.
- Only two orders for sinusoidal.

\( g = \frac{1}{s} \)

\( g \) - grating constant

incident collimated light

+4.
+3.
+2.
+1.
0.
-1.
-2.
-3.
-4.

diffraction orders
Finite width of real grating orders

- Interference function of a finite number \( N \) of periods

- Finite width of every order depends on \( N \)

- Sharp order direction only in the limit of \( N \to \infty \)

\[
I = \frac{\sin^2 \left( \frac{\pi \cdot g \cdot N \cdot \sin \theta}{\lambda} \right)}{\sin^2 \left( \frac{\pi \cdot g \cdot \sin \theta}{\lambda} \right)}
\]

\[
\sin \frac{\theta_{1/2}}{2} = \frac{\lambda}{4g \cdot N}
\]
Diffractive Elements

- Original lens height profile \( h(x) \)
- Wrapping of the lens profile: \( h_{\text{red}}(x) \) reduction on maximal height \( h_{2\pi} \)
- Digitalization of the reduced profile: \( h_q(x) \)
Diffracting surfaces

- Surface with grating structure:
  new ray direction follows the grating equation
- Local approximation in the case of space-varying grating width
  \[ \ddot{s}' = \frac{n}{n'} \cdot s + \frac{m \lambda g}{n'd} \cdot \hat{g} + \gamma \cdot \hat{e} \]
- Raytrace only into one desired diffraction order
- Notations:
  \( g \): unit vector perpendicular to grooves
  \( d \): local grating width
  \( m \): diffraction order
  \( e \): unit normal vector of surface
- Applications:
  - diffractive elements
  - line gratings
  - holographic components
Diffracting surfaces

- Local micro-structured surface

- Location of ray bending: macroscopic carrier surface

- Direction of ray bending: local grating micro-structure

- Independent degrees of freedom:
  1. shape of substrate determines the point of the ray bending
  2. local grating constant determines the direction of the bended ray
Diffraction grating
Classical grating with straight lines
Parameters: LP/mm, diffraction order
Substrate can be curved, lines are straight in the local coordinate system on the surface

Elliptical grating 1:
Similar, but grooves can be curved for projection onto x-y-plane,
Substrate can be aspheric

Elliptical grating 2:
Similar to 1, but curved lines defined by intersection of planes with asphere

Binary1
Substrate rotational symmetric asphere

\[ z = \frac{cr^2}{1 + \sqrt{1 - (1 + k)c^2 r^2}} + \alpha_1 r^2 + \alpha_2 r^4 + \alpha_3 r^6 + \alpha_4 r^8 + \alpha_5 r^{10} + \alpha_6 r^{12} + \alpha_7 r^{14} + \alpha_8 r^{16}, \]

Phase of binary element: extended polynomial, scaled on normalization radius in radiant

\[ \Phi = M \sum_{i=1}^{N} A_i E_i(x, y) \]
- Binary2
  Similar to 1, but phase only circular symmetric

- Binary3
  Substrate and phase circular symmetric
  Two different data sets on two ring zones

- Binary4
  Similar to 3, but several zones possible

\[ \Phi = M \sum_{i=1}^{N} A_i p^{2i} \]

\[ z_1 = \frac{c_1 r^2}{1 + \sqrt{1 - (1 + k_1)c_1^2 r^2}} + \sum_{i=1}^{N} \alpha_{1i} r^{2i}, \text{for } r \leq A_1, \]

\[ z_2 = z_o + \frac{c_2 r^2}{1 + \sqrt{1 - (1 + k_2)c_2^2 r^2}} + \sum_{i=1}^{N} \alpha_{2i} r^{2i}, \text{for } r > A_1, \]

\[ \Phi_1 = M_1 \sum_{i=1}^{N} \beta_1 r_{1i}^{2i}, \text{and } \rho_1 = \frac{r}{A_1}, \]

\[ \Phi_2 = \delta_0 + M_2 \sum_{i=1}^{N} \beta_2 r_{2i}^{2i}, \text{where } \rho_2 = \frac{r}{A_2}. \]

\[ z_j(r) = \frac{c_j r^2}{1 + \sqrt{1 - (1 + k_j)c_j^2 r^2}} + \sum_{i=1}^{Na} \alpha_{ji} r^{2i} + z_o. \]

\[ \Phi_j = \delta_0 + M_j \sum_{i=1}^{Np} \beta_{ji} r_{ji}^{2i}. \]
Diffraction grating
Classical grating with straight lines
Parameters: LP/mm, diffraction order
Substrate can be curved, lines are straight in the local coordinate system on the surface

Elliptical grating 1:
Similar, but grooves can be curved for projection onto x-y-plane,
Substrate can be aspheric

Elliptical grating 2:
Similar to 1, but curved lines defined by intersection of planes with asphere

Binary1
Substrate rotational symmetric asphere

\[ z = \frac{cr^2}{1 + \sqrt{1 - (1 + k)c^2 r^2}} + \alpha_1 r^2 + \alpha_2 r^4 + \alpha_3 r^6 + \alpha_4 r^8 + \alpha_5 r^{10} + \alpha_6 r^{12} + \alpha_7 r^{14} + \alpha_8 r^{16}, \]

Phase of binary element: extended polynomial, scaled on normalization radius in radiant

\[ \Phi = M \sum_{i=1}^{N} A_i E_i(x, y) \]
- Radial grating
  Grating with circular symmetry and a line spacing, which changes over the radius

- Variable line space grating
  Straight lines but unevenly separated

- Hologram 1

- Hologram 2

- Toroidal hologram

- Optically fabricated hologram
  Defined by corresponding lens systems to generate the interference with residual aberrations

- Toroidal grating
  Cylindrical surface with usual line grating structure

- Extended toroidal grating

\[ d(p) = A_0 + A_1 p^1 + A_2 p^{-1} + A_3 p^2 + A_4 p^{-2} + \ldots , \]
Raytracing in GRIN media

- Ray: in general curved line
- Numerical solution of Eikonal equation
- Step-based Runge-Kutta algorithm
  4th order expansion, adaptive step width
- Large computational times necessary for high accuracy

\[
\frac{d^2 \vec{r}}{dt^2} = n \cdot \nabla n = \vec{D} = \left( \begin{array}{c}
\frac{\partial n}{\partial x} \\
\frac{\partial n}{\partial y} \\
\frac{\partial n}{\partial z}
\end{array} \right)
\]
Description of GRIN media

- Analytical description of grin media by Taylor expansions of the function $n(x,y,z)$

$\n = n_{o,\lambda} + c_1 h + c_2 h^2 + c_3 h^4 + c_4 h^6 + c_5 h^8 + c_6 z + c_7 z^2 + c_8 z^3 + c_9 z^4$

$+ c_{10} x + c_{11} x^2 + c_{12} x^3 + c_{13} y + c_{14} y^2 + c_{15} y^3$

- Circular symmetry, nested expansion with mixed terms

$\n = n_{o,\lambda} + c_1 h^2 + c_2 h^4 + c_3 h^6 + c_4 h^8 + z \left( c_5 + c_6 h^2 + c_7 h^4 + c_8 h^6 + c_9 h^8 \right)$

$+ z^2 \left( c_{10} + c_{11} h^2 + c_{12} h^4 + c_{13} h^6 + c_{14} h^8 \right) + z^3 \left( c_{15} + c_{16} h^2 + c_{17} h^4 + c_{18} h^6 + c_{19} h^8 \right)$

- Circular symmetry only radial

$\n = n_{o,\lambda} \sqrt{1 + c_2 \left( c_1 h \right)^2 + c_3 \left( c_1 h \right)^4 + c_4 \left( c_1 h \right)^6 + c_5 \left( c_1 h \right)^8 + c_6 \left( c_1 h \right)^{10}}$

- Only axial gradients

$\n = n_{o,\lambda} \sqrt{1 + c_2 \left( c_1 z \right)^2 + c_3 \left( c_1 z \right)^4 + c_4 \left( c_1 z \right)^6 + c_5 \left( c_1 z \right)^8}$

- Circular symmetry, separated, wavelength dependent

$\n = n_{o,\lambda} + c_{1,\lambda} h^2 + c_{2,\lambda} h^4 + c_{3,\lambda} h^6 + c_{4,\lambda} h^8 + c_{5,\lambda} z + c_{6,\lambda} z^2 + c_{7,\lambda} z^3$
Curved ray path in inhomogeneous media

Different types of profiles

- Radial gradient rod lens
- Axial gradient rod lens
- Radial and axial gradient rod lens

\[ n(x,y,z) \]

\( n_{\text{entrance}}(y) \)

\( n_{\text{exit}}(y) \)

\[ n_{i} \quad n_{o} \]

\[ n(x,y,z) \]
Collecting radial selfoc lens

- Thick Wood lens with parabolic index profile
- Principal planes at 1/3 and 2/3 of thickness

\[ n(r) = n_0 - n_2 \cdot r^2 \]

- \( n_2 > 0 \) : collecting lens
- \( n_2 < 0 \) : negative lens
Gradient Lenses

- Types of lenses with parabolic profile
  \[ n(r) = n_0 - n_2 \cdot r^2 = n_0 \cdot \left(1 - n_r \cdot r^2\right) \]
  \[ = n_0 \cdot \left(1 - \frac{1}{2} A \cdot r^2\right) \]

- Pitch length
  \[ p = 2\pi \cdot \sqrt{\frac{n_0}{2n_2}} = \frac{2\pi}{\sqrt{2n_r}} \]
Description of Grin Media in Zemax

- **Gradient 1**
  \[ \begin{align*}
  n &= n_0 + n_{r2}r^2 + n_{r1}r, \\
  n^2 &= n_0 + n_{r2}r^2 + n_{r4}r^4 + n_{r6}r^6 + n_{r8}r^8 + n_{r10}r^{10} + n_{r12}r^{12}
  \end{align*} \]

- **Gradient 3**
  \[ \begin{align*}
  n &= n_0 + n_{r2}r^2 + n_{r4}r^4 + n_{r6}r^6 + n_{z1}z + n_{z2}z^2 + n_{z3}z^3
  \end{align*} \]

- **Gradient 4**
  \[ \begin{align*}
  n &= n_0 + n_{x1}x + n_{x2}x^2 + n_{y1}y + n_{y2}y^2 + n_{z1}z + n_{z2}z^2
  \end{align*} \]

- **Gradient 5**
  \[ z = \frac{cr^2}{1 + \sqrt{1 - (1 + k)c^2r^2}} + x\tan(\alpha) + y\tan(\beta) \]

- **Gradient 6**
  with dispersion
  \[ n = n_0 + n_1r^2 + n_2r^4 + n_3r^6 + n_4r^8 \]

- **Gradient 7**
  spherical shells
  \[ n = n_0 + \alpha(r - R) + \beta(r - R)^2, \text{ where} \]
  \[ r = \frac{R}{|R|} \sqrt{x^2 + y^2 + (R - z)^2} \]
Description of Grin Media in Zemax

- **GRADIUM**

\[ n = \sum_{i=0}^{11} n_i \left( \frac{z + \Delta z}{z_{max}} \right)^i \]

- **Gradient 9**
  iso-index lines as z-surfaces

\[ z = \frac{cr^2}{1 + \sqrt{1 - (1 + k)c^2r^2}} + x\tan(\alpha) + y\tan(\beta) \]

\[ n = n_0 \left[ 1.0 - \frac{A}{2}r^2 \right] \quad A(\lambda) = \left[ K_0 + \frac{K_1}{\lambda^2} + \frac{K_2}{\lambda^4} \right]^2 \]

- **Gradient 10**

\[ n = n_0 + n_{y1}y_a + n_{y2}y_a^2 + n_{y3}y_a^3 + n_{y4}y_a^4 + n_{y5}y_a^5 + n_{y6}y_a^6 \]

- **Grid gradient**
Description of Grin Media in Zemax

- Gradient 1
  \[ n = n_0 + n_{r_2} r^2 + n_{r_1} r, \]

- Gradient 2
  \[ n^2 = n_0 + n_{r_2} r^2 + n_{r_4} r^4 + n_{r_6} r^6 + n_{r_8} r^8 + n_{r_{10}} r^{10} + n_{r_{12}} r^{12} \]

- Gradient 3
  \[ n = n_0 + n_{r_2} r^2 + n_{r_4} r^4 + n_{r_6} r^6 + n_{z_1} z + n_{z_2} z^2 + n_{z_3} z^3 \]

- Gradient 4
  \[ n = n_0 + n_{x_1} x + n_{x_2} x^2 + n_{y_1} y + n_{y_2} y^2 + n_{z_1} z + n_{z_2} z^2 \]

- Gradient 5
  \[ z = \frac{c r^2}{1 + \sqrt{1 - (1 + k)c^2 r^2}} + x \tan(\alpha) + y \tan(\beta) \]

- Gradient 6
  with dispersion
  \[ n = n_0 + n_{1} r^2 + n_{2} r^4 + n_{3} r^6 + n_{4} r^8 \]
  \[ n_x = A_x + B_x \frac{\lambda^2}{\lambda} + C_x \frac{\lambda^2}{\lambda^2} + D_x \frac{\lambda^2}{\lambda^4} \]

- Gradient 7
  spherical shells
  \[ n = n_0 + \alpha(r - R) + \beta(r - R)^2, \text{ where} \]
  \[ r = \frac{R}{|R|} \sqrt{x^2 + y^2 + (R - z)^2} \]
Description of Grin Media in Zemax

- **GRADIUM**

\[ n = \sum_{i=0}^{11} n_i \left( \frac{z + \Delta z}{z_{max}} \right)^i \]

- **Gradient 9**
  iso-index lines as z-surfaces

\[ z = \frac{cr^2}{1 + \sqrt{1 - (1 + k)c^2r^2}} + x\tan(\alpha) + y\tan(\beta) \]

\[ n = n_0 \left[ 1.0 - \frac{A}{2} r^2 \right] \]

\[ A(\lambda) = \left[ K_0 + \frac{K_1}{\lambda^2} + \frac{K_2}{\lambda^4} \right]^2 \]

- **Gradient 10**

\[ n = n_0 + n_{y1}y_a + n_{y2}y_a^2 + n_{y3}y_a^3 + n_{y4}y_a^4 + n_{y5}y_a^5 + n_{y6}y_a^6 \]

- **Grid gradient**