Imaging and Aberration Theory

Lecture 5: Aberrations representations
2018-11-15
Herbert Gross
<table>
<thead>
<tr>
<th>No.</th>
<th>Date</th>
<th>Topic</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>18.10.</td>
<td>Paraxial imaging</td>
<td>paraxial optics, fundamental laws of geometrical imaging, compound systems</td>
</tr>
<tr>
<td>2</td>
<td>25.10.</td>
<td>Pupils, Fourier optics, Hamiltonian coordinates</td>
<td>pupil definition, basic Fourier relationship, phase space, analogy optics and mechanics, Hamiltonian coordinates</td>
</tr>
<tr>
<td>3</td>
<td>01.11.</td>
<td>Eikonal</td>
<td>Fermat principle, stationary phase, Eikonals, relation rays-waves, geometrical approximation, inhomogeneous media</td>
</tr>
<tr>
<td>4</td>
<td>08.11.</td>
<td>Aberration expansions</td>
<td>single surface, general Taylor expansion, representations, various orders, stop shift formulas</td>
</tr>
<tr>
<td>5</td>
<td>15.11.</td>
<td>Representation of aberrations</td>
<td>different types of representations, fields of application, limitations and pitfalls, measurement of aberrations</td>
</tr>
<tr>
<td>6</td>
<td>22.11.</td>
<td>Spherical aberration</td>
<td>phenomenology, sph-free surfaces, skew spherical, correction of sph, aspherical surfaces, higher orders</td>
</tr>
<tr>
<td>7</td>
<td>29.11.</td>
<td>Distortion and coma</td>
<td>phenomenology, relation to sine condition, aplanatic sytems, effect of stop position, various topics, correction options</td>
</tr>
<tr>
<td>8</td>
<td>06.12.</td>
<td>Astigmatism and curvature</td>
<td>phenomenology, Coddington equations, Petzval law, correction options</td>
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<tr>
<td>9</td>
<td>13.12.</td>
<td>Chromatical aberrations</td>
<td>Dispersion, axial chromatical aberration, transverse chromatical aberration, spherochromatism, secondary spectrum</td>
</tr>
<tr>
<td>10</td>
<td>20.12.</td>
<td>Sine condition, aplanatism and isoplanatism</td>
<td>Sine condition, isoplanatism, relation to coma and shift invariance, pupil aberrations, Herschel condition, relation to Fourier optics</td>
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<tr>
<td>11</td>
<td>10.01.</td>
<td>Wave aberrations</td>
<td>definition, various expansion forms, propagation of wave aberrations</td>
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<tr>
<td>12</td>
<td>17.01.</td>
<td>Zernike polynomials</td>
<td>special expansion for circular symmetry, problems, calculation, optimal balancing, influence of normalization, measurement</td>
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<td>13</td>
<td>24.01.</td>
<td>Point spread function</td>
<td>ideal psf, psf with aberrations, Strehl ratio</td>
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<td>14</td>
<td>31.01.</td>
<td>Transfer function</td>
<td>transfer function, resolution and contrast</td>
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<tr>
<td>15</td>
<td>07.02.</td>
<td>Additional topics</td>
<td>Vectorial aberrations, generalized surface contributions, Aldis theorem, intrinsic and induced aberrations, reversability</td>
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</tbody>
</table>
1. Stop shift formulas
2. Lens aberration contributions
3. Pupil aberrations
4. Representation of geometrical aberrations
5. Representation of wave aberrations
6. Miscellaneous
7. Aberration measurements
**Eikonal of 4th order**

\[ L_A^{(4)} = K \cdot \frac{(s - p)^4}{8p^4} (x^2 + y^2)^2 + S \cdot \frac{s^4}{8p^4} (x_p^2 + y_p^2)^2 + A \cdot \frac{s^2 \cdot (s - p)^2}{2p^4} (xx_p + yy_p)^2 \]

\[ + P \cdot \frac{s^2 \cdot (s - p)^2}{4p^4} (x^2 + y^2)(xx_p + yy_p) - D \cdot \frac{s \cdot (s - p)^3}{2p^4} (x^2 + y^2)(xx_p + yy_p) - C \cdot \frac{s^3 \cdot (s - p)}{2p^4} (x^2 + y^2)(xx_p + yy_p) \]

**Coefficients**

1. **Spherical aberration**

\[ K = -\frac{(n' - n)b}{R^3} - ns \cdot \left( \frac{1}{R s} - \frac{1}{s_p^2} \right)^2 + n's' \left( \frac{1}{R s'} - \frac{1}{s_p'^2} \right)^2 \]

2. **Astigmatism**

\[ S = -\frac{(n' - n)b}{R^3} - Q^2 \left( \frac{1}{ns} - \frac{1}{n's'} \right) \]

3. **Field curvature**

\[ P = -\frac{(n' - n)b}{R^3} - QQ_p \cdot \left( \frac{1}{ns} - \frac{1}{n's'} \right) + Q(Q - Q_p) \cdot \left( \frac{1}{ns_p} - \frac{1}{n's_p'} \right) \]

4. **Distortion**

\[ D = -\frac{(n' - n)b}{R^3} - Q_p^2 \cdot \left( \frac{1}{ns} - \frac{1}{n's'} \right) + Q_p(Q - Q_p) \cdot \left( \frac{1}{ns_p} - \frac{1}{n's_p'} \right) \]

5. **Coma**

\[ C = -\frac{(n' - n)b}{R^3} - QQ_p \cdot \left( \frac{1}{ns} - \frac{1}{n's'} \right) \]
Stop Shift Formulas

- If the stop is moved, the chief ray takes a modified way through the system.
- Approach: expansion of the surface coefficient formulas for small changes in the pupil position $p, p'$.
- The stop shift formulas shows the change of the Seidel coefficients due to this effect.
- Also possible:
  - set of formulas for object or image shift
  - applicable for curved objects
Stop Shift Formulas

- Stop shift formulas explicate with the help of the moving parameter

\[ \delta E = \frac{\bar{h}_{\text{new}} - \bar{h}_{\text{old}}}{h} \]

sph \[ S_I^* = S_I \]

coma \[ S_{II}^* = S_{II} + \delta E \cdot S_I \]

ast \[ S_{III}^* = S_{III} + \delta E \cdot S_{II} + \delta E^2 \cdot S_I \]

curv \[ S_{IV}^* = S_{IV} \]

dist \[ S_V^* = S_V + \delta E \cdot (3S_{III} + S_{IV}) + 3\delta E^2 \cdot S_{II} + \delta E^3 \cdot S_I \]

- Mix of aberration types due to stop shift: induced aberrations
Examples:
1. spherical aberration induces coma
2. coma induces astigmatism
Lens Contributions of Seidel

- In 3rd order (Seidel):
  Additive contributions of thin lenses (equal $\omega$) to the total aberration value (stop at lens position)

- Spherical aberration
  $X$: lens bending
  $M$: position parameter

- Coma

- Astigmatism

- Field curvature

- Distortion
  $D_{\text{lens}} = 0$
Lens Contributions of Seidel

- Spherical aberration

\[ S_{\text{lens}} = \frac{1}{32n(n-1)f^3} \left[ \frac{n^3}{n-1} + \frac{n+2}{n-1} \cdot \left( X - \frac{2(n^2-1)}{n+2} \cdot M \right)^2 - \frac{n^2(n-1)}{n+2} \cdot M^2 \right] \]

- Special impact on correction:
  1. Special quadratic dependence on bending \( X \)
     Minimum at
     \[ X_{\text{sph min}} = -\frac{2(n^2-1)}{n+2} \cdot M \]
  2. No correction for small \( n \) and \( M \)
  3. Correction for large
     \( n \): infrared materials
     \( M \): virtual imaging
     Limiting value
     \[ M_{s=0}^2 = \frac{n(n+2)}{(n-1)^2} \]
Photographic lens

- Incidence angles for chief and marginal ray
- Field dominant system
- Quasi symmetry can be seen at the surface contributions of field aberrations
- Symmetry disturbed for spherical aberration

![Photographic lens diagram]

![Graphs of aberrations]
Microscopic Objective Lens

- Incidence angles for chief and marginal ray
- Aperture dominant system
Microscopic Lens

- Large distance system
- Problems with large diameters
- Aplanatic front group
- Not corrected for curvature and distortion
- Astigmatic contributions of cemented surfaces corrected by rear group
- Sign of lateral chromatic aberration in front group
Lithographic Lens

- Large effect of mirror on field curvature

- Typical bulge structure shows the correction of field curvature according to the Petzval theorem
Primary Aberration Spot Shapes

- Simplified set of Seidel formulas:
  field point only in $y'$ considered

  \[
  \Delta y' = S' \cdot r_{p}^{3} \cos \varphi_{p} + C' \cdot y' \cdot r_{p}^{2} (2 + \cos 2\varphi_{p}) + (2A' + P') \cdot y'^{2} \cdot r'_{p} \cos \varphi_{p} + D' \cdot y'^{3}
  \]

  \[
  \Delta x' = S' \cdot r_{p}^{3} \sin \varphi_{p} + C' \cdot y' \cdot r_{p}^{2} \sin 2\varphi_{p} + P' \cdot y'^{2} \cdot r'_{p} \sin \varphi_{p}
  \]

- Spherical aberration $S$:
  circle

  \[
  \Delta y' = S' \cdot r_{p}^{3} \cos \varphi_{p} , \quad \Delta x' = S' \cdot r_{p}^{3} \sin \varphi_{p}
  \]

  \[
  \Delta x'^{2} + \Delta y'^{2} = S'^{2} \cdot r_{p}^{6}
  \]

- Coma:
  shifted circle

  \[
  \Delta y' = C' \cdot y' \cdot r_{p}^{2} (2 + \cos 2\varphi_{p}) , \quad \Delta x' = C' \cdot y' \cdot r_{p}^{2} \sin 2\varphi_{p}
  \]

  \[
  \Delta x'^{2} + (\Delta y' - 2C' \cdot y' \cdot r_{p}^{2})^{2} = C'^{2} \cdot y'^{2} \cdot r_{p}^{4}
  \]

- Astigmatism:
  focal line

  \[
  \Delta y' = 2A' \cdot y'^{2} \cdot r'_{p} \cos \varphi_{p} , \quad \Delta x' = 0
  \]

- Field curvature:
  circle

  \[
  \Delta y' = P' \cdot y'^{2} \cdot r'_{p} \cos \varphi_{p} , \quad \Delta x' = P' \cdot y'^{2} \cdot r'_{p} \sin \varphi_{p}
  \]

  \[
  \Delta x'^{2} + \Delta y'^{2} = P'^{2} \cdot y'^{4} \cdot r_{p}^{2}
  \]

- Distortion:
  shifted point

  \[
  \Delta y' = D' \cdot y'^{3} , \quad \Delta x' = 0
  \]
Primary Aberration Spot Shapes

- Schematically:

1) spherical aberration
2) coma
3) astigmatism
4) field curvature
5) distortion
Optical Image Formation

- **Perfect optical image:**
  All rays coming from one object point intersect in one image point
- **Real system with aberrations:**
  1. transverse aberrations in the image plane
  2. longitudinal aberrations from the image plane
  3. wave aberrations in the exit pupil
Representation of Geometrical Aberrations

- **Longitudinal aberrations** $\Delta s$

- **Transverse aberrations** $\Delta y$

![Diagram showing longitudinal and transverse aberrations](image)
Representation of Geometrical Aberrations

- **Angle aberrations $\Delta u$**

- **Wave aberrations $\Delta W$**

\[
\Delta s' = \frac{R}{y_p} \cdot \Delta y' = \frac{\Delta y'}{\sin u'} = -\frac{R^2}{y_p} \cdot \frac{\partial W(x_p, y_p)}{\partial y_p}
\]
Angle Aberrations

- Angle aberrations for a ray bundle: deviation of every ray from common direction of the collimated ray bundle
- Representation as a conventional spot diagram
- Quantitative spreading of the collimated bundle in mrad / °
Aperture Dependence of Longitudinal Aberration

- Typical representation:
  Longitudinal aberration as function of aperture (pupil coordinate)

- If correction at the edge: maximum residuum at the zone $1/\sqrt{2}$

- Typical: largest gradients at the edge

- Correcting aspheres or high NA of higher order:
  oscillatory behavior
Longitudinal Aberration Chart

- Spherical aberration
  - 4 colors
- Coma in zone
  - 4 colors
- Coma in full field
  - 4 colors
- Image shells/astigmatism
  - 4 colors
- Distortion
  - Main color
- Chromatic difference in magnification
- Secondary chromatic spherical aberration

Graphs showing various aberrations and their respective magnitudes and effects.
Pupil Sampling

- Ray plots
- Spot diagrams

![Diagram showing ray plots, spot diagrams, and pupil sampling](image-url)
Ray Selection Planes

- Tangential / sagittal / skew rays
- View along optical axis
Transverse Aberrations

- Typical low order polynomial contributions for: defocus, coma, spherical aberration, lateral color
- This allows a quick classification of real curves

\[ \Delta y' = K' \cdot r'_p \cos \varphi_p \]

\[ \Delta y' = S' \cdot r'^3_p \cos \varphi_p \]

\[ \Delta y' = C' \cdot y'_p \cdot r'^2_p \cdot (2 + \cos 2\varphi_p) \]
Interpretation of Transverse Aberrations

- Combinations of basic shapes

![Graphs showing combinations of basic shapes like spherical, defocus, coma, distortion, and their interactions.](Image)
Transverse Aberrations

- Classical aberration curves
- Strong relation to spot diagram
- Usually only linear sampling along the x-, y-axis
  - no information in the quadrant of the aperture

\[ \Delta y = \Delta x \]

\[ l = 486 \text{ nm} \]

\[ l = 588 \text{ nm} \]

\[ l = 656 \text{ nm} \]

\[ \lambda = 486 \text{ nm} \]

\[ \lambda = 588 \text{ nm} \]

\[ \lambda = 656 \text{ nm} \]
**Best Image plane**

- **Best resolution:**
  - bright central peak in spot
  - tangent at transverse aberration curve

- **Best contrast:**
  - mean straight line over complete pupil of transverse aberration curve
  - smallest maximal deviation

- Different criteria give slightly different best image planes

\[
\frac{\partial W_{rms}}{\partial \Delta z} = 0 \quad \frac{\partial D_s}{\partial \Delta z} = 0
\]
Transverse Aberrations

- Characteristic chart for the representation of transverse aberrations

![Graphs showing transverse aberrations](image)

wavelengths:
365 nm
480 nm
546 nm
644 nm
Pupil Aberration

- Characteristic chart for the representation of pupil aberration
- Distortion of the pupil grid from the entrance to the exit pupil
- Pupil aberration can be interpreted as the spherical aberration of the chief ray for the pupil imaging
Sine Condition

- Sine condition not fulfilled:
  - nonlinear scaling from entrance to exit pupil
  - spatial filtering on warped grid, nonlinear sampling of spatial frequencies
  - pupil size changes
  - apodization due to distortion
  - wave aberration could be calculated wrong
  - quantitative measure of offence against the sine condition (OSC):
    distortion of exit pupil grid

\[
D_p = \frac{x_{ap}}{f \cdot n \cdot \sin u} - 1
\]
- Photometric effect of pupil distortion: illumination changes at pupil boundary
- Effect induces apodization
- Sign of distortion determines the effect: outer zone of pupil brighter / darker
- Additional effect: absolute diameter of pupil changes

OSC and Apodization

![Graph showing intensity vs. rp with different pupil diameters and focus positions: -50 μm, -20 μm, focused, +20 μm, +50 μm. The graph includes labels for barrel and pincushion distortion.]
Variation of Chromatical Aberrations

- Representation of the image location as a function of the wavelength: axial chromatical shift
- Representation of the chromatical magnification difference with field height: lateral chromatical aberration
Spot Diagram

- All rays start in one point in the object plane
- The entrance pupil is sampled equidistant
- In the exit pupil, the transferred grid may be distorted
- In the image plane a spreaded spot diagram is generated
- Variation of field and color
- Scaling of size:
  1. Airy diameter (small circle)
  2. 2nd moment circle (larger circle, scales with wavelength)
  3. surrounding rectangle

<table>
<thead>
<tr>
<th>Color</th>
<th>Diagram</th>
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<tbody>
<tr>
<td>486 nm</td>
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<td>546 nm</td>
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<tr>
<td>656 nm</td>
<td><img src="image" alt="656 nm Diagram" /></td>
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</tbody>
</table>
Gaussian Moment of Spot

- Spot pattern with transverse aberrations $\Delta x_j$ and $\Delta y_j$
  1. centroid
    \[
    \langle \Delta x_S \rangle = \frac{1}{N} \sum_j \Delta x_j \quad \langle \Delta y_S \rangle = \frac{1}{N} \sum_j \Delta y_j
    \]
  2. 2nd order moment
    \[
    M_G = \langle \Delta r^2 \rangle = \frac{1}{N} \sum_j \left[ \left( \Delta x_j - \langle \Delta x_S \rangle \right)^2 + \left( \Delta y_j - \langle \Delta y_S \rangle \right)^2 \right]
    \]
  3. diameter
    \[
    D = 2 \cdot \sqrt{M_G}
    \]

- Generalized:
  Rays with weighting factor $g_j$:
  corresponds to apodization
  \[
  M_G = \langle \Delta r^2 \rangle = \frac{1}{N_G} \sum \left[ g_j \left( \Delta x_j - \langle \Delta x_S \rangle \right)^2 + \left( \Delta y_j - \langle \Delta y_S \rangle \right)^2 \right]
  \]

- Worst case estimation:
  size of surrounding rectangle $D_x = 2\Delta x_{\text{max}}, D_y = 2\Delta y_{\text{max}}$
Practical problem in analysis of classical spot diagrams: relation between deviations and pupil location is lost

Idea of Kingslake: transverse aberrations of spot points drawn in pupil intersection points

$\Delta x$ and $\Delta y$ at every surface in the pupil sampling grid

$\Delta r$ at all surfaces in the pupil sampling grid

Problems:
1. proper representation of quite different scales
2. distorted grid in case of induced aberrations

- Extension of Kingslakes representation for surface contributions

- Problem: compaction of high complexity, limited clearness
Aberrations of a Single Lens

- Single plane-convex lens,
  BK7, \( f = 100 \text{ mm}, \lambda = 500 \text{ nm} \)
- Spot as a function of field position
- Coma shape rotates according to circular symmetry
- Decrease of performance with the distance to the axis

- Example HMD without symmetry
Caustic of Spherical Aberration and Coma

- **negativ spherical aberration**
  - intrafocal: compact broadened spot with bright edge
  - extrafocal: ring structure

- **positiv spherical aberration**
  - intrafocal: ring structure with bright center
  - extrafocal: ring structure with bright outer ring

- **coma**
  - bending of caustic
  - shifted center of gravity

Ref: W. Singer
Definition of the peak valley value
Wave Aberrations

- Classification of wave aberrations for one image point: Zernike polynomials

- Mean root square of wave front error

\[ W_{rms} = \sqrt{\langle W^2 \rangle - \langle W \rangle^2} = \frac{1}{A_{Exp}} \iint [W(x_p, y_p) - W_{mean}(x_p, y_p)]^2 \, dx_p \, dy_p \]

- Normalization: size of pupil area

\[ A_{Exp} = \iint \, dx \, dy \]

- Worst case / peak-valley wave front error

\[ W_{pv} = \max\left[W_{\text{max}}(x_p, y_p) - W_{\text{min}}(x_p, y_p)\right] \]

- Generalized for apodized pupils (non-uniform illumination)

\[ W_{rms} = \sqrt{\frac{1}{A_{Exp}^{(w)}} \iint I_{Exp}(x_p, y_p) \cdot [W(x_p, y_p) - W_{\text{mean}}^{(w)}(x_p, y_p)]^2 \, dx_p \, dy_p} \]
Primary Aberrations

- Relation:
  wave / geometrical aberration

<table>
<thead>
<tr>
<th>Type</th>
<th>Wave aberration</th>
<th>Geometrical spot</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spherical aberration</td>
<td>$W = c_1 \cdot r^4$</td>
<td>$\Delta x' \propto c_1 \cdot r^3 \sin \varphi$</td>
</tr>
<tr>
<td>Symmetry to Periodicity</td>
<td>axis constant</td>
<td>$\Delta y' \propto c_1 \cdot r^3 \cos \varphi$</td>
</tr>
<tr>
<td>Coma</td>
<td>$W = c_2 \cdot yr^3 \cos \varphi$</td>
<td>$\Delta y' \propto c_2 \cdot yr^2 \cdot (2 + \cos 2\varphi)$</td>
</tr>
<tr>
<td>Symmetry to Periodicity</td>
<td>one plane</td>
<td>$\Delta x' \propto c_2 \cdot yr^2 \sin 2\varphi$</td>
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<tr>
<td>one period</td>
<td></td>
<td>one straight line</td>
</tr>
<tr>
<td>Astigmatism</td>
<td>$W = c_3 \cdot y^2r^2 \cos^2 \varphi$</td>
<td>$\Delta x' = 0$</td>
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<tr>
<td>Symmetry to Periodicity</td>
<td>two planes</td>
<td>$\Delta y' \propto c_3 \cdot y^2r \cos \varphi$</td>
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<td>two period</td>
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<td>two straight lines</td>
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<tr>
<td>Field curvature (sagittal)</td>
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<tr>
<td>Symmetry to Periodicity</td>
<td>axis constant</td>
<td>$\Delta x' \propto c_4 \cdot y^2r \sin \varphi$</td>
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<td>point 1 period</td>
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<td>one period</td>
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<td>Distortion</td>
<td>$W = c_5 \cdot y^3r \cos \varphi$</td>
<td>$\Delta x' = 0$</td>
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<td>Symmetry to Periodicity</td>
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<td>$\Delta y' \propto c_5 \cdot y^3$</td>
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<tr>
<td>one period</td>
<td></td>
<td>one straight line</td>
</tr>
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Ref: H. Zügge
Typical Variation of Wave Aberrations

- Microscopic objective lens:
  Changes of rms value of wave aberration with wavelength

- Common practice:
  1. Diffraction limited on axis for main part of the spectrum
  2. Requirements relaxed in the outer field region
  3. Requirement relaxed at the blue edge of the spectrum

- Representation of the wave aberration with field position
Typical Variation of Wave Aberrations

- Representation of the wave aberration for defocussing at several field points
  - decrease of performance with field height
  - field curvature

- Wavefront over the pupil as surface
Typical Variation of Wave Aberrations

- Representation of the wave aberration as a function of field and wavelength for a microscopic lens.

- Analysis:
  1. diffraction limited correction near to axis for medium wavelength range
  2. no flattening
  3. blue edge more critical than red edge
Zernike Coefficients per Surface

- Contributions of the lower Zernike coefficients per surface, In logarithmic scale not additive (Fringe convention)

- Error in additivity due to numerical reasons for astigmatism
- Effect of induced aberrations and grid distortion in the range of $\lambda / 20$ in this case
TMA System

- Example system: plane symmetric TMA system nearly diffraction limited correction for a small field of view
  - $M_1$: off axis asphere, $M_2, M_3$: freeforms
- F-number 1.8, field $-1^\circ$ ...$+1^\circ$

<table>
<thead>
<tr>
<th>field angles x/y</th>
<th>$x = -1^\circ$</th>
<th>$x = 0^\circ$</th>
<th>$x = +1^\circ$</th>
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<td><img src="image8" alt="Image" /></td>
<td><img src="image9" alt="Image" /></td>
</tr>
</tbody>
</table>
- Surface contributions of every mirror with parabasal reference pupil rescaling neglected

- Dominating astigmatism

- Sum of wave aberration not additive, difference due to induced aberrations
- Low order Zernikes as a function of the field position

- Completly different distributions, Complete characterization gives a huge amount of detailed information.

- Also analytical solution for lower orders provided in the literature

PSD Ranges

- Typical impact of spatial frequency ranges on PSF
- Low frequencies: loss of resolution classical Zernike range
- High frequencies: Loss of contrast statistical
- Large angle scattering
- Mif spatial frequencies: complicated, often structured false light distributions
- The spatial frequency determines the effect of the wave front aberration

- Characteristic ranges, scaled on the diameter of the pupil:
  - figure error: Zernike causes resolution loss
  - midfrequency range
  - high frequency: roughness causes contrast loss
Hartmann Shack Wavefront Sensor

- Typical setup for component testing

- Lenslet array

![Diagram of the setup](image)

- Fiber illumination
- Collimator
- Beam-splitter
- Detector
- Lenslet array
- Telescope for adjustment of the diameter
- Test surface

![Images of subaperture and point spread function](image)

- Subaperture
- Point spread function

2-dimensional lenslet array
Spot Pattern of a HS - WFS

- Aberrations produce a distorted spot pattern
- Calibration of the setup for intrinsic residual errors
- Problem: correspondence of the spots to the subapertures

a) spherical aberration
b) coma
c) trefoil aberration
Hartmann Method

- Similar to Hastmann Shack Method with simple hole mask and two measuring planes
- Measurement of spot center position as geometrical transverse aberrations
- Problems: broadening by diffraction

\[ s'_{y} = s'_{1} + \left( s'_{2} - s'_{1} \right) \cdot \frac{y_{1}}{y_{1} + y_{2}} \]
- Real pinhole pattern with signal
- Problems with cross talk and threshold
Testing with Twyman-Green Interferometer

- Short common path, sensible setup
- Two different operation modes for reflection or transmission
- Always factor of 2 between detected wave and component under test

1. mode:
   - lens tested in transmission
   - auxiliary mirror for auto-collimation

2. mode:
   - surface tested in reflection
   - auxiliary lens to generate convergent beam
Interferograms of Primary Aberrations

Spherical aberration 1 $\lambda$

Astigmatism 1 $\lambda$

Coma 1 $\lambda$

-1  -0.5  0  +0.5  +1
Defocussing in $\lambda$
Critical definition of the interferogram boundary and the Zernike normalization radius in reality