Consider an optical system which images an off-axis object point at height \( y \) and suffers from Petzval field curvature \( S_{IV} \) and astigmatism \( S_{III} \) only (wave aberration coeff. in Seidel notation).

a) Show that Petzval field curvature can be understood as field dependent defocus \( d_f = \frac{S_{IV}}{4} y^2 \).

b) Calculate the location of the sagittal, tangential and ideal image surface in the presence of astigmatism and Petzval field curvature and sketch their field dependence.

c) Calculate the distance of the sagittal, tangential and ideal image of a given point object if the system is aberrated by \( \lambda/4 \) each of astigmatism and Petzval field curvature (\( \lambda = 0.5 \mu m \)). The exit pupil has a diameter of 1cm and is located at a distance of 10cm to the image plane.

**Hint:** Consider the wave-aberration function \( W(y, \rho, \phi) \) for astigmatism and Petzval field curvature for a fixed \( y \). A defocus term \( W(\rho, \phi) = d_f \rho^2 \) is related to a shift of the image plane of \( \Delta z = -\frac{BR^2}{nD_{XP}^2} d_f \), where \( R \) and \( D_{XP} \) are the reference radius and the diameter of the exit pupil.

**Exercise 5-2: Achromatic Prism**

Consider the refraction of a paraxial ray by a thin glass prism with index \( n(\lambda) \) and apex angle \( \epsilon \). Due to the dispersion of the glass, the refraction angles change with the wavelength \( \lambda \). Therefore, incident white light is split into different colors.

(a) Calculate the deflection angle \( \alpha \) for an incident ray of wavelength \( \lambda \) in paraxial approximation.

(b) Describe a possibility to measure the Abbe number \( \nu \) of a glass using such a prism.

Using two prisms of different material, the angular width of the spectrum can be minimized.

(c) Describe a corresponding setup consisting of two thin prisms made of BK7 and SF5 which has equal deflection angles for the F and C line. Estimate the remaining angular width of the spectrum by calculating the difference in the deflection angles of the F and d line.

(d) Compare with a thin prism made of BK7 which has the same deflection angle for the d line.

<table>
<thead>
<tr>
<th>glass</th>
<th>F (486nm)</th>
<th>d (587nm)</th>
<th>C (656nm)</th>
<th>( \nu_d )</th>
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<tbody>
<tr>
<td>BK7</td>
<td>1.5224</td>
<td>1.5168</td>
<td>1.5143</td>
<td>64.17</td>
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<tr>
<td>SF5</td>
<td>1.6875</td>
<td>1.6727</td>
<td>1.6666</td>
<td>32.25</td>
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</tbody>
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