Imaging and Aberration Theory

Lecture 7: Distortion and coma
2018-11-29
Herbert Gross
<table>
<thead>
<tr>
<th>Week</th>
<th>Date</th>
<th>Topic</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>18.10.</td>
<td>Paraxial imaging</td>
<td>paraxial optics, fundamental laws of geometrical imaging, compound systems</td>
</tr>
<tr>
<td>2</td>
<td>25.10.</td>
<td>Pupils, Fourier optics, Hamiltonian coordinates</td>
<td>pupil definition, basic Fourier relationship, phase space, analogy optics and mechanics, Hamiltonian coordinates</td>
</tr>
<tr>
<td>3</td>
<td>01.11.</td>
<td>Eikonal</td>
<td>Fermat principle, stationary phase, Eikonals, relation rays-waves, geometrical approximation, inhomogeneous media</td>
</tr>
<tr>
<td>4</td>
<td>08.11.</td>
<td>Aberration expansions</td>
<td>single surface, general Taylor expansion, representations, various orders, stop shift formulas</td>
</tr>
<tr>
<td>5</td>
<td>15.11.</td>
<td>Representation of aberrations</td>
<td>different types of representations, fields of application, limitations and pitfalls, measurement of aberrations</td>
</tr>
<tr>
<td>6</td>
<td>22.11.</td>
<td>Spherical aberration</td>
<td>phenomenology, sph-free surfaces, skew spherical, correction of sph, aspherical surfaces, higher orders</td>
</tr>
<tr>
<td>7</td>
<td>29.11.</td>
<td>Distortion and coma</td>
<td>phenomenology, relation to sine condition, aplanatic systems, effect of stop position, various topics, correction options</td>
</tr>
<tr>
<td>8</td>
<td>06.12.</td>
<td>Astigmatism and curvature</td>
<td>phenomenology, Coddington equations, Petzval law, correction options</td>
</tr>
<tr>
<td>9</td>
<td>13.12.</td>
<td>Chromatical aberrations</td>
<td>Dispersion, axial chromatical aberration, transverse chromatical aberration, spherochromatism, secondary spectrum</td>
</tr>
<tr>
<td>10</td>
<td>20.12.</td>
<td>Sine condition, aplanatism and isoplanatism</td>
<td>Sine condition, isoplanatism, relation to coma and shift invariance, pupil aberrations, Herschel condition, relation to Fourier optics</td>
</tr>
<tr>
<td>11</td>
<td>10.01.</td>
<td>Wave aberrations</td>
<td>definition, various expansion forms, propagation of wave aberrations</td>
</tr>
<tr>
<td>12</td>
<td>17.01.</td>
<td>Zernike polynomials</td>
<td>special expansion for circular symmetry, problems, calculation, optimal balancing, influence of normalization, measurement</td>
</tr>
<tr>
<td>13</td>
<td>24.01.</td>
<td>Point spread function</td>
<td>ideal psf, psf with aberrations, Strehl ratio</td>
</tr>
<tr>
<td>14</td>
<td>31.01.</td>
<td>Transfer function</td>
<td>transfer function, resolution and contrast</td>
</tr>
<tr>
<td>15</td>
<td>07.02.</td>
<td>Additional topics</td>
<td>Vectorial aberrations, generalized surface contributions, Aldis theorem, intrinsic and induced aberrations, reversibility</td>
</tr>
</tbody>
</table>
1. Geometry of coma spot
2. Coma-dependence on lens bending, stop position and spherical aberration
3. Point spread function with coma
4. Distortion
5. Examples
Ray Caustic of Coma

- A sagittal ray fan forms a groove-like surface in the image space.

- Tangential ray fan for coma: caustic.
Building of Coma Spot

- Coma aberration: for oblique bundles and finite aperture due to asymmetry
- Special problem: coma grows linear with field size $y$
- Systems with large field of view: coma hard to correct
- Relation of spot circles and pupil zones as shown
Coma deviation, elimination of the azimuthal dependence: circle equation

- Diameter of the circle and position variation with \( r_p^2 \)
  - Every zone of the circle generates a circle in the image plane

- All circles together form a comet-like shape

- The chief ray intersection point is at the tip of the cone

- The transverse extension of the cone shape has a ratio of 2:3
  - the meridional extension is enlarged and gives a poorer resolution
Coma

- Ray trace properties
- Double speed azimuthal growth between pupil and image
- Sagittal coma smaller than tangential coma

\[ \Delta y_{\text{tan}} = 3 \cdot \Delta y_{\text{sag}} \]
Wavefront and Spot for Coma

- Coma Seidel transverse aberrations

\[
\Delta y' = S' \cdot r_p^3 \cos \theta_p + C' \cdot y' \cdot r_p^{12} (2 + \cos 2\theta_p) + (2A' + P') \cdot y' \cdot r_p^{12} \cos \theta_p + D' \cdot y_p^{13}
\]

\[
\Delta x' = S' \cdot r_p^3 \sin \theta_p + C' \cdot y' \cdot r_p^{12} \sin 2\theta_p + P' \cdot y' \cdot r_p^{12} \sin \theta_p
\]

- Wavefront for coma

\[
W = r_p^3 \cos \theta_p = x_p^2 y_p + y_p^3
\]

with

\[
x_p = r_p \sin \theta_p \quad , \quad y_p = r_p \cos \theta_p
\]

- Relationship

\[
\frac{\partial W}{\partial y_p} = -\frac{\Delta y'}{R}, \quad \frac{\partial W}{\partial x_p} = -\frac{\Delta x'}{R}
\]

- Here

\[
\Delta x' = -R \frac{\partial W}{\partial x_p} = -2R \cdot (x_p y_p) = -2R r_p^2 \sin \theta_p \cos \theta_p = -R r_p^2 \sin 2\theta_p
\]

\[
\Delta y' = -R \frac{\partial W}{\partial y_p} = -R \cdot (x_p^2 + 3y_p^2) = -R r_p^2 \left( 2 + \cos 2\theta_p \right)
\]
Tangential and Sagittal Coma

- 2 terms of tangential transverse aberration:
  - Sagittal coma depends on \( x_p \), describes the asymmetry
  - Tangential coma depends on \( y_p \), corresponds to spherical aberration under skew conditions larger by a factor of 3

- Only asymmetry removed with sine condition: sagittal coma vanishes
- Occurrence of coma: skew chief ray and finite aperture
- Asymmetry between upper and lower coma ray
- Bended plane of sagittal coma rays
Coma

- Typical representations of coma
- Cubic curve in wavefront cross section
- Quadratic function in transverse aberrations

Ref: H. Zügge
Tangential vs Sagittal Resolution

- Asymmetry of the PSF
- Coma 3rd creates a 2:3 diameter pattern
- Usually coma oriented towards the axis, then $\text{MTF}_S > \text{MTF}_T$ (lower row)

Coma Orientation

- Orientation of the coma shape: distinction between
  1. outer coma, tip towards optical axis
  2. inner coma, tip outside

- Orientation of the coma spot is always rotating with the azimuthal angle of the considered field point
Bending of a Lens

- Bending a single lens with stop at the lens
- Variation of the primary aberrations
- The stop position is important for the off-axis aberrations
- Typical changes:
  1. coma linear
  2. chromatical magnification linear
  3. spherical aberration quadratically
- Lens with remote stop
- Not all of the aberrations spherical, astigmatism and coma can be corrected by bending simultaneously
- Zero correction for coma and astigmatism possible (depends on stop position)
- Spherical aberration not correctable
Inner and Outer Coma

- Effect of lens bending on coma
- Sign of coma: inner/outer coma

Ref: H. Zügge
Decomposition of coma:
1. part symmetrical around chief ray: skew spherical aberration
\[ \Delta y_{\text{skewsph}} = \frac{\Delta y_{\text{upcom}} + \Delta y_{\text{lowcom}}}{2} \]
2. asymmetrical part: tangential coma
\[ \Delta y_{\text{tangcoma}} = \frac{\Delta y_{\text{upcom}} - \Delta y_{\text{lowcom}}}{2} \]

Skew spherical aberration:
- higher order aberration
- caustic symmetric around chief ray
Geometrical Coma Spot

- Geometrical calculated spot intensity
  - There is a step at the lower circle boundary
  - The peak lies in the apex point
  - The centroid lies at the lower circle boundary
  - The minimal rms radius is

\[
I(x, y) = \begin{cases} 
  I_{\text{Exp}} \cdot a^3 \cdot \frac{1}{2A_c \cdot R} \cdot \frac{1}{\sqrt{x^2 - 3y^2}} & \text{inside largest circle} \\
  I_{\text{Exp}} \cdot a^3 \cdot \frac{1}{A_c \cdot R} \cdot \frac{1}{\sqrt{x^2 - 3y^2}} & \text{else inside coma shape}
\end{cases}
\]

\[
r_{\text{rms}} = \sqrt{\frac{2}{3}} \cdot \frac{R \cdot A_c}{a}
\]
Transverse aberrations in the case of coma and defocus

\[ \Delta \tilde{H} = -\frac{1}{L} \left[ W_{131} r_p^2 \tilde{H} + 2 \cdot (2W_{020} + W_{131} H r_p^2 \cos \theta) \cdot \tilde{r}_p \right] \]

Two deviations:
1st term along field vector
2nd term along pupil vector

Zonal curve for different defocus values: Limacon of Pascal \( H = 2(b + a \cos \theta) \)

Special cases cardio / double circle in focal point

\( \frac{b}{a} = 2.0 \)

\( \frac{b}{a} = 1.5 \)

\( \frac{b}{a} = 1.0 \)

\( \frac{b}{a} = 0.8 \)

\( \frac{b}{a} = 0.4 \)

\( \frac{b}{a} = 0.2 \)

\( \frac{b}{a} = 0.05 \)

\( \frac{b}{a} = 0.0 \)
- PSF with coma
- The 1st diffraction ring is influenced very sensitive

$W_{31} = 0.03 \lambda$
$W_{31} = 0.06 \lambda$
$W_{31} = 0.09 \lambda$
$W_{31} = 0.15 \lambda$
Psf with Coma

\[ I(x) \]

\[ W_{13} = 0.3 \lambda \quad W_{13} = 1.0 \lambda \quad W_{13} = 2.4 \lambda \quad W_{13} = 5.0 \lambda \quad W_{13} = 10.0 \lambda \]

Ref: Francon, Atlas of optical phenomena
- Change of Zernike coma coefficient
  - peak height reduced
  - peak position constant due to tilt component
  - distribution becomes asymmetrical

- Change of Seidel coma coefficient
  - peak height reduced
  - peak position moving
  - distribution becomes asymmetrical
Separation of the peak and the centroid position in a point spread function with coma.
From the energetic point of view coma induces distortion in the image.

- $c_7 = 0.3$
- $c_7 = 0.5$
- $c_7 = 1$

centroid
- Defocus: centroid moves on a straight line (line of sight)
- Peak of intensity moves on a curve (bananicity)
Line of Sight

- Centroid of the psf intensity

- Elementary physical argument: The centroid has to move on a straight line: line of sight

- Wave aberrations with odd order:
  - centroid shifted
  - peak and centroid are no longer coincident

\[ y_s(z) = \frac{2 \cdot z}{D_{Exp}} \cdot \sum_{n=1,3,5,...} \sqrt{2(n + 1)} \cdot c_{n1} \]

\[ x_s(z) = \frac{\iint x \cdot I(x, y, z) dx dy}{\iint I(x, y, z) dx dy} = \frac{1}{P} \cdot \iint x \cdot I(x, y, z) dx dy \]
Higher Order Aberrations

- Sine condition fulfilled:
  linear coma removed
- Remaining aberrations of higher order:
  1. Oblique tangential spherical
     \[ W = c_3 \cdot y^2 \cdot r_p^3 \cdot y_p^2 \]
     \[ \Delta x' = -\frac{1}{n' u'} c_3 \cdot y^2 \cdot r_p^3 \cdot \sin \varphi_p \cos^2 \varphi_p \]
     \[ \Delta y' = -\frac{1}{n' u'} c_3 \cdot y^2 \cdot r_p^3 \cdot \cos \varphi_p \left(1 + \cos^2 \varphi_p \right) \]
  2. Oblique sagittal spherical
     \[ W = c_2 \cdot y^2 \cdot r_p^4 \]
     \[ \Delta x' = -\frac{1}{n' u'} 4c_1 \cdot y^2 \cdot r_p^3 \cdot \sin \varphi_p \]
     \[ \Delta y' = -\frac{1}{n' u'} c_2 \cdot y^2 \cdot r_p^3 \cdot \cos \varphi_p \]
  3. Elliptical coma
     \[ W = c_4 \cdot y^3 \cdot r_p^2 \cdot y_p + c_5 \cdot y^3 \cdot y_p^3 \]
     \[ \Delta x' = -\frac{1}{n' u'} c_4 \cdot y^3 \cdot r_p^2 \cdot \sin 2\varphi_p \]
     \[ \Delta y' = -\frac{1}{n' u'} \left[ c_4 \cdot y^3 \cdot r_p^2 \left(2 + \cos 2\varphi_p \right) + 3c_5 \cdot y^3 \cdot r_p^2 \cdot \cos^2 \varphi_p \right] \]
The lens contribution of coma is given by
if the stop is located at the lens.

Therefore the coma can be corrected by bending the lens.

The optimal bending is given by
and corrects the 3rd order coma completely.

The stop shift equation for coma is given by
with the normalized ratio of the chief ray height
to the marginal ray height.

If the spherical aberration \( S_1 \) is not corrected, there is a natural stop position with vanishing coma.

If the spherical aberration is corrected (for example by an aspheric surface), the coma doesn't change with the stop position.

\[
C_{lens} = \frac{1}{4ns' f^2} \left[ \frac{n+1}{n-1} X - (2n+1)M \right]
\]

\[
X = \frac{(2n+1)(n-1)}{n+1} \cdot M
\]

\[
S_{II}^* = S_{II} + \delta E \cdot S_I
\]

\[
\delta E = \frac{\bar{h}_{new} - \bar{h}_{old}}{h}
\]
Coma Correction: Achromate

- Bending of an achromate
  - optimal choice: small residual spherical aberration
  - remaining coma for finite field size
- Splitting achromate:
  - additional degree of freedom:
  - better total correction possible
  - high sensitivity of thin air space
- Aplanatic glass choice:
  - vanishing coma
- Cases:
  a) simple achromate, sph corrected, with coma
  b) simple achromate, coma corrected by bending, with sph
  c) other glass choice: sph better, coma reversed
  d) splitted achromate: all corrected
  e) aplanatic glass choice: all corrected

Ref: H. Zügge
Coma-free Stop Position

- Example
- The front stop position of a single lens is shifted
- The 3rd order Seidel coefficient as well as the Zernike coefficient vanishes at a certain position of the stop
Combined effect, aspherical case prevents correction

Ref: H. Zügge
Coma Correction: Symmetry Principle

- Perfect coma correction in the case of symmetry
- But magnification $m = -1$ not useful in most practical cases

<table>
<thead>
<tr>
<th>Symmetry principle</th>
<th>Image height: $y' = 19$ mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pupil section:</td>
<td>meridional sagittal</td>
</tr>
<tr>
<td>Transverse Aberration:</td>
<td>$\Delta y'$ 0.5 mm $\Delta y'$ 0.5 mm</td>
</tr>
</tbody>
</table>

(a)

(b)

Ref: H. Zügge
Correction of Coma

Single surface

\[ C = 4 \cdot \frac{R - s' + p'}{s' - R} \cdot S \]

Coma \( C = 0 \)

- if spherical is corrected
- if chief ray is not refracted, stop at the center of curvature

Lens / System

→ Options for Coma correction

- Correct spherical
- Stop shift
- Lens Bending (around the stop)
- Symmetry

Spherical aberration, astigmatism, Coma cancelled by symmetry

Ref: B. Böhme
Image Degradation by Coma

- Imaging of a bar pattern with a coma of $0.4\lambda$ in $x$ and $y$
- Structure size near the diffraction limit
- Asymmetry due to coma seen in comparison of edge slopes
Coma Truncation by Vignetting

without vignettierung

with vignettierung

Ref: H. Zügge
Distortion Example: 10%

- What is the type of degradation of this image?
- Sharpness good everywhere!

Ref: H. Zügge
Distortion Example: 10%

- Image with sharp but bended edges/lines
- No distortion along central directions

Ref: H. Zügge
Distortion

- Distortion: change of magnification over the field
- Corresponds to spherical aberration of the chief ray
- Measurement: relative change of image height

\[ V = \frac{y_{\text{real}} - y_{\text{ideal}}}{y_{\text{ideal}}} \]

- No image point blurr
  only geometrical shape deviation
- Sign of distortion:
  1. \( V < 0 \) : barrel,
     lens with stop in front
  2. \( V > 0 \) : pincushion,
     lens with rear stop
TV Distortion

- Conventional definition of distortion
  \[ V = \frac{\Delta y}{y} \]

- Special definition of TV distortion
  \[ V_{TV} = \frac{\Delta H}{H} \]

- Measure of bending of lines

- Acceptance level strongly depends on kind of objects:
  1. geometrical bars/lines: 1% is still critical
  2. biological samples: 10% is not a problem

- Digital detection with image post processing: un-distorted image can be reconstructed
Distortion

- Purely geometrical deviations without any blur
- Distortion corresponds to spherical aberration of the chief ray
- Important is the location of the stop: defines the chief ray path
- Two primary types with different sign:
  1. barrel, $D < 0$
     - front stop
  2. pincushion, $D > 0$
     - rear stop
- Definition of local magnification changes

\[
D = \frac{y'_{\text{real}} - y'_{\text{ideal}}}{y'_{\text{ideal}}}
\]
Distortion and Stop Position

- Sign of distortion of a single lens depends on stop position
- Ray bending of chief ray determines the distortion

<table>
<thead>
<tr>
<th>Lens</th>
<th>Stop</th>
<th>Distortion</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>positive lens</td>
<td>rear stop</td>
<td>D &gt; 0</td>
<td>Tele lens</td>
</tr>
<tr>
<td>negative lens</td>
<td>front stop</td>
<td>D &gt; 0</td>
<td>Loupe</td>
</tr>
<tr>
<td>positive lens</td>
<td>front stop</td>
<td>D &lt; 0</td>
<td>Retro focus lens</td>
</tr>
<tr>
<td>negative lens</td>
<td>rear stop</td>
<td>D &lt; 0</td>
<td>reversed Binocular</td>
</tr>
</tbody>
</table>

Ref: H. Zügge
Distortion of Higher Order

- Combination of distortion of 3rd and 5th order:
- Bended lines with turning points
- Typical result for corrected/compensated distortion
- Non-symmetrical systems:
  - Generalized distortion types
- Correction complicated
- Distortion occurs, if the magnification depends on the field height $y$
- In the special case of an invariant location $p'$ of the exit pupil: the tangent of the angle of the chief ray should be scaled linear
- Airy tangent condition: necessary but not sufficient condition for distortion correction:
- This corresponds to a corrected angle of the pupil imaging form entrance to exit pupil

\[
\frac{\tan \omega'}{\tan \omega} = \text{const}
\]
Reasons of Distortion

- Second possibility of distortion:
  the pupil imaging suffers from longitudinal spherical aberration
- The location of the exit pupil than depends on the field height
- With the simple relations
  \[ y = p \cdot \tan w, \quad y' = p' \cdot \tan w' \]

we have the general expression
for the magnification

\[ m(y) = \frac{y'}{y} = \frac{p'(y) \cdot \tan w'}{p \cdot \tan w} = \left( \frac{p_o' + \Delta p'(y)}{p} \right) \cdot \tan w' \]

- For vanishing distortion:
  1. the tan-condition is fulfilled (chief ray angle)
  2. the spherical aberration of the pupil imaging is corrected (chief ray intersection point)
Distortion of a Retrofocus System

Retro focus systems:

crystal distortion

Barrel distortion:

negatives front group

stop

positive rear group

barrel distortion

20 %
Keystone distortion

- Tilting object
- Tilted image system
- Principal plane
- Optical axis

\[ \theta \quad \theta' \]
\[ h \quad y \quad s \]
\[ h' \quad y' \quad s' \]
Distortion

- Visual impression of distortion on real images
- Visibility only at straight edges
- Edge through the center are not affected
Fish-Eye-Lens

- Example lens with 210° field of view
- Distortion types
Head Mounted Display

- Commercial system: Zeiss Cinemizer
- Critical performance of distortion due to asymmetry
Head-Up Display

- Refractive 3D-system
- Free-formed prism
- Field dependence of coma, distortion and astigmatism
- One coma nodal point
- Two astigmatism nodal points
Distortion Correction

- Correction the distortion of a retro focus type photographic lens by additional splitted lens

  a) distortion not corrected: 15%

  b) distortion corrected by - + : 2.5%

  c) distortion corrected by + - : 2.5%

Ref: A. Herkommer
Distortion in Spectrometer

- Spatial distortion, keystone: bent exit slit
- Spectral distortion, smile
Pupil Imaging

- **a)** EnP regular
  - Ray weight of source
  - No aiming

- **b)** Pupil regular
  - Correct pupil boundary

- **c)** ExP regular
  - Correct PSF and W calculation

Wavefront sampling
PSF calculation
Apodization