4.4.5 Photonic Crystal Slabs

- Structures with 2D periodicity and a finite thickness
- The finite thickness introduces qualitatively new behaviour as compared to ideal 2D PhC
- PhC slabs can confine light by a combination of index guiding and band gaps.
- Examples: A rod slab \((r = 0.2a, \ d = 2a)\) and a hole slab \((r = 0.3a, \ d = 0.6a)\)
• The in-plane wave vector $\mathbf{k}_\parallel = (k_x, k_y)$ is conserved due to periodicity, but the vertical wave vector $k_z$ is not conserved.

• The system is invariant under reflections through the $z = 0$ plane $\rightarrow$ Modes will be either TE-like or TM-like

• In the projected band structure, the extended modes form a light cone for $\omega > c|\mathbf{k}_\parallel|$. Below the light cone appear discrete guided bands. The respective and gaps are incomplete.
• The size of the gap depends on the slab thickness
• It is ideal, when the fundamental index-guided mode is well confined, but no higher order modes are yet supported → the ideal thickness is approximately \( \frac{\lambda}{2n_{\text{eff}}} \)
• We can form a waveguide by reducing the radius of all rods in a particular row.

• Fabrication example (typically EBL with proximity correction):

• Removing a row of rods completely, as we did for 2D PhCs, the mode would not be guided, as it would not be vertically confined by index guiding

• Generally, PhC slabs cannot confine light to the low-index medium
$E_z$-field cross sections of the reduced-radius line defect for $r = 0.14a$ and $\frac{k_x a}{2\pi} = 0.42$
• The rod slab always requires a substrate.
• The influence of the substrate: It breaks the $z = 0$ reflection symmetry $\rightarrow$ TE and TM modes can couple and destroy any TM/TE-only bandgap.
• Modes that were confined in such a gap become leaky and radiate within the plane of periodicity.
• The polarization mixing can be reduced by etching the substrate together with the rods.
• Alternatively, the symmetry can be restored by adding a superstrate on top of the slab with similar optical properties as the substrate.
• In the hole slab, on the other hand, we can form a waveguide by removing a row of holes.

• This waveguide has a series of guided modes.

• Symmetry in the $y = 0$ plane means we can classify these modes as even or odd under reflection in this plane.
• Fabrication example:

A free-standing membrane is achieved by underetching

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• $H_z$ field cross sections for the missing-hole waveguide at $k_x = \frac{\pi}{a}$, only (c) is shown at $k_x = 0.32 \frac{\pi}{a}$

• If two defect modes couple to each other at an intersection point, this leads to an anti-crossing with drastic changes in the field pattern with $k_x$.
• If bends are introduced, the discrete translational symmetry in waveguide direction is broken and the wavevector along the waveguide is no longer conserved.

• Due to the presence of the bandgap there can be no losses into the slab

• However, light can be reflected. Refelctions can be minimized by design

• Point defects in PhC slabs can localize light at the defect site
• However, due to the presence of the light cone, these modes are always leaky $\rightarrow$ resonances
• Example: Reduction of the $\varepsilon$ of a rod and its nearest neighbours from 12 to 9:

$Q \approx 13000$

$Q$ is reduced drastically by the presence of a substrate
• To maximize $Q$ for an incomplete gap, one can trade off localization for loss: The more delocalized the mode is in space, the more localized its Fourier transform can be, allowing most of the Fourier components to lie inside the light cone and not radiate.

Example: radiative lifetime of resonant cavity modes in a rod slab formed by decreasing the $\varepsilon$ of the rods at the defect sites.
• Another strategy to maximize $Q$ is by cancelling the dominant component of the radiation by a forced balance of scattered fields with opposite signs, e.g. by using the contributions from lattice sites adjacent to the defect.

• The advantage of this technique is that the mode volume $V$ can be kept small upon increasing $Q$.

• Reason for striving for high $Q$ and small $V$ : Spontaneous emission enhancement (compare nanoantennas):
  **Purcell factor:**

\[
F_0 = \frac{\gamma}{\gamma_0} \propto \frac{Q}{V}
\]

• Definition of the mode volume depends on the problem.

• For spontaneous emission in PhC cavities $V = \int (d^3r \varepsilon |E|^2) / \text{max}(\varepsilon |E|^2)$
4.4.6 Photonic Crystal Fibres

- propagation perpendicular to the periodic plane
- infinite homogeneous extension in the propagation direction
- confinement in air is possible
- Presentation by Martin Kafula
4.5 3D Photonic Crystals

• Periodicity along three different axes
• A complete photonic bandgap for all propagation directions can be achieved
• Light can be localized in all three dimensions
• Representation of several 3D lattices in a cubic supercell with lattice constant $a$.

Blue: simple cubic
Add dark red: face-dentred cubic
Add also pink: diamond lattice
4.5.1 3D Photonic Crystals in nature

In animals: The butterflies Morpho Rhetenor und Parides Sesostris

In geology: natural opals
Silica spheres arranged hexagonally in layers

Taken from: J.B. Pendry, Current Science 76, 1311 (1999)
4.5.2 Complete Bandgap materials

- A variety of three-dimensional structures with a complete photonic bandgap (cPBG) has been discovered.

- Some examples:
  - Woodpile structure
  - Inverse Woodpile
  - Slanted pores
  - Inverse Opal
  - Square spirals
  - Circular spirals
Example: The woodpile structure

- Stacking sequence ABCD
- Face-centred cubic lattice for $\frac{c}{a} = \sqrt{2}$
- Formation of a bandgap possible if the index contrast exceeds 1.9.
• Band structure of a silicon woodpile structure for $d = 0.25 \alpha$, $\epsilon = 11.76$, aspect ratio $\frac{h}{d} = 2$

• 14.4% complete photonic bandgap
Introduction of defects in woodpiles

- Defect plane
- Line defect
- Point defect
- Point defect

Dielectric defects

- Air defect
A point defect in a woodpile structure: Calculated bandstructure, electric field profile and transmission, Defect $Q \approx 30000$
A line defect in a woodpile structure: Calculated bandstructure and transmission
4.5.3 Fabrication of 3D PhCs

• In theory, 3D PhCs have outstanding properties. For example:
  • The $Q$-factor of a point defect can in principle be made arbitrarily large by increasing the number of stacked layers
  • Light can be localized in three dimensions in air by a point air defect
  • Light can in principle be guided along sequences of point defects with complex trajectories and small bend radii thanks to the complete PBG.
  • Spontaneous emission can be completely suppressed in the band gap.
  • However, in reality their performance is drastically limited by the quality of the fabrication, as 3D nanofabrication is extremely challenging, in particular for high-index materials.
Example 1: Stacking of 2D slabs

• First, individual layers of the woodpile structure are fabricated by electron beam lithography and reactive ion etching.

• Then the layers are stacked onto each other using a micromanipulator.

• Highest quality 3D Photonic Crystals
• Defect placement possible


• Extreme requirements for alignment
• Extremely slow (up to months for one sample)
Example 2: Deposition by sedimentation

- Rather slow technique
- Resulting crystals are not perfect in quality
- No complete bandgap
Exp. 3: DLW and silicon (double) inversion

- Reminder: Direct laser writing → 3D resolution, 3D writing capability, high flexibility
- Problem: photoresists typically have too low refractive index for a cPBG → transfer the structure to high index material, e.g. silicon ($n \approx 3.5$)

$\Rightarrow$ high index contrast $\Rightarrow$ 3D PBG @ optical wavelengths

Silicon Woodpile – Fabrication Example

- Oblique-view electron micrograph of silicon double inverted woodpile structure (FIB cut)
- Experimental parameters:
  \( a = 566 \text{ nm}, \ c = 1.15*a, \ d = 0.365*a, \ \chi = 1.46 \)

Silicon Woodpile – Band Structure Calculation

• Calculation parameters:
  a = 566.3 nm, c = 1.15*a, d = 0.365*a, χ = 1.46, ε = 9.9

→ 6.9% PBG including 1.55 μm