Imaging and Aberration Theory

Lecture 8: Astigmatism and field curvature
2012-12-14
Herbert Gross

Winter term 2012
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<td>Sensitivity in 3rd order, structure of a system, superposition and induced aberrations, analysis of optical systems, lens contributions, Sine condition, isoplanatism, sine condition, Herschel condition, isoplanatism, relation to coma and shift invariance, pupil aberrations, relation to Fourier optics and phase space</td>
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1. Geometrical astigmatism
2. Point spread function for astigmatism
3. Field curvature
4. Petzval theorem
5. Correcting field curvature
6. Examples
Astigmatism

- Reason for astigmatism:
  chief ray passes a surface under an oblique angle,
  the refractive power in tangential and sagittal section are different
- The astigmatism is influenced by the stop position
- A tangential and a sagittal focal line is found in different distances
- Tangential rays meet closer to the surface
- In the midpoint between both focal lines:
  circle of least confusion
Beam cross section in the case of astigmatism:

- Elliptical shape transforms its aspect ratio
- degenerate into focal lines in the focal plane distances
- special case of a circle in the midpoint: smallest spot
Primary Aberration Spot Shape for Astigmatism

- Seidel formulas for field point only in $y'$ considered
  \[
  \Delta y' = S' \cdot r_p^3 \cos \varphi_p + C' \cdot y' \cdot r_p^2 (2 + \cos 2\varphi_p) + (2A' + P') \cdot y'^2 \cdot r_p \cos \varphi_p + D' \cdot y'^3 \\
  \Delta x' = S' \cdot r_p^3 \sin \varphi_p + C' \cdot y' \cdot r_p^2 \sin 2\varphi_p + P' \cdot y'^2 \cdot r_p \sin \varphi_p
  \]

- Field curvature: circle
  \[
  \Delta y' = P' \cdot y'^2 \cdot r_p \cos \varphi_p, \quad \Delta x' = P' \cdot y'^2 \cdot r_p \sin \varphi_p
  \]
  \[
  \Delta x'^2 + \Delta y'^2 = P'^2 \cdot y'^4 \cdot r_p^2
  \]

- Astigmatism: focal line
  \[
  \Delta y' = 2A' \cdot y'^2 \cdot r_p \cos \varphi_p, \quad \Delta x' = 0
  \]

- General spot with defocus: wave aberration
  \[
  W(x, y) = c_{20} \cdot (x^2 + y^2) + c_{22} \cdot y^2
  \]
  \[
  \Delta x = -R \cdot \frac{\partial W}{\partial x} = -2Rc_{20}x, \quad \Delta y = -R \cdot \frac{\partial W}{\partial y} = -2Ry \cdot (c_{20} + c_{22})
  \]
  \[
  \frac{\Delta x^2}{(2Rc_{20})^2} + \frac{\Delta y^2}{[2Rr \cdot (c_{20} + c_{22})]^2} = 1
  \]
  \[
  \frac{\Delta x^2}{(Rc_{22})^2} + \frac{\Delta y^2}{(Rc_{22})^2} = 1
  \]

Transverse aberrations

Circle/ring in the pupil $r^2 = x^2 + y^2$

delivers an elliptical spot in the image

Special case for $c_{20} = -c_{22}/2$:
circle of least confusion
Astigmatism

- At sagittal focus
- Defocus only in tangentall cross section

Ref: H. Zügge
Astigmatism

- At median focus
- Anti-symmetrical defocus in T-S-cross section

Ref: H. Zügge
Astigmatism

- Imaging of a polar grid in different planes
  - Tangential focus:
    - blur in azimuthal direction
    - rings remain sharp
  - Sagittal focus:
    - blur in radial direction
    - spokes remain sharp
Astigmatism

- Behavior of a local position of the field:
  1. good resolution of horizontal structures
  2. bad resolution of vertical structures
- Imaging of polar diagram shows not the classical behaviour of the complete field
Coddington Equations

- For an oblique ray, the effective curvatures of the spherical surface depend on azimuth.
- There are two focal points for sagittal / tangential aperture rays.
- This splitting occurs already for infinitesimal aperture angles around the chief ray.
- Intersection lengths along the chief ray: Coddington equations.

\[
\frac{n'\cos^2 i'}{l'_{\tan}} - \frac{n\cos^2 i}{l_{\tan}} = \frac{n'\cos i' - n\cos i}{R}, \quad \frac{n'}{l'_{sag}} - \frac{n}{l_{sag}} = \frac{n'\cos i' - n\cos i}{R}
\]

- Right side: oblique power of the spherical interface.
- The sagittal image must be located on the auxiliary axis by symmetry.

Diagrams showing the relationships between the chief ray, auxiliary ray, tangential and sagittal images, and the surface properties.
Wavefront for Astigmatism with Defocus

- Astigmatic wavefront with defocus
- Purely cylindrical for focal lines in x/y
- Purely toroidal without defocus: circle of least confusion
- Zernike coefficients $c$ in $\lambda$
- Pure astigmatism
- Shape is not circular symmetric due to finite width of focal lines
Ref: Francon, Atlas of optical phenomena
- Bending effects astigmatism
- For a single lens 2 bending with zero astigmatism, but remaining field curvature
Field Curvature and Image Shells

- Imaging with astigmatism:
  Tangential and sagittal image shell sharp depending on the azimuth
- Difference between the image shells: astigmatism
- Astigmatism corrected:
  It remains one curved image shell,
  Bended field: also called Petzval curvature
- System with astigmatism:
  Petzval sphere is not an optimal
  surface with good imaging resolution
- No effect of lens bending on curvature,
  important: distribution of lens
  powers and indices
Astigmatisms and Curvature of Field

- Image surfaces:
  1. Gaussian image plane
  2. tangential and sagittal image shells (curved)
  3. mean image shell of best sharpness
  4. Petzval shell, arteficial, not a good image

- Seidel theory:
  \[
  \Delta s'_{\text{tan}} -\Delta s'_{\text{pet}} = 3 \cdot (\Delta s'_{\text{sag}} - \Delta s'_{\text{pet}})
  \]
  \[
  \Delta s'_{\text{pet}} = \frac{3\Delta s'_{\text{sag}} - \Delta s'_{\text{tan}}}{2}
  \]

- Astigmatism is difference
  \[
  \Delta s'_{\ast \text{ast}} = \Delta s'_{\text{tan}} - \Delta s'_{\text{sag}}
  \]

- Best image shell
  \[
  \Delta s'_{\text{best}} = \frac{\Delta s'_{\text{sag}} + \Delta s'_{\text{tan}}}{2}
  \]
Different possibilities for the correction of astigmatism and field curvature

Two independent aberrations allow 4 scenarious

- a) bended image plane
   - residual astigmatism
- b) bended image plane
   - corrected astigmatism
- c) flattened image plane
   - residual astigmatism
- d) flattened image plane
   - corrected astigmatism
Petzval Shell

- The Petzval shell is not a desirable image surface
- It lies outside the S- and T-shell:

\[ \Delta s_{pet}' = \frac{3\Delta s_{sag}' - \Delta s_{tan}'}{2} \]

- The Petzval curvature is a result of the Seidel aberration theory

\( \Delta s'_{ast} < 0 \)
\( \Delta s'_{sag} < \Delta s'_{pet} \)
\( \Delta s'_{tan} < \Delta s'_{sag} \)

\( \Delta s_{ast}' = 0 \)
\( \Delta s_{sag}' = \Delta s_{pet}' \)
\( \Delta s_{tan}' = \Delta s_{pet}' \)

\( \Delta s_{ast}' = -(2/3)\Delta s_{pet}' \)
\( \Delta s_{sag}' = (2/3)\Delta s_{pet}' \)
\( \Delta s_{tan}' = 0 \)

\( \Delta s_{ast}' = -\Delta s_{pet}' \)
\( \Delta s_{sag}' = (1/2)\Delta s_{pet}' \)
\( \Delta s_{tan}' = -\Delta s_{sag}' \)

\( \Delta s_{ast}' = -2\Delta s_{pet}' \)
\( \Delta s_{sag}' = 0 \)
\( \Delta s_{tan}' = -2\Delta s_{pet}' \)
- The image splits into two curved shells in the field
- The two shells belong to tangential / sagittal aperture rays
- There are two different possibilities for description:
  1. sag and tan image shell
  2. difference (astigmatism) and mean (medial image shell) of sag and tan

Ref: H. Zügge
Field Curvature

- Focussing into different planes of a system with field curvature
- Sharp imaged zone changes from centre to margin of the image field
- The image is generated on a curved shell
- In 3rd order, this is a sphere
- For a single refracting surface, the Petzval radius is given by

\[ r_p = -\frac{nr}{n' - n} \]

- For a system of several lenses, the Petzval curvature is given by

\[ \frac{1}{r_p} = -n' \sum_k \frac{1}{n_k \cdot f_k} \]
Petzval Theorem for Field Curvature

- Petzval theorem for field curvature:
  1. formulation for surfaces
    \[ \frac{1}{R_{ptz}} = -n_m' \sum_k \frac{n_k'^{-} - n_k}{n_k \cdot n_k'^{-} \cdot r_k} \]
  2. formulation for thin lenses (in air)
    \[ \frac{1}{R_{ptz}} = -\sum_j \frac{1}{n_j \cdot f_j} \]

- Important: no dependence on bending

- Natural behavior: image curved towards system

- Problem: collecting systems with \( f > 0 \):
  If only positive lenses:
  \( R_{ptz} \) always negative

- Optical system
  - Object plane
  - Ideal image plane
  - Real image shell
  - Optical system
Petzval Theorem

- Elementary derivation by a momocentric system of three surfaces:
  interface surface with r, object and image surface
- Consideration of a skew auxiliary axis
  \[ a = s - r \quad , \quad a' = s' - r \]
- Imaging condition
  \[ \frac{1}{n's'} - \frac{1}{ns} = \frac{n' - n}{nn'r} \]
- For the special case of a flat object gives
  \[ \frac{1}{R_p} - \frac{n' - n}{nr} \]
Petzval Theorem for Field Curvature

- **Goal:** vanishing Petzval curvature and positive total refractive power for multi-component systems

\[
R_{ptz} = - \sum_j \frac{1}{n_j \cdot f_j}
\]

\[
\frac{1}{f} = \sum_j \frac{h_j}{h_i} \cdot \frac{1}{f}
\]

- **Solution:**
  
  General principle for correction of curvature of image field:

  1. **Positive lenses with:**
     - high refractive index
     - large marginal ray heights
     - gives large contribution to power and low weighting in Petzval sum

  2. **Negative lenses with:**
     - low refractive index
     - small marginal ray heights
     - gives small negative contribution to power and high weighting in Petzval sum
Flattening Meniscus Lenses

- Possible lenses / lens groups for correcting field curvature
- Interesting candidates: thick meniscus shaped lenses

1. Hoeghs mensicus: identical radii
   - Petzval sum zero
   - remaining positive refractive power

   \[
   \frac{1}{R_{ptz}} = - \sum_k \frac{n_k' - n_k}{n_k \cdot n_k' \cdot r_k} = - \frac{1}{n \cdot f} + \left( \frac{n - 1}{n} \right)^2 \cdot \frac{d}{r_1 r_2}
   \]

   \[
   F' = \frac{(n - 1)^2 d}{n \cdot r^2}
   \]

2. Concentric meniscus,
   - Petzval sum negative
   - weak negative focal length
   - refractive power for thickness d:

   \[
   r_2 = r_1 - d
   \]

   \[
   \frac{1}{R_{ptz}} = \frac{(n - 1) \cdot d}{n r_1 \cdot (r_1 - d)}
   \]

   \[
   F' = - \frac{(n - 1)d}{nr_1(r_1 - d)}
   \]

3. Thick meniscus without refractive power
   Relation between radii

   \[
   r_2 = r_1 - d \cdot \frac{n - 1}{n}
   \]

   \[
   \frac{1}{R_{ptz}} = \frac{(n - 1)^2 \cdot d}{nr_1 \left[ nr_1 - d \cdot (n - 1) \right]} > 0
   \]
Correcting Petzval Curvature

- Group of meniscus lenses

- Effect of distance and refractive indices

\[ \frac{1}{R_{\text{pet}}} = \frac{5}{d} = 15 \text{ mm} \]

\[ \text{SF66} / d = 15 \text{ mm} \]

\[ \frac{K_5}{d} = 25 \text{ mm} \]

Ref: H. Zügge
Correcting Petzval Curvature

- Triplet group with + - +

- Effect of distance and refractive indices

Ref: H. Zügge
Flattening Field Lens

Effect of a field lens for flattening the image surface

1. Without field lens
   curved image surface

2. With field lens
   image plane
Conic Sections

- Explicit surface equation, resolved to $z$

Parameters: curvature $c = 1 / R$

conic parameter $\kappa$

- Influence of $k$ on the surface shape

\[
z = \frac{c(x^2 + y^2)}{1 + \sqrt{1 - (1 + \kappa)c^2(x^2 + y^2)}}
\]

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<th>Surface shape</th>
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<td>$\kappa = -1$</td>
<td>paraboloid</td>
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<tr>
<td>$\kappa &lt; -1$</td>
<td>hyperboloid</td>
</tr>
<tr>
<td>$\kappa = 0$</td>
<td>sphere</td>
</tr>
<tr>
<td>$\kappa &gt; 0$</td>
<td>oblate ellipsoid (disc)</td>
</tr>
<tr>
<td>$0 &gt; \kappa &gt; -1$</td>
<td>prolate ellipsoid (cigar)</td>
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- Relations with axis lengths $a, b$ of conic sections

\[
\begin{align*}
\kappa &= \left(\frac{a}{b}\right)^2 - 1 \\
c &= \frac{b}{a^2} \\
b &= \frac{1}{|c(1 + \kappa)|} \\
a &= \frac{1}{c\sqrt{|1 + \kappa|}}
\end{align*}
\]

- Radii of curvature

\[
R_T = \frac{1}{c} \cdot \left(1 - \kappa c^2 x^2\right)^{3/2}, \quad R_S = \frac{1}{c} \cdot \left(1 - \kappa c^2 x^2\right)^{1/2}
\]
Local Radii of Curvature

- Principal radii of curvature in and perpendicular to the plane of incidence

\[ R_{h1} = R \cdot \cos \theta , \quad R_{h2} = \frac{R}{\cos \theta} \]

- Projection of the principal radii of curvatur into an arbitrary plane with azimuthal angle \( \theta \):

\[
\begin{align*}
\frac{1}{R_{\parallel}} &= \frac{\cos^2 \theta}{R_{h1}} + \frac{\sin^2 \theta}{R_{h2}} \\
\frac{1}{R_{\perp}} &= \frac{\cos^2 \theta}{R_{h2}} + \frac{\sin^2 \theta}{R_{h1}}
\end{align*}
\]
- Plane parallel plate: image location changed, intersection length increased

\[ \Delta s' = \frac{n-1}{n} d \approx \frac{d}{3} \]

- Reflection prisms work optically as plane parallel plate inside optical systems

- Finite numerical aperture:
  Generation of spherical aberration
  Application: cover glass in microscopy

\[ \Delta s'_{sph} = \frac{d \cdot (n^2 - 1) \sin^2 u}{2n^3} \]

- Non-parallel ray path:
  Generation of astigmatism
  Application: Prism positions in collimated beam path preferred

\[ \Delta s'_{ast} = \frac{d \cdot (n^2 - 1) \cdot \sin^2 w}{n^3} \]
Astigmatism of Oblique Mirrors

- Mirror with finite incidence angle: effective focal lengths
  \[ f_{\text{tan}} = \frac{R \cdot \cos i}{2} \quad f_{\text{sag}} = \frac{R}{2 \cos i} \]

- Mirror introduces astigmatism
  \[ \Delta s'_{\text{ast}} = \frac{s^2 \cdot R \cdot \sin^2 i}{2 \cos i \cdot \left( s - \frac{R \cos i}{2} \right) \cdot \left( s - \frac{R}{2 \cos i} \right)} \]

- Parametric behavior of scales astigmatism
Field Curvature of a Mirror

- Mirror: opposite sign of curvature than lens
- Correction principle: field flattening by mirror
Microscope Objective Lens

- Possible setups for flattening the field
- Goal:
  - reduction of Petzval sum
  - keeping astigmatism corrected
- Three different classes:
  1. No effort
  2. Semi-flat
  3. Completely flat

![Diagram of possible setups for flattening the field with different classes: no effort, semi-flat, and completely flat. The diagram includes labeled setups a) to f) with corresponding configurations.](image-url)
An achromate is typically corrected for axial chromatical aberration.

The achromatization condition for two thin lenses close together reads:

\[ \frac{F_1}{v_1} + \frac{F_2}{v_2} = 0 \]

The Petzval sum usually is negative and the field is curved:

\[ \frac{1}{R_p} = -\sum_j \frac{1}{n_j f_j} \]

A flat field is obtained, if the following condition is fulfilled:

\[ \frac{F_1}{n_1} + \frac{F_2}{n_2} = 0 \]

This gives the special condition of simultaneous correction of achromatization and flatness of field:

\[ \frac{v_1}{v_2} = \frac{n_1}{n_2} \]
New Achromate

- This condition corresponds to the requirement to find two glasses on one straight line in the glass map.

- The solution is well known as simple photographic lens (landscape lens).
Field Curvature

- Correction of Petzval curvature in photographic lens Tessar
- Positive lenses: green $n_j$ small
- Negative lenses: blue $n_j$ large
- Correction principle: special choice of refractive indices

$$\frac{1}{R} = - \sum_j \frac{F_j}{n_j}$$

- Cemented component: New Achromate
- Spherical aberration not correctable in the New Achromate
Asymmetrical Anastigmatic Doublets

- Antiplanet

- Protar

- Dagor

- Orthostigmat
Field Curvature

- Correction of Petzval field curvature in lithographic lens for flat wafer

- Positive lenses: green \( h_j \) large, \( n_j \) small

- Negative lenses: blue \( h_j \) small, \( n_j \) large

- Correction principle: certain number of bulges

\[
\frac{1}{R} = -\sum_j \frac{F_j}{n_j}
\]

\[
F = \sum_j \frac{h_j}{h_1} \cdot F_j
\]
Field Flatness

- Principle of multi-bulges to reduce Petzval sum

\[
\frac{1}{r_p} = -n' \cdot \sum_k \frac{1}{n_k \cdot f_k}
\]

- Seidel contributions show principle
Field Flatness

- Effect of mirror on Petzval sum
- Flatness of field for catadioptric lenses
- Symmetrical system
- Astigmatism corrected
- Field curvature remains
Astigmatism of Eyeglasses with Rotating Eye

- Rotating eye: Astigmatism
- Coddington equations:
  Elliptical line with vanishing astigmatism:
  Tscherning ellipses

![Diagram of astigmatism with Coddington equations and Tscherning ellipses](image)

- Wollaston lens
- Ostwald lens
- No astigmatism

![Graph of refractive power](image)
Semiconductor laser sources show two types of astigmatism:
1. elliptical anamorphic aperture (not a real astigmatism)
2. z-variation of the internal source points in case of index guiding

In laser physics, the quadratic astigmatic beam shape is not considered to be a degradation, the $M^2$ is constant, it can be corrected by a simple cylindrical optic.
- Anamorphotic or cylindrical/toroidal system are used to get a circular profile form a semiconductor laser
- Example: laser beam collimator
45° Effects of Anamorphotic Systems

- Example of two aspherical cylindrical lenses with different focal lengths
- For high numerical apertures: no longer additivity and decoupling of x-and y-cross section
- The wavefront shows the deviations in the 45° directions
Decentered System

- Real systems with centering errors:
  - non-circular symmetric surface on axis
  - astigmatism on axis
  - point spread function no longer circular symmetric
Astigmatism due to Fabrication Errors

- Real surfaces with varying radius of curvature
- Toroidal surface shape with $R_{\text{max}}, R_{\text{min}}$
- Astigmatic effects on axis
- Irregularity errors in tolerancing measured by interferometry as ring difference
Astigmatic Correction by Clocking

- Two lens surfaces nearby with equal astigmatism due to manufacturing errors
- Azimuthal rotation of one lens against the other compensates astigmatism in first approximation
- This clocking procedure is used in practice for adjusting sensitive systems
Example Microscopic lens

Adjusting:
1. Axial shifting lens: focus
2. Clocking: astigmatism
3. Lateral shifting lens: coma

Ideal: Strehl $D_S = 99.62\%$
With tolerances: $D_S = 0.1\%$
After adjusting: $D_S = 99.3\%$

Ref.: M. Peschka
- Sucessive steps of improvements

Ref.: M. Peschka
Summary

- Astigmatism is caused by the different effective radii of curvature in the cross sections of a skew chief ray
- Crossed sagittal / tangential focal lines forms two sharp image shells
- The image locations are given by the Coddington equations and are already obtained in an infinitesimal environment of the chief ray
- Oblique mirror and prisms in converging beams generate astigmatism
- The mean image shell between S and T image shell defines a curved image
- The flatness of the image shell is given in third order, if the Petzval theorem is fulfilled
- The flattening can be obtained by thick meniscus lenses
- In particular oblique mirrors show a positive image curvature
- In real systems with manufacturing deviations, astigmatism can be observed already on axis