Imaging and Aberration Theory

Lecture 9: Chromatical aberrations
2018-12-14
Herbert Gross
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1. Material dispersion
2. Partial dispersion
3. Anomalous partial dispersion
4. Axial chromatical error
5. Achromatic
6. Apochromates
7. Spherochromatism
8. Chromatical variation of magnification
9. Examples
Dispersion and Abbe number

- Description of dispersion:

  Abbe number

  \[ \nu(\lambda) = \frac{n(\lambda) - 1}{n_F' - n_C'} \]

- Visual range of wavelengths: typically d,F,C or e,F’,C’ used

  \[ \nu_e = \frac{n_e - 1}{n_F' - n_C'} \]

- Typical range of glasses
  \[ \nu_e = 20 \ldots 100 \]

- Two fundamental types of glass:
  Crown glasses:
  \( n \) small, \( \nu \) large, dispersion low
  Flint glasses:
  \( n \) large, \( \nu \) small, dispersion high
Abbe Number and Achromatization

- Curvatures $c_j$ of the radii of a lens

- Focal power at the center wavelength $e$ for a thin lens

- Difference in focal powers for outer wavelengths $F', C'$

with the Abbe number

- Focal length at the center wavelength

- Difference of the focal lengths for outer wavelengths

- Achromatization condition for two thin lenses close together

\[ c_1 = \frac{1}{r_1}, \quad c_2 = \frac{1}{r_2} \]

\[ F_e = (n_e - 1)(c_1 - c_2) = (n_e - 1) \cdot \Delta c \]

\[ \Delta F = F_{F'} - F_{C'} = (n_{F'} - n_{C'}) \cdot \Delta c = \frac{n_{F'} - n_{C'}}{n_e - 1} \cdot (n_e - 1) \Delta c = \frac{F_e}{\nu_e} \]

\[ \nu_e = \frac{n_e - 1}{n_{F'} - n_{C'}} \]

\[ f_e = \frac{1}{F_e} = \frac{1}{(n_e - 1) \Delta c} \]

\[ \Delta f = f_{F'} - f_{C'} = \frac{n_{C'} - n_{F'}}{(n_{F'} - 1)(n_{C'} - 1) \Delta c} \approx \frac{n_{C'} - n_{F'}}{(n_e - 1)^2 \Delta c} = - \frac{f_e}{\nu_e} \]

\[ \Delta F = \frac{F_1}{\nu_1} + \frac{F_2}{\nu_2} = \frac{1}{f_1 \nu_1} + \frac{1}{f_2 \nu_2} = 0 \]
Usual representation of glasses:
- diagram of refractive index vs dispersion $n(\nu)$

- Left to right:
  - Increasing dispersion
  - Decreasing Abbe number
Material with different dispersion values:
- Different slope and curvature of the dispersion curve
- Stronger change of index over wavelength for large dispersion
- Inversion of index sequence at the boundaries of the spectrum possible
Atomic Model of Dispersion

- Atomic model for the refractive index: oscillator approach of atomic field interaction

\[
(n_r + i \cdot n_i)^2 = \frac{N e^2}{2\pi \cdot c \varepsilon_0 m} \sum_j f_j \lambda^2 \lambda_j^2 + i \gamma_j \lambda \lambda_j^2
\]

- Sellmeier dispersion formula: corresponding function

\[
n^2 = A + \sum_j \frac{B_j \cdot \lambda^2}{\lambda^2 - C_j}
\]

- Special case of coupled resonances: example quartz, degenerated oscillators

\[
n^2 = A + \frac{B_0 \cdot \lambda_0^4}{(\lambda^2 - \lambda_0^2)^2} + \sum_{j=1}^{4} \frac{B_j \cdot \lambda^2}{\lambda^2 - C_j}
\]
Dispersionsformeln

- Schott-Formel
  Empirische
- Selmeier
  Basierend auf Oszillatormodell
- Bausch-Lomb
  Empirische
- Herzberger
  Basierend auf Oszillatormodell
- Hartmann
  Basierend auf Oszillatormodell

\[ n = \sqrt{a_o + a_1 \lambda^2 + a_2 \lambda^{-2} + a_3 \lambda^{-4} + a_4 \lambda^{-6} + a_5 \lambda^{-8}} \]

\[ n(\lambda) = \sqrt{A + B \frac{\lambda^2}{\lambda^2 - \lambda_1^2} + C \frac{\lambda^2}{\lambda^2 - \lambda_2^2}} \]

\[ n(\lambda) = \sqrt{A + B \lambda^2 + C \lambda^4 + \frac{D}{\lambda^2} + \frac{E \lambda^2}{(\lambda^2 - \lambda_o^2)} + \frac{F \lambda^2}{\lambda^2 - \lambda_o^2}} \]

\[ n(\lambda) = a_o + a_1 \lambda^2 + \frac{a_2}{\lambda^2 - \lambda_o^2} + \frac{a_3}{(\lambda^2 - \lambda_o^2)^2} \]

mit \( \lambda_o = 0.168 \mu m \)

\[ n(\lambda) = a_o + \frac{a_1}{a_3 - \lambda} + \frac{a_4}{a_5 - \lambda} \]
Relative partial dispersion:

- Change of dispersion slope with $\lambda$.
- Different curvature of dispersion curve.

Definition of local slope for selected wavelengths relative to secondary colors:

$$P_{\lambda_1, \lambda_2} = \frac{n(\lambda_1) - n(\lambda_2)}{n_F - n_C}$$

Special $\lambda$-selections for characteristic ranges of the visible spectrum:

- $\lambda = 656 / 1014$ nm far IR
- $\lambda = 656 / 852$ nm near IR
- $\lambda = 486 / 546$ nm blue edge of VIS
- $\lambda = 435 / 486$ nm near UV
- $\lambda = 365 / 435$ nm far UV
The relative partial dispersion changes approximately linear with the dispersion for glasses

\[ P_{\lambda_1, \lambda_2} = a_{\lambda_1, \lambda_2} \cdot \nu_d + b_{\lambda_1, \lambda_2} \]

- Nearly all glasses are located on the normal line in a P-\( \nu \)-diagram
- The slope of the normal line depends on the selection of wavelengths
- Glasses apart from the normal line shows anomalous partial dispersion \( \Delta P \)

\[ P_{\lambda_1, \lambda_2} = a_{\lambda_1, \lambda_2} \cdot \nu_d + b_{\lambda_1, \lambda_2} + \Delta P_{\lambda_1, \lambda_2} \]

these material are important for chromatical correction of higher order
Partial Dispersion

Anormal partial dispersion and normal line

$p_{g,F}$

$0.5000$ $0.5375$ $0.5750$ $0.6125$ $0.6500$

$90$ $80$ $70$ $60$ $50$ $40$ $30$ $20$ $10$ $0$
- There are some special glasses with a large deviation from the normal line.
- Of special interest: long crowns and short flints.
Anomalous Partial Dispersion

- Normal glasses:
  Partial dispersion changes linear with Abbe number

- Definition of P depends on selected wavelengths

- Normal line defined by F2 and K7

- Deviation from linear behavior: anomalous partial dispersion $\Delta P$

\[
P_{\lambda_1 \lambda_2} = a_{\lambda_1 \lambda_2} \cdot v_d + b_{\lambda_1 \lambda_2} + \Delta P_{\lambda_1 \lambda_2}
\]

- The value of $\Delta P$ depends on the wavelength selection

- Typical $\Delta P$ considered at the red and the blue end of the visible spectrum

- Large deviation values $\Delta P$ are necessary for apochromatic chromatical correction

\[
\begin{align*}
P_{C,t} &= 0.5450 + 0.004743 \cdot v_d \\
P_{C,s} &= 0.4029 + 0.002331 \cdot v_d \\
P_{F,e} &= 0.4884 + 0.000526 \cdot v_d \\
P_{g,F} &= 0.6438 + 0.001682 \cdot v_d \\
P_{i,g} &= 1.7241 + 0.008382 \cdot v_d
\end{align*}
\]
Arrows in the glass map: indication of the deviation from the normal line

Vertical component: at the red horizontal: at the blue end of the spectrum

\[ P_{\lambda_1\lambda_2} = a_{\lambda_1\lambda_2} \cdot v_d + b_{\lambda_1\lambda_2} + \Delta P_{\lambda_1\lambda_2} \]
Chromatical Aberrations

- Axial chromatical aberration:
  - dispersion of marginal ray
  - different image locations

- Transverse chromatical aberration:
  - dispersion of chief ray
  - different image sizes
1. Primary/1st order chromatical aberrations:
   - axial chromatical aberration
     error of the marginal ray by dispersion
   - transverse chromatical aberration
     error of the chief ray by dispersion

2. Higher order chromatical aberrations:
   - secondary spectrum
     residual axial error, if only selected wavelength are coinciding
   - spherochromatism
     chromatical variation of the spherical aberration, observed in an achromate
   - chromatical variation of all monochromatic aberrations
     e.g. astigmatism, coma, pupil location,...
Various cases of chromatic aberration correction

- a) axial and lateral color corrected
- b) axial color corrected
- c) lateral color corrected
- d) no color corrected
Axial chromatical aberration:

- Higher refractive index in the blue results in a shorter intersection length for a single lens.
- The colored images are defocussed along the axis.
- Definition of the error: change in image location /
  intersection length.
- Correction needs several glasses with different dispersion.
- Single lens: normal dispersion
  blue intersection length is shorter
  than red.
- Notations:
  1. CHL = chromatical longitudinal
  2. AXCL = axial chromatic.

\[ \Delta s'_{CHL} = s'_{F'} - s'_{C'} \]
- Longitudinal chromatical aberration for a single lens
- Best image plane changes with wavelength

Ref: H. Zügge
Secondary Spectrum

- Simple achromatization / first order correction:
  - two glasses with different dispersion
  - equal intersection length for outer wavelengths (blue F', red C')

- Residual deviation for middle wavelength (green e):
  secondary spectrum

\[ \Delta s'_{SS} = s'_\lambda - s'_C = -f', \frac{P^{(1)}_{\lambda,C'} - P^{(2)}_{\lambda,C'}}{V_1 - V_2} \]
Achromate: Basic Formulas

- **Idea:**
  1. Two thin lenses close together with different materials
  2. Total power
     \[ F = F_1 + F_2 \]

- **Achromatic correction condition**
  \[ \frac{F_1}{\nu_1} + \frac{F_2}{\nu_2} = 0 \]

- **Individual power values**
  \[ F_1 = \frac{1}{1 - \frac{\nu_2}{\nu_1}} \cdot F \]
  \[ F_2 = \frac{1}{1 - \frac{\nu_1}{\nu_2}} \cdot F \]

- **Properties:**
  1. One positive and one negative lens necessary
  2. Two different sequences of plus (crown) / minus (flint)
  3. Large \( \nu \)-difference relaxes the bendings
  4. Achromatic correction independent from bending
  5. Bending corrects spherical aberration at the margin
  6. Aplanatic coma correction for special glass choices
  7. Further optimization of materials reduces the spherical zonal aberration
- Compensation of axial colour by appropriate glass choice
- Chromatical variation of the spherical aberrations: spherochromatism (Gaussian aberration)
- Therefore perfect axial color correction (on axis) are often not feaseable

\[
\begin{align*}
\text{Achromate} \\
\text{(a)} & \quad \text{BK7} \\
\text{(b)} & \quad \text{BK7} \quad \text{F2} \\
n &= 1.5168 \quad 1.6200 \\
\nu &= 64.17 \quad 36.37 \\
F &= 2.31 \quad -1.31 \\
\end{align*}
\]
Achromate

- Achromate
- Longitudinal aberration
- Transverse aberration
- Spot diagram

\[ r_p \]

\[ 486 \text{ nm} \]  
\[ 587 \text{ nm} \]  
\[ 656 \text{ nm} \]
Achromate: Correction

- Cemented achromate:
  6 degrees of freedom:
  3 radii, 2 indices, ratio $\nu_1/\nu_2$

- Correction of spherical aberration:
  diverging cemented surface with
  positive spherical contribution
  for $n_{\text{neg}} > n_{\text{pos}}$

- Choice of glass: possible goals
  1. aplanatic coma correction
  2. minimization of spherochromatism
  3. minimization of secondary spectrum

- Bending has no impact on chromatical correction:
  is used to correct spherical aberration at the edge

- Three solution regions for bending
  1. no spherical correction
  2. two equivalent solutions
  3. one aplanatic solution, very stable
Bending of an Achromate - Aplanatic Case

- $\Delta n$ too small: no spherical correction
- Aplanatic case:
  - same zero points of bending for coma and spherical aberration
  - only one solution for spherical
- Large $\Delta n$:
  - Two solutions for bending with corrected spherical correction
  - no coincidence with coma zero point
- Appropriate glass combinations for aplanatic correction
- Case of NA = 0.1 with Rayleigh range $R_u = 0.0587$ mm
- Comparison of residual aberrations

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<th>Flint</th>
<th>Crown</th>
<th>Vendor crown</th>
<th>coma $z_8$</th>
<th>zonal spherical [mm]</th>
<th>secondary spectrum [mm]</th>
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<td>SF66</td>
<td>FEL4</td>
<td>Hoya</td>
<td>$-7 \times 10^{-9}$</td>
<td>-0.0148</td>
<td>0.0586</td>
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<tr>
<td>SF15</td>
<td>BAL15</td>
<td>Ohara</td>
<td>$9 \times 10^{-8}$</td>
<td>-0.0182</td>
<td>0.0487</td>
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<tr>
<td>SF6</td>
<td>N-BALF5</td>
<td>Schott</td>
<td>$2 \times 10^{-7}$</td>
<td>-0.0169</td>
<td>0.0531</td>
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<td>SF57</td>
<td>KZF2</td>
<td>Schott</td>
<td>$-2 \times 10^{-7}$</td>
<td>-0.0180</td>
<td>0.0600</td>
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<td>SF58</td>
<td>S-TIL2</td>
<td>Ohara</td>
<td>$-3 \times 10^{-6}$</td>
<td>-0.0170</td>
<td>0.0578</td>
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<td>SF1</td>
<td>ADC2</td>
<td>Hoya</td>
<td>$2 \times 10^{-7}$</td>
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<td>0.0492</td>
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<td>SF11</td>
<td>ADF1</td>
<td>Hoya</td>
<td>$5 \times 10^{-7}$</td>
<td>-0.0156</td>
<td>0.0648</td>
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Achromatic Solutions in the Glass Diagram

large $v$-differences give relaxed bendings

large $n$-differences give relaxed bendings
Correction axial color requires a larger \( \nu \)-difference in the glass map:
if the difference \( \Delta \nu \) becomes smaller, the axial focal powers are increasing

Correction of the spherical aberration requires a significant smaller \( n \) in the positive crown lens

Achromate

\[
\begin{align*}
D_n &< 0 \\
D_n &\text{very small} \\
D_n &\text{small} \\
\text{good solution} \\
\end{align*}
\]
Optimization of Achromatic Glasses

- For one given flint a line indicates the usefull crown glasses and vice versa.
- Perfect aplanatic line of corresponding glasses (corrected for coma).
- Condition:

\[
  n_2 \left( \frac{1}{r_2} - \frac{1}{s'_2} \right) \cdot \left[ \frac{n_1 + 1}{n_1} \cdot \frac{\nu_1}{\nu_2} - \frac{n_2 + 1}{n_2} \right] = -\frac{\nu_2}{\nu_1 - \nu_2} \cdot \left[ (n_1 - 1) \cdot \left( \frac{\nu_1}{\nu_2} \right)^2 + \frac{\nu_1}{\nu_2} - \frac{n_2}{n_2 - 1} \right]
\]
Achromate

- Residual aberrations of an achromate
- Clearly seen:
  1. Distortion
  2. Chromatical magnification
  3. Astigmatism
Surface and Lens contribution of Axial Color

- Considering the Abbe invariant

\[ Q_j = Q'_j \]

\[ n'_j \left( \frac{1}{r_j} - \frac{1}{s'_j} \right) = n_j \cdot \left( \frac{1}{r_j} - \frac{1}{s_j} \right) \]

- Derivative after the wavelength

\[
\frac{dn'_j}{d\lambda} \cdot \left( \frac{1}{r_j} - \frac{1}{s'_j} \right) + \frac{n'_j}{s_j} \cdot \frac{ds'_j}{d\lambda} = \frac{dn_j}{d\lambda} \cdot \left( \frac{1}{r_j} - \frac{1}{s_j} \right) + \frac{n_j}{s_j} \frac{ds_j}{d\lambda}
\]

\[ \omega_j = \frac{h_j}{h_1} \quad \omega_{j+1} = \omega_j \cdot \frac{s_{j+1}}{s'_j} \]

- Summing over all surfaces of a system with the marginal ray height ratio and the propagation of the ratio

\[
\Delta s'_{CHL} = -\frac{s'^2_N}{n'_N \omega_N^2} \sum_j \omega_j^2 \cdot \left[ \left( \frac{1}{r_j} - \frac{1}{s'_j} \right) \cdot \Delta \chi n'_j - \left( \frac{1}{r_j} - \frac{1}{s_j} \right) \cdot \Delta \chi n_j \right]
\]

\[
\Delta s'^2_{CHL} = -\frac{s'^2_n}{n'_N \omega_N^2} \sum_{j=1}^{N} \omega_j^2 \cdot Q_j \cdot \left[ \frac{n'_j - 1}{n'_j v'_j} \right] \cdot \left[ \frac{n_j - 1}{n_j v_j} \right] = -\frac{s'^2_N}{n'_N \omega_N^2} \sum_{j=1}^{N} K^2_{CHL}
\]

with the surface contribution coefficient

\[
K^2_{CHL} = \omega^2_j Q_j \cdot \left( \frac{n'_j - 1}{n'_j v'_j} - \frac{n_j - 1}{n_j v_j} \right)
\]
General Achromatization

- Contribution of a thin lens to the axial chromatical aberration
- Axial chromatical aberration of a system of thin lenses
- Condition of achromatization of a system of lenses
- Special case of lenses close together
- Condition of apochromatic (polychromatic) correction with the partial relative dispersion

\[ K_{lens}^{CHL} = \frac{\omega_j^2 \cdot F_j}{v_j} = \frac{\omega_j^2}{f'_j \cdot v_j} \]

\[ \Delta s_{CHL}' = - \frac{s'^2}{\omega_n^2} \cdot \sum_j \omega_j^2 \cdot \frac{F_j}{v_j} \]

\[ \sum_j \omega_j^2 \cdot \frac{F_j}{v_j} = 0 \]

\[ \sum_j \frac{F_j}{v_j} = 0 \]

\[ \sum_j \omega_j^2 \cdot \frac{P_j \cdot F_j}{v_j} = 0 \]
Dialyt-Achromat

- Dialyt approach: Achromatization with two lenses at finite distance
  \[ k = \frac{t}{f'_a} \]
- Scaling parameter \( k \):
- With finite marginal ray height
- Focal length condition
  \[ \frac{1}{f} = \frac{1}{f_a} + \frac{1-k}{f_b} \]
- Achromatization
  \[ \frac{y_a^2}{f_a \cdot v_a} + \frac{y_b^2}{f_b \cdot v_b} = 0 \]
- Focal lengths of the lenses
  \[ f_a = f \cdot \left[ 1 - \frac{v_b}{v_a (1-k)} \right] \]
  \[ f_b = f \cdot (1-k) \cdot \left[ 1 - \frac{v_a (1-k)}{v_b} \right] \]
- Lens distance
  \[ d = k \cdot \left[ 1 - \frac{v_b}{v_a \cdot (1-k)} \right] \cdot f \]
Dialyt Achromat

- Usage of only one glass material with achromatic correction: dialyt achromate
- No real imaging possible
- Parameters:
  \[ f_a = \frac{k f}{k - 1} \]
  \[ f_b = -k \cdot f \cdot (k - 1) \]
  \[ t = \frac{k^2}{k - 1} \cdot f \]
- Setup
Axial Color Correction with Schupman Lens

- Special layout of dialyte approach according to Schupman
- Mirror guarantees real imaging

\[ f_1 = 300 \text{ mm} \]
\[ f_2 = -100 \text{ mm} \]
Axial Colour: Apochromate

- Choice of at least one special glass
- Correction of secondary spectrum: anomalous partial dispersion
- At least one glass should deviate significantly from the normal glass line

Diagram showing waveguide properties with wavelengths 656 nm, 588 nm, 486 nm, and 436 nm.
- Focal power condition
- Achromatic condition
- Secondary spectrum
- Curvatures of lenses
  \[ c = \frac{1}{r_1} - \frac{1}{r_2} \]
- Parameter E

\[ F = F_1 + F_2 + F_3 \]
\[ \frac{F_1}{v_1} + \frac{F_2}{v_2} + \frac{F_3}{v_3} = 0 \]
\[ \frac{P_1 \cdot F_1}{v_1} + \frac{P_2 \cdot F_2}{v_2} + \frac{P_3 \cdot F_3}{v_3} = 0 \]

\[ c_a = \frac{1}{f \cdot E \cdot (v_a - v_c)} \cdot \frac{P_b - P_c}{n_{a,\lambda 1} - n_{a,\lambda 3}} \]
\[ c_b = \frac{1}{f \cdot E \cdot (v_a - v_c)} \cdot \frac{P_c - P_a}{n_{b,\lambda 1} - n_{b,\lambda 3}} \]
\[ c_c = \frac{1}{f \cdot E \cdot (v_a - v_c)} \cdot \frac{P_a - P_b}{n_{c,\lambda 1} - n_{c,\lambda 3}} \]

\[ E = \frac{1}{v_a - v_c} \left[ v_a \cdot (P_b - P_c) + v_b \cdot (P_c - P_a) + v_c \cdot (P_a - P_b) \right] \]

- The 3 materials are not allowed to be on the normal line
- The triangle of the 3 points should be large: small \( c_j \) give relaxed design
Preferred glass selection for apochromates
Effect of different materials

Axial chromatical aberration changes with wavelength

Different levels of correction:
1. No correction: lens, one zero crossing point
2. Achromatic correction:
   - coincidence of outer colors
   - remaining error for center wavelength
   - two zero crossing points
3. Apochromatic correction:
   - coincidence of at least three colors
   - small residual aberrations
   - at least 3 zero crossing points
   - special choice of glass types with anomalous partial dispertion necessity

Axial Colour: Achromate and Apochromate

![Diagram showing different correction levels with wavelength and zero crossing points.](chart.png)
Spherochromatism

- Spherochromatism: variation of spherical aberration with wavelength, Alternative notation: Gaussian chromatical error

- Individual curve of spherical aberration with color

- Conventional achromate:
  - coinciding image location for red (C’) and blue (F’) on axis (paraxial)
  - differences and secondary spectrum for green (e)
  - but different intersection lengths for finite aperture rays

- Better balancing with half spherochromatism on axis
Split of cemented surface: reduced zonal residual aberration possible

Larger distance of air gap: reduced spherochromatism

Correction principle: Different ray heights at second lens and different dependencies on ray heights:
Focus \[ \sim \omega^2 \]
Spherical aberration \[ \sim \omega^4 \]

Ref: D. Ochse
New Achromate

- Conventional achromate:
  strong bending of image shell, typical

\[ R_{ptz} = -1.3 \cdot f' \]

- Special selection of glasses:
  1. achromatization
     \[ \frac{F_1}{v_1} + \frac{F_2}{v_2} = 0 \]
  2. Petzval flattening
     \[ \frac{F_1}{n_1} + \frac{F_2}{n_2} = 0 \]

- Residual field curvature:
  \[ \frac{1}{R_{ptz}} = -\frac{1}{v_2 - v_1} \cdot \left( \frac{v_1}{n_1} - \frac{v_2}{n_2} \right) \cdot \frac{1}{f'} \]

- Combined condition
  \[ \frac{v_1}{v_2} = \frac{n_1}{n_2} \]

- But usually no spherical correction possible
This condition corresponds to the requirement to find two glasses on one straight line through the origin in the glass map

\[ \Delta = \frac{n_1 - n_2}{V_1 - V_2} = 0 \]

Examples:
- K5 / PSK51: \( \Delta = 0.00007 \)
- SF2 / N-LASF40: \( \Delta = 0.0060 \)
- LLF1 / LAK33: \( \Delta = 0.00033 \)

The solution is well known as simple photographic lens (landscape lens)
Principles of Glass Selection in Optimization

- Design rules for glass selection

- Different design goals:
  1. Color correction:
     large dispersion difference desired
  2. Field flattening:
     large index difference desired

Ref: H. Zügge
- Nearly equal refractive indices
- Difference in Abbe number not larger than 30

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Burried Surfaces

- Cemented component with plane outer surfaces
- For center wavelength only plane parallel plate, not seen in collimated light
- Curved cemeneted surface:
  - dispersion for outer spectral wavelengths
  - color correction without disturbing the main wavelength

- Example

![Diagram](image)
Lateral Color Aberration

- Dispersion of the chief ray deviation in the lens
- Effect resembles the dispersion of a prism in the upper part of the lens
- In the image plane, the differences in the colored ray angles cause changes in the ray height
- The lateral color aberrations corresponds to a change of magnification with the wavelength
Lateral chromatical aberration:
Higher refractive index in the blue results in a stronger ray bending of the chief ray for a single lens
The colored images have different size, the magnification is wavelength dependent
Definition of the error: change in image height/magnification
Correction needs several glasses with different dispersion
The aberration strongly depends on the stop position

\[
\Delta y'_{CHV} = y'_F - y'_C
\]

\[
\Delta \bar{y}'_{CHV} = \frac{y'_F - y'_C}{y'_e}
\]
Surface and Lens contribution of Lateral Color

- If the imaging of the entrance to the exit pupil suffers from axial chromatical aberrations, this delivers an error of the exit pupil location and also of the chief ray angle: chromatical lateral aberration

- Transverse chromatical aberration of a lens system

- Surface contribution coefficient of lateral color

- Corresponding lens summation formula

\[
\Delta y'_{\text{CHV}} = y'_n \cdot \sum_{j=1}^{n} H_j^{\text{CHV}}
\]

\[
H_j^{\text{CHV}} = \frac{s_1 \cdot s_{p1}}{s_{p1} - s_1} \cdot \omega_j \omega_{pj} Q_{pj} \cdot \left[ \left( \frac{n'_{j} - 1}{n'_{j} v'_{j}} \right) - \left( \frac{n_j - 1}{n_j v_j} \right) \right]
\]

\[
\Delta y'_{\text{CHV}} = y'_n \cdot \frac{s_1 \cdot s_{p1}}{s_{p1} - s_1} \sum_{j=1}^{n} \omega_j \omega_{pj} Q_{pj} \cdot \left( \frac{n_j - 1}{n_j v_j} \right)
\]

\[
\Delta y'_{\text{CHV}} = y'_n \cdot \frac{s_1 \cdot s_{p1}}{s_{p1} - s_1} \sum_{j=1}^{n} \omega_j \omega_{pj} \frac{F_j}{v_j}
\]
Lateral Color Correction: Principle of Symmetry

- Perfect symmetrical system: magnification $m = -1$
- Stop in centre of symmetry
- Symmetrical contributions of wave aberrations are doubled (spherical)
- Asymmetrical contributions of wave aberration vanishes $W(-x) = -W(x)$
- Easy correction of:
  - coma, distortion, chromatical change of magnification

![Diagram](image-url)
Chromatic Variation of Magnification

Representation of CHV:
1. Spot diagram
2. Magnification $m(\lambda)$
3. Transverse aberration:
   - offset of chief ray reference

![Diagram showing spot diagram, chromatical magnification difference, transverse aberration curves, and field height graphs.](attachment:image.png)
Lateral color \((LAC)\) for two different stop positions \(a\) and \(b\) (with stop sizes that define the same marginal ray) relates to longitudinal color \((LOC)\) like this:

\[
LAC^a = LAC^b + \Delta q \cdot LOC
\]

Where \(\Delta q\) is the “stop shift parameter” which is the same for every surface \(j\)

\[
\Delta q = \frac{h_{pj}^a - h_{pj}^b}{h_j}
\]

- If there is longitudinal color in the system, there will be a stop position for which lateral color vanishes
- If there is no longitudinal color, lateral color is independent of the stop position

Ref: D. Ochse
Example system with four N-BK7 lenses corrected for e-line
Goal: Correct also for C' and F' lines

1. Ensure the system has longitudinal color

2. Find the stop position for which lateral color vanishes

3. Correct longitudinal color there and move stop back

Ref: D. Ochse
Impression of CHV in real images
- Typical colored fringes blue/red at edges visible
- Color sequence depends on sign of CHV
Chromatical Difference in Magnification

Color rings are hardly seen due to colored image
Lateral shift of colored psf positions

Ref: J. Kaltenbach
Axial Chromatical Aberration

Special effects near black-white edges

Ref: J. Kaltenbach