Imaging and Aberration Theory

Lecture 6: Spherical aberration

2019-11-22

Herbert Gross
<table>
<thead>
<tr>
<th>Week</th>
<th>Date</th>
<th>Topic</th>
<th>Details</th>
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<tbody>
<tr>
<td>1</td>
<td>18.10</td>
<td>Paraxial imaging</td>
<td>paraxial optics, fundamental laws of geometrical imaging, compound systems</td>
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<td>Pupils, Fourier optics, Hamiltonian coordinates</td>
<td>pupil definition, basic Fourier relationship, phase space, analogy optics and mechanics, Hamiltonian coordinates</td>
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<td>Eikonal</td>
<td>Fermat principle, stationary phase, Eikonals, relation rays-waves, geometrical approximation, inhomogeneous media</td>
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<td>Aberration expansions</td>
<td>single surface, general Taylor expansion, representations, various orders, stop shift formulas</td>
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<td>5</td>
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<td>Representation of aberrations</td>
<td>different types of representations, fields of application, limitations and pitfalls, measurement of aberrations</td>
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<td>6</td>
<td>22.11</td>
<td>Spherical aberration</td>
<td>phenomenology, sph-free surfaces, skew spherical, correction of sph, aspherical surfaces, higher orders</td>
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<td>7</td>
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<td>Distortion and coma</td>
<td>phenomenology, relation to sine condition, aplanatic systems, effect of stop position, various topics, correction options</td>
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<td>Astigmatism and curvature</td>
<td>phenomenology, Coddington equations, Petzval law, correction options</td>
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<td>9</td>
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<td>Chromatical aberrations</td>
<td>Dispersion, axial chromatical aberration, transverse chromatical aberration, spherochromatism, secondary spectrum</td>
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<td>10</td>
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<td>Sine condition, aplanatism and isoplanatism</td>
<td>Sine condition, isoplanatism, relation to coma and shift invariance, pupil aberrations, Herschel condition, relation to Fourier optics</td>
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<td>Wave aberrations</td>
<td>definition, various expansion forms, propagation of wave aberrations</td>
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<td>12</td>
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<td>Zernike polynomials</td>
<td>special expansion for circular symmetry, problems, calculation, optimal balancing, influence of normalization, measurement</td>
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<td>13</td>
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<td>Point spread function</td>
<td>ideal psf, psf with aberrations, Strehl ratio</td>
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<td>31.01</td>
<td>Transfer function</td>
<td>transfer function, resolution and contrast</td>
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<td>15</td>
<td>07.02</td>
<td>Additional topics</td>
<td>Vectorial aberrations, generalized surface contributions, Aldis theorem, intrinsic and induced aberrations, reversability</td>
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</table>
1. Spherical aberration representations
2. Single lens spherical aberration
3. Aplanatic surfaces
4. Higher order spherical aberration
5. Correction of spherical aberration
6. Aspherical surfaces
Spherical Aberration: Angle of Incidence

- Spherical aberration: non-linearity of the law of refraction for finite angles of incidence $i$
- Example single plano-convex lens:
  1. bad orientation (red ray):
     \[
     \sin i = i - \frac{i^3}{6}
     \]
  2. optimal orientation (green ray):
     approximation for refractive index $n=1.5$
     \[
     2 \cdot \sin \frac{i}{2} = i - \frac{2}{6} \cdot \left(\frac{i}{2}\right)^3 = i - \frac{i^3}{24}
     \]
- Spherical aberration differs by a factor of 4
Spherical Aberration

- Spherical aberration:
  - On axis, circular symmetry
- Perfect focussing near axis: paraxial focus
- Real marginal rays: shorter intersection length (for single positive lens)
- Optimal image plane: circle of least rms value
Spherical Aberration

- Single positive lens
- Paraxial focal plane near axis, Largest intersection length
- Shorter intersection length for rim ray and outer aperture zones
Spherical Aberration: Best Image Location

- Spherical wave aberration
  \[ W = A_d r_p^2 + A_s r_p^4 \]
- Best image location by choice of defocus parameter \( A_d \)
- Several solutions dependent on criterion

<table>
<thead>
<tr>
<th></th>
<th>defocus ( A_d ) ([A_s])</th>
<th>maximum spotradius ([8 \text{ R } A_s])</th>
<th>rms spotradius ([8 \text{ R } A_s])</th>
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<tbody>
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<td>0</td>
<td>1</td>
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<tr>
<td>Medium image position</td>
<td>-1</td>
<td>0.5</td>
<td>0.204</td>
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<td>Smallest rms of spot</td>
<td>-1.333</td>
<td>0.333</td>
<td>0.167</td>
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<td>Smallest diameter</td>
<td>-1.5</td>
<td>0.25</td>
<td>0.177</td>
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<td>Marginal image position</td>
<td>-2</td>
<td>0.385</td>
<td>0.289</td>
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</table>
Spherical Aberration

- Different representations
- At paraxial Gaussian image location as reference

Ref: H. Zügge
Spherical Aberration

- Different representations
- At best plane, with optimal defocus

Ref: H. Zügge
- Point spread function for different amounts of spherical aberration and defocus
- Unsymmetrical behavior around the image plane (ring vs. compact profile)
- Symmetrical behavior for change of sign in $c_4$ and $c_9$
Caustic with Spherical aberration

- Definition of spherical aberration according to
  1. Zernike, with appropriate defocussing
  2. Seidel, only higher order power

- Examples of the changing intensity distribution in the image plane for different aberration coefficients with amplitude above 0.3

- The behavior is completely symmetric relative to the sign of the coefficient c

- Larger aberrations give a smaller central peak and growing rings with large extent
Growing spherical aberration shows an asymmetric behavior around the nominal image plane for defocussing.

- $c_9 = 0$
- $c_9 = 0.3$
- $c_9 = 0.7$
- $c_9 = 1$
Spherical Corrected Surface

- Seidel contribution of spherical aberration with
  \[ \omega_j = \frac{h_j}{h_1}, \quad Q_j = n_j \cdot \left( \frac{1}{R_j} - \frac{1}{s_j} \right) \]

- Result

- Vanishing contribution:
  1. first bracket: vertex ray
  2. second bracket: concentric
  3. bracket: aplanatic surface

- Discussion with the Delano formula
  \[ S_j = \omega_j^4 Q_j^2 \left( \frac{1}{n_j' s_j'} - \frac{1}{n_j s_j} \right) \]
  \[ S_j = \left( \frac{h_j}{h_1} \right)^4 \cdot n_j^2 \cdot \left( \frac{1}{R_j} - \frac{1}{s_j} \right)^2 \cdot \left( \frac{1}{n_j' s_j'} - \frac{1}{n_j s_j} \right) \]

- \( h_j = 0 \)
- \( R_j = s_j \)
- \( n_j' s_j' = n_j s_j \)

- Discussion with the Delano formula

- \[ \Delta s_{S_{PH}}' = \Delta s_{S_{PH}} \cdot \frac{n_1 U_1 \sin u_1}{n_k' U_k' \sin u_k'} + \sum_j \frac{n_j}{n_j'} \cdot h \cdot \sin \frac{i'-i}{2} \cdot \frac{2i \cdot \sin \frac{i'-u}{2}}{U_j' \sin u_j'} \]

- 2. concentric corresponds to \( i' = i \)
- 3. aplanatic condition corresponds to \( i' = u \)
Delano’s Representation of Spherical Aberration

- Paraxial optics: Delano relation
  \[ n' \cdot q' \cdot U' = n \cdot q \cdot U + n \cdot i \cdot (Q' - Q) \]

- Real ray comparison:
  Delano surface contribution

Surface contribution grows with
1. ratio of refractive indices
2. height of the marginal ray
3. Influence of ray bending angle

- Influence of ray bending angle

Aplanatic Surfaces with Vanishing Spherical Aberration

- Aplanatic surfaces: zero spherical aberration:
  1. Ray through vertex: \( s' = s = 0 \)
  2. concentric
  3. Aplanatic: \( s' = s \) and \( u = u' \)
  \[ ns = n's' \]

- Condition for aplanatic surface:
  \[ r = \frac{ns}{n + n'} = \frac{n's'}{n + n'} = \frac{ss'}{s + s'} \]

- Virtual image location

- Applications:
  1. Microscopic objective lens
  2. Interferometer objective lens

![Diagram showing the linear growing aberrations with deviation of object location and no shift invariance in z.](attachment:image.png)
Aberration Generation

- Dependence of the aberration surface contribution on the geometry

\[ S_I = -\sum A^2_j \cdot h_j \cdot \Delta \left( \frac{u_j}{n_j} \right) = -\sum n_j^2 i^2_j \cdot h_j \cdot \left( \frac{u'_j}{n'_j} - \frac{u_j}{n_j} \right) \]

- Driving forces for aberrations are:
  1. spherical aberrations grows linear with the field height \( h \)
  2. spherical aberration grows quadratical with the incidence angle \( i^2 \)
  3. no impact, if the aplanatic condition is fulfilled
Aberration Dependence on Incidence Point

- Usual argumentation: spherical aberration grows with
  1. ray height \( h \)
  2. incidence angle \( i \)
  3. ray bending angle \( \delta \)
  4. stronger curvature \( c \) of the surface

- But: no impact in case of
  1. vertex intersection
  2. concentric transition
  3. aplanatic constellation

- Parameter selection:
  1. the ray bending dependence corresponds to incidence \( i \):
  \[ \delta = u' - u = i' - i = i \cdot \left( \frac{n'}{n} - 1 \right) \propto i \]
  2. to get rid of the curvature it make sense to use the center angle \( \varphi \) instead of \( h \) as a normalization: \( h = \frac{\varphi}{c} \)
  3. therefore it make sense to calculate the spherical aberration as a function of
    - incidence \( i \)
    - center angle \( \varphi \)
  4. the dependence on \( \delta \) and \( h \) is then automatically covered
- Plot of spherical aberration as a function of $i$ and $\delta$: three lines with vanishing aberration

![Graph showing spherical aberration as a function of angle of incidence and center angle.](image)
Aberration Dependence on Incidence Point

- If $i$ or $\phi$ is growing:
  - move along a line through the origin
  - aberrations are continuously growing with $i$ in between the three zero-cases
  - depending on the value of $c$, the spherical aberration is positive or negative
Aplanatic Lenses

- Aplanatic lenses
- Combination of one concentric and one aplanatic surface: zero contribution of the whole lens to spherical aberration
- Not useful:
  1. aplanatic-aplanatic
  2. concentric-concentric
     bended plane parallel plate, nearly vanishing effect on rays

\[ \begin{align*}
\text{A-A} & : \quad \text{parallel offset} \\
\text{A-C} & : \quad \text{convergence enhanced} \\
\text{C-C} & : \quad \text{no effect} \\
\text{C-A} & : \quad \text{convergence reduced}
\end{align*} \]
Spherical Corrected Singlets

- Exact surface without spherical aberration
- Approach with Fermat principle
  \[ n \cdot z + n' \cdot \sqrt{(f - z)^2 + y^2} = f \cdot n' \]
- Result: depending on ratio of refractive indices
  \[
  \left( \frac{z - \frac{n'f}{n+n'}}{\frac{n'+n'}{n'f}} \right)^2 - \frac{n+n'}{n-n'} \cdot y^2 = 1
  \]
- Ray bending at one surface only: very sensitive component
Single Lens free of Spherical aberration

- Object location at infinity
- Refraction with only one surface: exact analytical solution

a) Rear surface
   exact hyperbola
   front surface plane
   \( \kappa_2 = -2.35 \)

b) Front surface
   exact ellipsoid
   rear surface
   concentric
   \( \kappa_1 = -0.434 \)
Elliptical Surface and Aplanatic Case

- Meniscus lens with concentric rear surface
- Change of front radius $R_1 = 6 / 8 / 10\ mm$
- Spherical aberration $c_9$ for variation of conic constant $\kappa$
- Generally corrected for elliptical shape
- Special case of aplanatic spherical surface

![Graph showing spherical aberration $c_9$ vs. conic constant $\kappa$ for different radii $R_1$. The graph has lines for $R_1 = 6$, $R_1 = 8$, and $R_1 = 10$. The aplanatic case is indicated by a red line. The graph also shows the transition from prolate to oblate ellipsoid.]
Aplanatic Surface

- Spherical aberration vanishes for all orders

- Aplanatic lens at high NA:
  effective real NA is higher than paraxial

- Further possible aplanatic lenses
  1. less practical importance
  2. used in microscopic objective front lens
Selection of Expansion Height for a Lens

- Usual selection of expansion height for Seidel formulas: Vertex plane height $h_0$

- Thin Lens with small $X$:
  - height $h_1$, $h_2$, $h_0$ nearly the same
  - no severe problems

- Thin lens with large bending:
  - large height differences
  - changes by reverted lens
  - additional option: height $h_p$ in the principal plane

- Reversed lens:
  - now $h_{02}$ gives a difference in higher orders than $h_{01}$
Microscopic Lens

- Microscopic lens
  100x1.3 oil / Kuda
  Patent US 5978147
- Huge intermediate aberrations
- Large higher orders mainly at cemented surfaces with small $\Delta n$
- Front group: alternating sign due to $r$ below/above aplanatic radius

![Diagram of a microscope system with labeled surfaces and wavefront corrections.](image-url)

**Graphs:**

- **sph [a.u.]:**
  - 3rd order
  - Higher orders
  - Rear group
  - Middle group
  - Front group
  - Single lenses

- **sph-cumul [a.u.]:**
  - $W = 144 \lambda$
  - $W = 64 \lambda$
  - $W = 84 \lambda$
  - Exact
  - Corrected

**Equation:**

$$W = 144 \lambda$$
■ High-NA microscopic lens: 100x0.95 air
■ Differences of ideal vs real MR ray heights up to 25%
■ Consequence:
  - Seidel surface contributions identical for both ray directions
  - Difference of exact spherical surface contributions depending on direction of rays
  - Sensitivity evaluation depends on orientation of the system

a) low \(\rightarrow\) high side direction

b) high \(\rightarrow\) low side direction
### Bending of a Lens

- **Bending:** change of shape for invariant focal length
- **Parameter of bending**

\[ X = \frac{r_1 + r_2}{r_2 - r_1} \]

![Diagram showing various types of lenses with different values of X ranging from X < -1 to X > 1.](image)
Magnification / Conjugation Parameter

- Magnification/conjugation parameter $M$:
defines ray path through the lens

\[
M = \frac{U' + U}{U' - U} = \frac{1 + m}{1 - m} = \frac{2f}{s} + 1 = \frac{2f}{s'} - 1
\]

- Special cases:
  1. $M = 0$ : symmetrical 4f-imaging setup
  2. $M = -1$: object in front focal plane
  3. $M = +1$: object in infinity

- The parameter $M$ strongly influences the aberrations
Lens Contributions of Seidel

- Spherical aberration
  \[ S_{lens} = \frac{1}{32n(n-1)f^3} \left[ \frac{n^3}{n-1} + \frac{n+2}{n-1} \left( X - \frac{2(n^2-1)}{n+2} \cdot M \right)^2 - \frac{n^2(n-1)}{n+2} \cdot M^2 \right] \]

- Special impact on correction:
  1. Special quadratic dependence on bending \( X \)
     Minimum at
     \[ X_{sph\,min} = - \frac{2(n^2-1)}{n+2} \cdot M \]
  2. No correction for small \( n \) and \( M \)
  3. Correction for large
     \( n \): infrared materials
     \( M \): virtual imaging
     Limiting value
     \[ M_{s=0}^2 = \frac{n(n+2)}{(n-1)^2} \]
Spherical Aberration of a bended Lens

- Separation of the spherical aberration of a thin lens into the contributions of the two surfaces
- Effect of bending, represented by the curvature $c_1$ of the front surface

\[ i_1 = 0 \quad i_2 = 0 \]

first surface

second surface

lens total
• Changes of the incidence angles at the front and the rear surface of a bended lens
• Figure without sign of incidence angle
• Angle at the second surface depends on the refractive index
- Spherical aberration and focal spot diameter as a function of the lens bending (for n=1.5)
- Optimal bending for incidence averaged incidence angles
- Minimum larger than zero: usually no complete correction possible
## Lens of Best Shape

- Optimal bending of a focusing lens for collimated input: $M = +1$

$$X_{sph \text{ min}} = - \frac{2(n^2 - 1)}{n + 2} M$$

- Plan convex lens for $n = 1.686$
- Higher indices: meniscus shape

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<tr>
<th>index $n$</th>
<th>bending $X$</th>
<th>shape</th>
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<tbody>
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<td>1.4</td>
<td>-0.565</td>
<td>bi convex</td>
</tr>
<tr>
<td>1.686</td>
<td>-1</td>
<td>plan convex</td>
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<tr>
<td>2</td>
<td>-1.5</td>
<td>meniscus</td>
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<td>3</td>
<td>-3.2</td>
<td>mensicus</td>
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<td>4</td>
<td>-5</td>
<td>meniscus</td>
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<tr>
<td>10</td>
<td>-16.5</td>
<td>meniscus</td>
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</table>
Spherical Aberration Correction by Bending

- Fixed plano-convex lens $L_2$
- Correction of spherical aberration by bended negative lens $L_1$
- Compensation of negative SPH$_1$ by positive SPH$_2$
- Two solutions with different sign of bending
- Solutions asymmetric due to shift of principal plane
- Small nonlinearity due to growing height at lens 2
- Adapted distance to keep total focal length constant
- Diagram:
  - merit function spot size as function distance and bending

Diagram:
- Solution 1
  - $\rho = -0.0073$
- Solution 2
  - $\rho = 0.0022$
  - $\rho = 0.0063$
  - $\rho = -0.0073$

Graph:
- Lens $L_2$ fixed (negative)
- Corrected solution 1 (bending negative)
- Corrected solution 2 (bending positive)
G-Sum Formula

- Alternative formula for the 3rd order spherical aberration of a thin lens:

  G-sum formula of Conrady

\[
\Delta s'_{sph} = -\frac{h^4}{n'u'} \cdot \sum G_n c^n - G_2 c^2 c_1 + G_3 c^2 v + G_4 cc_1^2 - G_5 c c_1 v + G_6 c v^2
\]

- Vergence / object distance

\[v = \frac{1}{s}\]

- Curvatures

\[c = \frac{n-1}{f'}, \quad c_1 = \frac{1}{r_1}\]

- G-factors for refractive index

\[G_1 = \frac{1}{2} \cdot n^2 \cdot (n-1) \quad , \quad G_2 = \frac{1}{2} \cdot (2n+1) \cdot (n-1)\]

\[G_3 = \frac{1}{2} \cdot (3n+1) \cdot (n-1) \quad , \quad G_4 = \frac{1}{n} \cdot (n+2) \cdot (n-1)\]

\[G_5 = \frac{2}{n} \cdot (n^2 - 1) \quad , \quad G_6 = \frac{1}{2n} \cdot (3n+2) \cdot (n-1)\]
**Spherical Aberration**

- Lens bending for $n = 1.5$

![Diagram showing spherical aberration with scale and range of real imaging](image)
- Lens Bending for $n = 1.9$
In particular for large $|X|, |M|$ the higher order spherical aberration of a thin lens is not well described by 3rd order due to large incidence angles $i_1, i_2$.

The parabolic behavior of a spherical correction in 3rd order is too simple.
Spherical Correction of a Single Lens

- Spherical aberration of a single lens
- Optimal bending for given $n$ and $M$
  \[ X_{sph\text{min}} = - \frac{2(n^2 - 1)}{n + 2} M \]
- Correction possible only for virtual imaging with
  \[ M_{s=0} = \frac{\sqrt{n(n+2)}}{n-1} \]
  \[ n > \frac{M^2 + 1 + \sqrt{3M^2 + 1}}{M^2 - 1} \]

<table>
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<tr>
<th>$M$</th>
<th>$n_{\text{min}}$</th>
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<tr>
<td>2</td>
<td>2.869</td>
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<td>1.914</td>
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<td>5</td>
<td>1.447</td>
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Change of the object distance for
1. plano convex lens
2. biconvex lens
3. achromate

Variation of spherical aberration with the distance
Mainly quadratic
Spherical Aberration of an Achromate

- Achromatic condition does not contain the curvature:
- Bending can be used to correct for spherical aberration at the edge for the center wavelength
- Index difference must be small enough to allow for spherical correction
- Complex dependence on indices and Abbe number ratio

\[ \Delta n_{\text{max}} = \frac{n_1}{n_2} \]

\( n_1 = 1.4 \)
\( n_1 = 1.5 \)
\( n_1 = 1.6 \)
Spherical Aberration: Zone Error

- Wave aberration of 6th order
- Zonal coefficient $A_z$
- Spherical aberration with correction at the edge
  $A_z = - A_s$
- Smallest wave aberration at zone
- Smallest spot diameter for $A_d = -3/2A_s$ at

$$\Delta s'_{\text{min}} = - \frac{3A_s}{\sin^2 u'} = \frac{3}{4} \Delta s'_R$$

with diameter

$$D_{\text{min}} = \frac{2A_s}{\sin u'} = - \frac{1}{2} \Delta s'_R \sin u'$$

$$= \frac{1}{4} D_{\text{parax}}$$

$$W = A_d r_p^2 + A_s r_p^4 + A_z r_p^6$$

$$\Delta s'_R = 0$$

$$r_p = 1/\sqrt{2} = 0.707$$
Spherical Correction / Higher Orders

- Partial correction of residual spherical aberration by 5th order or 5th and 7th order
- Different (alternating) sign of coefficients
- Residual total error significantly smaller
- Large gradients at the edge
- 3rd and 5th order compensation: residual zonal error at $\frac{1}{\sqrt{2}}=0.707$ of the pupil radius

a) 2 orders

b) 3 orders
Higher Order Circular Symmetric Zernikes

- Zernike function with circular symmetry with growing order
- Normalized to $+1$ in centre and at the edge
- Growing spatial frequencies
- Highest slope at the edge of the pupil
- Microscope Lens
- Zernike spectrum shows large higher contributions

From: H. Zügge
Merte Surface

- Small difference in refractive index
- Growing higher order contributions
Spherical Aberration Correction

- Correction of spherical aberration by splitting the ray bending
- Optimal bending of lenses
- Splitting of lenses
- Smooth reducing of spherical aberration or marginal correction

\[ W_{\text{rms}} = 5.21 \lambda \]
\[ W_{\text{rms}} = 1.91 \lambda \]
\[ W_{\text{rms}} = 0.91 \lambda \]
\[ W_{\text{rms}} = 0.221 \lambda \]
\[ W_{\text{rms}} = 0.168 \lambda \]
\[ W_{\text{rms}} = 0.026 \lambda \]
\[ W_{\text{rms}} = 0.0159 \lambda \]
\[ W_{\text{rms}} = 0.0001 \lambda \]
<table>
<thead>
<tr>
<th>Nr</th>
<th>System</th>
<th>Spot-$\varnothing$ geo. [(\mu\text{m})]</th>
<th>Spot-$\varnothing$ rms [(\mu\text{m})]</th>
<th>Wave ab. rms [(\lambda)]</th>
<th>Strehl ratio [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Plano convex lens</td>
<td>366</td>
<td>206</td>
<td>5.21</td>
<td>1.7</td>
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<tr>
<td>2</td>
<td>Dublett of plano convex lenses</td>
<td>136</td>
<td>76.8</td>
<td>1.91</td>
<td>5.2</td>
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<tr>
<td>3</td>
<td>Dublett with meniscus lenses</td>
<td>63.9</td>
<td>36.2</td>
<td>0.903</td>
<td>12.8</td>
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<tr>
<td>4</td>
<td>Dublett, edge corrected</td>
<td>26.1</td>
<td>13.5</td>
<td>0.221</td>
<td>23.1</td>
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<td>5</td>
<td>Achromate , cemented</td>
<td>21.4</td>
<td>11.1</td>
<td>0.168</td>
<td>35.8</td>
</tr>
<tr>
<td>6</td>
<td>Achromate , spitted</td>
<td>5.1</td>
<td>2.54</td>
<td>0.024</td>
<td>97.3</td>
</tr>
<tr>
<td>7</td>
<td>Achromate with mesniscus</td>
<td>2.94</td>
<td>1.48</td>
<td>0.0167</td>
<td>98.8</td>
</tr>
<tr>
<td>8</td>
<td>Four lens system optimized</td>
<td>0.008</td>
<td>0.005</td>
<td>0.0001</td>
<td>100.0</td>
</tr>
</tbody>
</table>
Correction of Spherical Aberration

- Comparison of lenses for the same aperture:
  1. single lens with optimal bending
  2. Dublett with optimal bending
  3. Triplet with optimal bending
  4. Triplet with aplanatic-concentric meniscus lenses
  5. Triplet with compensating negative lens
  6. Four lenses
General Aspherical Surface

- Conic surface as basic shape
- Additional correction of the sag by a Taylor expansion
  Only even powers: no kink at \( r=0 \)
- Mostly rotational symmetric shape considered
- Problems with this representation:
  1. added contributions not orthogonal, bad performance during optimization
  2. non-normalized representation, coefficients depend on absolute size of the diameter (very small high order coefficients for large diameters)
  3. Oscillatory behavior, large residual slope error can occur
  4. in optics slope and not sag is relevant
  5. the coefficients can not be measured/are hard to control, tolerancing is critical and complicated
  6. the added sag is along \( z \), more important is a correction perpendicular to the surface for strong aspheres

\[
z(x, y) = \frac{c(x^2 + y^2)}{1 + \sqrt{1 - (1 + \kappa) \cdot c^2 (x^2 + y^2)}} + \sum_{k=2}^{k_{\text{max}}} a_k \cdot (x^2 + y^2)^k
\]

\[
z(r) = \frac{c \cdot r^2}{1 + \sqrt{1 - (1 + \kappa) \cdot c^2 r^2}} + \sum_{k=2}^{k_{\text{max}}} a_k \cdot r^{2k}
\]
Aspheres: Correction of Higher Order

- Correction at discrete sampling
- Large deviations between sampling points
- Larger oscillations for higher orders
- Better description: slope, defines ray bending

Corrected points with $y' = 0$

residual spherical transverse aberrations

paraxial range

$y' = c\frac{dz_A}{dy}$

perfect correcting surface

points with maximal angle error

corrected points residual angle deviation

real asphere with oscillations

$A$
Aspherical Expansion Order

- Improvement by higher orders
- Generation of high gradients
Forbes Aspheres

- New representation of aspherical expansions according to Forbes (2007)

\[ z(r) = \frac{c \cdot r^2}{1 + \sqrt{1 - (1 + \kappa) \cdot c^2 r^2}} + \sum_{k=2}^{k_{\text{max}}} a_k \cdot Q_k(r^2) \]

- Special polynomials \( Q_k(r^2) \):
  1. Contributions are orthogonal slope
  2. tolerancing is easily measurable
  3. optimization has better performance
  4. usually fewer coefficients are necessary
  5. use of normalized radial coordinate makes coefficients independent on diameter

- Two different versions possible:
  a) strong aspheres: deviation defined along \( z \)
  b) mild aspheres: deviation defined perpendicular to the surface
New representations of Forbes

Typical shape of contributions of the 6 lowest correction terms

- a) strong
- b) mild
Aspherical Single Lens

- Correction on axis and field point
- Field correction: two aspheres
Lithographic Projection: Improvement by Aspheres

- Considerable reduction of length and diameter by aspherical surfaces
- Performance equivalent
- 2 lenses removable

a) NA = 0.8 spherical

b) NA = 0.8, 8 aspherical surfaces

31 lenses

-9%

29 lenses

-13%

Ref: W. Ulrich
Best Position of Aspheres

- Location depending on correction target:
  - spherical: pupil plane
  - coma and astigmatism: field plane
- No effect on Petzval curvature

![Graph showing aspherical effect, pupil, and image positions with spherical, coma, and astigmatism distortion curves.]