Imaging and Aberration Theory

Lecture 7: Coma and Distortion
2013-12-12
Herbert Gross

Winter term 2013
<table>
<thead>
<tr>
<th></th>
<th>Date</th>
<th>Topic</th>
<th>Notes</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>24.10.</td>
<td>Paraxial imaging</td>
<td>paraxial optics, fundamental laws of geometrical imaging, compound systems</td>
</tr>
<tr>
<td>2</td>
<td>07.11.</td>
<td>Pupils, Fourier optics,</td>
<td>pupil definition, basic Fourier relationship, phase space, analogy optics and mechanics, Hamiltonian coordinates</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Hamiltonian coordinates</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>14.11.</td>
<td>Eikonal</td>
<td>Fermat Principle, stationary phase, Eikonals, relation rays-waves, geometrical approximation, inhomogeneous media</td>
</tr>
<tr>
<td>4</td>
<td>21.11.</td>
<td>Aberration expansion</td>
<td>single surface, general Taylor expansion, representations, various orders, stop shift formulas</td>
</tr>
<tr>
<td>5</td>
<td>28.11.</td>
<td>Representations of aberrations</td>
<td>different types of representations, fields of application, limitations and pitfalls, measurement of aberrations</td>
</tr>
<tr>
<td>6</td>
<td>05.12.</td>
<td>Spherical aberration</td>
<td>phenomenology, sph-free surfaces, skew spherical, correction of sph, aspherical surfaces, higher orders</td>
</tr>
<tr>
<td>7</td>
<td>12.12.</td>
<td>Distortion and coma</td>
<td>phenomenology, relation to sine condition, aplanatic systems, effect of stop position, various topics, correction options</td>
</tr>
<tr>
<td>8</td>
<td>19.12.</td>
<td>Astigmatism and curvature</td>
<td>phenomenology, Coddington equations, Petzval law, correction options</td>
</tr>
<tr>
<td>9</td>
<td>09.01.</td>
<td>Chromatical aberrations</td>
<td>Dispersion, axial chromatical aberration, transverse chromatical aberration, spherochromatism, secondary spectrum</td>
</tr>
<tr>
<td>10</td>
<td>16.01.</td>
<td>Further reading on aberrations</td>
<td>sensitivity in 3rd order, structure of a system, analysis of optical systems, lens contributions, Sine condition, isoplanatism, sine condition, Herschel condition, relation to coma and shift invariance, pupil aberrations, relation to Fourier optics</td>
</tr>
<tr>
<td>11</td>
<td>23.01.</td>
<td>Wave aberrations</td>
<td>definition, various expansion forms, propagation of wave aberrations, relation to PSF and OTF</td>
</tr>
<tr>
<td>12</td>
<td>30.01.</td>
<td>Zernike polynomials</td>
<td>special expansion for circular symmetry, problems, calculation, optimal balancing, influence of normalization, recalculation for offset, ellipticity, measurement</td>
</tr>
<tr>
<td>13</td>
<td>06.02.</td>
<td>Miscellaneous</td>
<td>Intrinsic and induced aberrations, Aldi theorem, vectorial aberrations, partial symmetric systems</td>
</tr>
</tbody>
</table>
1. Geometry of coma spot
2. Coma-dependence on lens bending, stop position and spherical aberration
3. Point spread function with coma
4. Distortion
5. Examples
- Occurrence of coma: skew chief ray and finite aperture
- Asymmetry between upper and lower coma ray
- Bended plane of sagittal coma rays
Ray Caustic of Coma

- A sagittal ray fan forms a groove-like surface in the image space.

- Tangential ray fan for coma: caustic.
Building of Coma Spot

- Coma aberration: for oblique bundles and finite aperture due to asymmetry
- Special problem: coma grows linear with field size \( y \)
- Systems with large field of view: coma hard to correct
- Relation of spot circles
  and pupil zones as shown
Coma

- Coma deviation, elimination of the azimuthal dependence: circle equation

- Diameter of the circle and position variation with $r_p^2$
  Every zone of the circle generates a circle in the image plane

- All circles together form a comet-like shape

- The chief ray intersection point is at the tip of the cone

- The transverse extension of the cone shape has a ratio of 2:3
  the meridional extension is enlarged and gives a poorer resolution

![Diagram of coma with circle equations and ray intersections]
Coma

- Ray trace properties
- Double speed azimuthal growth between pupil and image
- Sagittal coma smaller than tangential coma

\[ \Delta y_{\text{tan}} = 3 \cdot \Delta y_{\text{sag}} \]
Coma

- Typical representations of coma
- Cubic curve in wavefront cross section
- Quadratic function in transverse aberrations

Ref: H. Zügge
Coma Orientation

- Orientation of the coma shape: distinction between
  1. outer coma, tip towards optical axis
  2. inner coma, tip outside

- Orientation of the coma spot is always rotating with the azimuthal angle of the considered field point
Bending of a Lens

- Bending a single lens

- Variation of the primary aberrations

- The stop position is important for the off-axis aberrations

- Typical changes:
  1. coma linear
  2. chromatical magnification linear
  3. spherical aberration quadratically

![Diagram of aberrations](image-url)
Lens with Remote Stop

- Lens with remote stop
- Not all of the aberrations spherical, astigmatism and coma can be corrected by bending simultaneously
- Zero correction for coma and astigmatism possible (depends on stop position)
- Spherical aberration not correctable
- Effect of lens bending on coma
- Sign of coma: inner/outer coma

Ref: H. Zügge
The lens contribution of coma is given by
\[ C_{\text{Lens}} = \frac{1}{4ns'f^2} \left[ \frac{n+1}{n-1} X - (2n+1)M \right] \]
if the stop is located at the lens.

Therefore the coma can be corrected by bending the lens.

The optimal bending is given by
\[ X = \frac{(2n+1)(n-1)}{n+1} \cdot M \]
and corrects the 3rd order coma completely.

The stop shift equation for coma is given by
\[ S_{II}^* = S_{II} + \delta E \cdot S_I \]
with the normalized ratio of the chief ray height to the marginal ray height.

If the spherical aberration \( S_I \) is not corrected, there is a natural stop position with vanishing coma.

If the spherical aberration is corrected (for example by an aspheric surface), the coma doesn't change with the stop position.
Coma-free Stop Position

- Example
- The front stop position of a single lens is shifted
- The 3rd order Seidel coefficient as well as the Zernike coefficient vanishes at a certain position of the stop

Coma-free Stop Position

<table>
<thead>
<tr>
<th>$t_1$ (mm)</th>
<th>$A_c$</th>
<th>$C_8$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>-5.0</td>
<td>0.50</td>
</tr>
<tr>
<td>114.3</td>
<td>1.0</td>
<td>-0.5</td>
</tr>
<tr>
<td>150</td>
<td>-5.0</td>
<td>0.50</td>
</tr>
</tbody>
</table>

Coma-free Stop Position

$A_c = 114.3$ $C_8 = 114.5$
## Influence of Stop Position on Coma

Achromat 4/100, $w = 10^\circ$, $y' = 17.6$

<table>
<thead>
<tr>
<th>Tranverse aberr. $\pm 2.0$</th>
<th>Spot $w = 10^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w = 0^\circ$</td>
<td>$w = 10^\circ$</td>
</tr>
<tr>
<td>$\Delta y'$</td>
<td>$\Delta y'$</td>
</tr>
</tbody>
</table>

Ref: H. Zügge
Coma Correction: Achromate

- **Bending of an achromate**
  - optimal choice: small residual spherical aberration
  - remaining coma for finite field size
- **Splitting achromate:**
  - additional degree of freedom:
  - better total correction possible
  - high sensitivity of thin air space
- **Aplanatic glass choice:**
  - vanishing coma
- **Cases:**
  a) simple achromate, sph corrected, with coma
  b) simple achromate, coma corrected by bending, with sph
  c) other glass choice: sph better, coma reversed
  d) splitted achromate: all corrected
  e) aplanatic glass choice: all corrected

---

<table>
<thead>
<tr>
<th>Achromat bending</th>
<th>Image height:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$y' = 0\ mm$</td>
</tr>
<tr>
<td></td>
<td>meridional</td>
</tr>
<tr>
<td>Transverse Aberration:</td>
<td>$\Delta y'$</td>
</tr>
<tr>
<td>------------------</td>
<td>-----------</td>
</tr>
<tr>
<td></td>
<td>0.05 mm</td>
</tr>
</tbody>
</table>

**Pupil section:** meridional meridional sagittal

**Image height:** $y' = 0\ mm y' = 2\ mm$

**Wave length:** 0.486 0.588 0.656

Ref: H. Zügge
Achromat 4/100, \( w = 10^\circ \), \( y' = 17.6 \), (Asphäre für Sph. Aberr. = 0)

<table>
<thead>
<tr>
<th>Asphäre</th>
<th>Transverse aberr. + 2.0</th>
<th>Spot</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( w = 0^\circ )</td>
<td>( w = 10^\circ )</td>
</tr>
<tr>
<td></td>
<td>( \Delta y' )</td>
<td>( \Delta y' )</td>
</tr>
</tbody>
</table>
- Combined effect, aspherical case prevents correction

<table>
<thead>
<tr>
<th>Plano-convex element exhibits spherical aberration</th>
<th>Sagittal coma ( \Delta y' ) 0.5 mm</th>
<th>Spherical aberration corrected with aspheric surface</th>
<th>Sagittal coma ( \Delta y' ) 0.5 mm</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Diagram 1" /></td>
<td><img src="image2.png" alt="Diagram 2" /></td>
<td><img src="image3.png" alt="Diagram 3" /></td>
<td><img src="image4.png" alt="Diagram 4" /></td>
</tr>
<tr>
<td><img src="image5.png" alt="Diagram 5" /></td>
<td><img src="image6.png" alt="Diagram 6" /></td>
<td><img src="image7.png" alt="Diagram 7" /></td>
<td><img src="image8.png" alt="Diagram 8" /></td>
</tr>
<tr>
<td><img src="image9.png" alt="Diagram 9" /></td>
<td><img src="image10.png" alt="Diagram 10" /></td>
<td><img src="image11.png" alt="Diagram 11" /></td>
<td><img src="image12.png" alt="Diagram 12" /></td>
</tr>
</tbody>
</table>

Ref: H. Zügge
Coma Correction: Symmetry Principle

- Perfect coma correction in the case of symmetry
- But magnification \( m = -1 \) not useful in most practical cases

<table>
<thead>
<tr>
<th>Symmetry principle</th>
<th>Image height: ( y' = 19 \text{ mm} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pupil section:</td>
<td>meridional sagittal</td>
</tr>
<tr>
<td>Transverse Aberration:</td>
<td>( \Delta y'' ) ( 0.5 \text{ mm} )</td>
</tr>
</tbody>
</table>

(a) \( \Delta y'' \)
(b) \( \Delta y'' \)

Ref: H. Zügge
Geometrical Coma Spot

- Geometrical calculated spot intensity
  - The is a step at the lower circle boundary
- The peak lies in the apex point
- The centroid lies at the lower circle boundary
- The minimal rms radius is

\[
I(x, y) = \begin{cases} 
  \frac{I_{Exp} \cdot a^3}{2A_c \cdot R} \cdot \frac{1}{\sqrt{x^2 - 3y^2}} & \text{inside largest circle} \\
  \frac{I_{Exp} \cdot a^3}{A_c \cdot R} \cdot \frac{1}{\sqrt{x^2 - 3y^2}} & \text{else inside coma shape}
\end{cases}
\]

\[
r_{rms} = \sqrt{\frac{2}{3}} \cdot \frac{R \cdot A_c}{a}
\]
Psf for Coma Aberration

- PSF with coma
- The 1st diffraction ring is influenced very sensitive

$W_{31} = 0.03 \lambda$
$W_{31} = 0.06 \lambda$
$W_{31} = 0.09 \lambda$
$W_{31} = 0.15 \lambda$
PsF with Coma

\[ I(x) \]

\begin{align*}
W_{13} &= 0.3 \lambda \\
W_{13} &= 1.0 \lambda \\
W_{13} &= 2.4 \lambda \\
W_{13} &= 5.0 \lambda \\
W_{13} &= 10.0 \lambda
\end{align*}

Ref: Francon, Atlas of optical phenomena
Transversal Psf with Coma

- Change of Zernike coma coefficient
  - peak height reduced
  - peak position constant due to tilt component
  - distribution becomes asymmetrical

- Change of Seidel coma coefficient
  - peak height reduced
  - peak position moving
  - distribution becomes asymmetrical
- Separation of the peak and the centroid position in a point spread function with coma
- From the energetic point of view coma induces distortion in the image
- Defocus: centroid moves on a straight line (line of sight)
- Peak of intensity moves on a curve (bananicity)
- Centroid of the psf intensity
- Elementary physical argument: The centroid has to move on a straight line: line of sight
- Wave aberrations with odd order:
  - centroid shifted
  - peak and centroid are no longer coincident

\[ x_s(z) = \frac{\iint x \cdot I(x, y, z) \, dx \, dy}{\iint I(x, y, z) \, dx \, dy} = \frac{1}{P} \cdot \iint x \cdot I(x, y, z) \, dx \, dy \]

\[ y_s(z) = \frac{2 \cdot z}{D_{ExP}} \cdot \sum_{n=1,3,5,...} \sqrt{2(n+1)} \cdot c_{n1} \]
Image Degradation by Coma

- Imaging of a bar pattern with a coma of 0.4 \( \lambda \) in x and y
- Structure size near the diffraction limit
- Asymmetry due to coma seen in comparison of edge slopes

<table>
<thead>
<tr>
<th>object</th>
<th>Psf with 0.4( \lambda ) x-coma</th>
<th>image</th>
<th>Psf with 0.4( \lambda ) y-coma</th>
<th>image</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="object" /></td>
<td><img src="image2" alt="psf_x-coma" /></td>
<td><img src="image3" alt="image" /></td>
<td><img src="image4" alt="psf_y-coma" /></td>
<td><img src="image5" alt="image" /></td>
</tr>
<tr>
<td><img src="image6" alt="psf_enlarged" /></td>
<td><img src="image7" alt="image_x/y_section" /></td>
<td></td>
<td><img src="image8" alt="psf_enlarged" /></td>
<td><img src="image9" alt="image_x/y_section" /></td>
</tr>
</tbody>
</table>
Coma Truncation by Vignetting

without vignettierung

with vignettierung

tangential / sagittal

Ref: H. Zügge
Distortion Example: 10%

- What is the type of degradation of this image?
- Sharpness good everywhere!
Distortion Example: 10%

- Image with sharp but bended edges/lines
- No distortion along central directions

Ref: H. Zügge
Distortion

- Distortion. change of magnification over the field
- Corresponds to spherical aberration of the chief ray
- Measurement: relative change of image height

\[ V = \frac{y_{\text{real}} - y_{\text{ideal}}}{y_{\text{ideal}}} \]

- No image point blur
  only geometrical shape deviation
- Sign of distortion:
  1. \( V < 0 \): barrel,
     lens with stop in front
  2. \( V > 0 \): pincushion,
     lens with rear stop
Conventional definition of distortion

\[ V = \frac{\Delta y}{y} \]

Special definition of TV distortion

\[ V_{TV} = \frac{\Delta H}{H} \]

Measure of bending of lines

Acceptance level strongly depends on kind of objects:
1. geometrical bars/lines: 1% is still critical
2. biological samples: 10% is not a problem

Digital detection with image post processing: un-distorted image can be reconstructed
Distortion

- Purely geometrical deviations without any blur
- Distortion corresponds to spherical aberration of the chief ray
- Important is the location of the stop: defines the chief ray path
- Two primary types with different sign:
  1. barrel, $D < 0$
     - front stop
  2. pincushion, $D > 0$
     - rear stop
- Definition of local magnification changes

$$D = \frac{y'_{\text{real}} - y'_{\text{ideal}}}{y'_{\text{ideal}}}$$
Distortion and Stop Position

- Sign of distortion of a single lens depends on stop position
- Ray bending of chief ray determines the distortion

<table>
<thead>
<tr>
<th>Lens</th>
<th>Stop</th>
<th>Distortion</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>positive lens</td>
<td>rear stop</td>
<td>D &gt; 0</td>
<td>Tele lens</td>
</tr>
<tr>
<td>negative lens</td>
<td>front stop</td>
<td>D &gt; 0</td>
<td>Loupe</td>
</tr>
<tr>
<td>positive lens</td>
<td>front stop</td>
<td>D &lt; 0</td>
<td>Retro focus lens</td>
</tr>
<tr>
<td>negative lens</td>
<td>rear stop</td>
<td>D &lt; 0</td>
<td>reversed Binocular</td>
</tr>
</tbody>
</table>

Ref: H. Zügge
Distortion of Higher Order

- Combination of distortion of 3rd and 5th order:
- Bended lines with turning points
- Typical result for corrected/compensated distortion
Non-symmetrical systems:
Generalized distortion types
Correction complicated
- Distortion occurs, if the magnification depends on the field height $y$
- In the special case of an invariant location $p'$ of the exit pupil: the tangent of the angle of the chief ray should be scaled linear
- Airy tangent condition: necessary but not sufficient condition for distortion correction:
- This corresponds to a corrected angle of the pupil imaging from entrance to exit pupil

\[ \frac{\tan w'}{\tan w} = const \]
Reasons of Distortion

- Second possibility of distortion: the pupil imaging suffers from longitudinal spherical aberration
- The location of the exit pupil then depends on the field height
- With the simple relations
  \[ y = p \cdot \tan w, \quad y' = p' \cdot \tan w' \]

we have the general expression for the magnification

\[ m(y) = \frac{y'}{y} = \frac{p'(y) \cdot \tan w'}{p \cdot \tan w} = \left( \frac{p_o'}{p} + \frac{\Delta p'(y)}{p} \right) \cdot \frac{\tan w'}{\tan w} \]

- For vanishing distortion:
  1. the tan-condition is fulfilled (chief ray angle)
  2. the spherical aberration of the pupil imaging is corrected (chief ray intersection point)
Distortion of a Retrofocus System

Retro focus systems:

barrel distortion

- negatives
- front group
- stop
- positive
- rear group

barrel distortion

20%
Keystone distortion

- Tilting the object will result in a tilted image.
- The principal plane is the plane containing the optical axis and the image plane.
- The system's optical parameters affect the distortion.

\[
\begin{align*}
\theta & \to \theta' \\
h & \to h' \\
s & \to s' \\
y & \to y' \\
\end{align*}
\]
Distortion

- Visual impression of distortion on real images
- Visibility only at straight edges
- Edge through the center are not affected
Fish-Eye-Lens

- Example lens with 210° field of view
Fish-Eye-Lens

- Distortion types
Head Mounted Display

- Commercial system: Zeiss Cinemizer
- Critical performance of distortion due to asymmetry
- Refractive 3D-system
- Free-formed prism
- Field dependence of coma, distortion and astigmatism
- One coma nodal point
- Two astigmatism nodal points

![Diagram of Head-Up Display](image)