Imaging and Aberration Theory

Lecture 6: Spherical aberration
2012-11-23
Herbert Gross
<table>
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<tr>
<th>No.</th>
<th>Date</th>
<th>Topic</th>
<th>Details</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>19.10.</td>
<td>Paraxial imaging</td>
<td>paraxial optics, fundamental laws of geometrical imaging, compound systems</td>
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<td>2</td>
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<td>Pupils, Fourier optics, Hamiltonian coordinates</td>
<td>pupil definition, basic Fourier relationship, phase space, analogy optics and mechanics</td>
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<td>3</td>
<td>02.11.</td>
<td>Eikonal</td>
<td>Fermat Principle, stationary phase, Eikonals, relation rays-waves, geometrical approximation, inhomogeneous media</td>
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<tr>
<td>4</td>
<td>09.11.</td>
<td>Aberration expansion</td>
<td>single surface, general Taylor expansion, representations, various orders, stop shift formulas</td>
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<td>5</td>
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<td>Representations of aberrations</td>
<td>different types of representations, fields of application, limitations and pitfalls, measurement of aberrations</td>
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<td>6</td>
<td>23.11.</td>
<td>Spherical aberration</td>
<td>phenomenology, sph-free surfaces, skew spherical, correction of sph, aspherical surfaces, higher orders</td>
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<td>7</td>
<td>07.12.</td>
<td>Distortion and coma</td>
<td>phenomenology, relation to sine condition, aplanatic systems, effect of stop position, various topics, correction options</td>
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<td>8</td>
<td>14.12.</td>
<td>Astigmatism and curvature</td>
<td>phenomenology, Coddington equations, Petzval law, correction options</td>
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<td>9</td>
<td>21.12.</td>
<td>Chromatical aberrations, Sine condition, isoplanasy</td>
<td>Dispersion, axial chromatical aberration, transverse chromatical aberration, sine condition, Herschel condition, isoplanays, relation to coma and shift invariance, pupil aberrations, relation to Fourier optics and phase space</td>
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<tr>
<td>10</td>
<td>11.01.</td>
<td>Surface contributions</td>
<td>sensitivity in 3rd order, structure of a system, superposition and induced aberrations, analysis of optical systems, lens contributions</td>
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<td>11</td>
<td>18.01.</td>
<td>Wave aberrations</td>
<td>definition, various expansion forms, propagation of wave aberrations, relation to PSF and OTF</td>
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<tr>
<td>12</td>
<td>25.01.</td>
<td>Zernike polynomials</td>
<td>special expansion for circular symmetry, problems, calculation, optimal balancing, influence of normalization, recalculation for offset, ellipticity, measurement</td>
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<tr>
<td>13</td>
<td>01.02.</td>
<td>Miscellaneous</td>
<td>Aldi theorem, telecentric case, afocal case, aberration balancing, Delano diagram, Scheimpflug imaging, Fresnel lenses, statistical aberrations</td>
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<tr>
<td>14</td>
<td>08.02.</td>
<td>Vectorial aberrations</td>
<td>Introduction, special cases, actual research, anamorphotic, partial symmetric</td>
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</table>
1. Pending topics
2. Spherical aberration representations
3. Aplanatic surfaces
4. Single lens spherical aberration
5. Correction of spherical aberration
6. Higher order spherical aberration
7. Aspherical surfaces
Typical Variation of Wave Aberrations

- Representation of the wave aberration as a function of field and wavelength for a microscopic lens

- Analysis:
  1. diffraction limited correction near to axis for medium wavelength range
  2. no flattening
  3. blue edge more critical than red edge
The spatial frequency determines the effect of the wave front aberration.

Characteristic ranges, scaled on the diameter of the pupil:
- figure error: Zernike causes resolution loss
- midfrequency range
- high frequency: roughness causes contrast loss

![Graph showing log I(r) against r/\text{airy} for different g/D values.](image)
- Power spectral density of the perturbation
- Three typical frequency ranges, scaled by diameter D

\begin{align*}
\log A^2_{\text{Four}} & \quad \text{oscillation of the polishing machine} \\
1/D & \quad 12/D \\
\text{long range low frequency figure Zernike} & \quad \text{mid frequency} \quad \text{micro roughness} \quad 1/\lambda
\end{align*}

\text{limiting line slope } m = -1.5\ldots-2.5
Spherical Aberration: Angle of Incidence

- Spherical aberration: non-linearity of the law of refraction for finite angles of incidence $i$
- Example single plano-convex lens:
  1. bad orientation (red ray):
     \[ \sin i = i - \frac{i^3}{6} \]
  2. optimal orientation (green ray):
     approximation for refractive index $n=1.5$
     \[ 2 \cdot \sin \frac{i}{2} = i - \frac{2}{6} \left( \frac{i}{2} \right)^3 = i - \frac{i^3}{24} \]
- Spherical aberration differs by a factor of 4
Spherical Aberration

- Spherical aberration:
  - On axis, circular symmetry
- Perfect focussing near axis: paraxial focus
- Real marginal rays: shorter intersection length (for single positive lens)
- Optimal image plane: circle of least rms value
Spherical Aberration

- Single positive lens
- Paraxial focal plane near axis, largest intersection length
- Shorter intersection length for rim ray and outer aperture zones
Spherical Aberration: Best Image Location

- Spherical wave aberration

\[ W = A_d r_p^2 + A_s r_p^4 \]

- Best image location by choice of defocus parameter \( A_d \)

- Several solutions dependent on criterion

<table>
<thead>
<tr>
<th></th>
<th>defocus ( A_d ) ( [A_s] )</th>
<th>maximum spotradius ( [8 R A_s] )</th>
<th>rms spotradius ( [8 R A_s] )</th>
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<tbody>
<tr>
<td>Paraxial image position</td>
<td>0</td>
<td>1</td>
<td>0.5</td>
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<tr>
<td>Medium image position</td>
<td>-1</td>
<td>0.5</td>
<td>0.204</td>
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<tr>
<td>Smallest rms of spot</td>
<td>-1.333</td>
<td>0.333</td>
<td>0.167</td>
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<tr>
<td>Smallest diameter</td>
<td>-1.5</td>
<td>0.25</td>
<td>0.177</td>
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<tr>
<td>Marginal image position</td>
<td>-2</td>
<td>0.385</td>
<td>0.289</td>
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</tbody>
</table>
Spherical Aberration

- Different representations
- At paraxial Gaussian image location as reference

Ref: H. Zügge
Spherical Aberration

- Different representations
- At best plane, with optimal defocus

Primary spherical aberration with compensating defocus

Wave aberration
- Tangential
  - $2\lambda$
- Sagittal
  - $2\lambda$

Transverse ray aberration
- $\Delta y' = 0.01$ mm
- $\Delta x = 0.01$ mm
- $\Delta y' = 0.01$ mm

Modulation Transfer Function (MTF)
- MTF at paraxial focus
- MTF through focus for 100 cycles per mm

Geometrical spot through focus

Ref: H. Zügge
Psf for Spherical Aberration

- Point spread function for different amounts of spherical aberration and defocus
- Unsymmetrical behavior around the image plane (ring vs. compact profile)
- Symmetrical behavior for change of sign in $c_4$ and $c_9$
Delano‘s Representation of Spherical Aberration

- Paraxial optics: Delano relation
  \[ n' \cdot q' \cdot U' = n \cdot q \cdot U + n' \cdot i \cdot (Q' - Q) \]

- Real ray comparison:
  Delano surface contribution
  \[ \Delta s'_{\text{SPH}} = \Delta s_{\text{SPH}} \cdot \frac{n U_1 \sin u_1}{n' U'_1 \sin u'_1} + \sum_j \frac{(Q - Q') \cdot i \cdot n_j}{n' U'_j \sin u'_j} \]

- Third order approximation
  \[ \Delta s'_{\text{SPH-3}} = \Delta s_{\text{SPH}} \cdot \frac{n^2 U^2_1}{n' U^2'_1} + \sum_j \frac{n_j h_j \cdot I_j \cdot (I_j + U'_j) \cdot (I'_j - I_j)}{2n'_j U^2_j} \]

- Influence of ray bending angle
Delano’s Representation of Spherical Aberration

- Delano surface contribution

\[
\Delta s'_{SPH} = \Delta s_{SPH} \cdot \frac{n_1 U_1 \sin u_1}{n'_k U'_k \sin u'_k} + \sum_j \frac{n_j}{n'_j} \cdot h \cdot \sin i' - i \cdot \frac{2 \cdot \sin \frac{i' - u}{2}}{U'_j \sin u'_j}
\]

- Third order contribution grows with
  1. ratio of refractive indices
  2. height of the marginal ray
  3. Influence of ray bending angle
Spherical Corrected Surface

- Seidel contribution of spherical aberration with
  \[ \omega_j = \frac{h_j}{h_1}, \quad Q_j = n_j \cdot \left( \frac{1}{R_j} - \frac{1}{s_j} \right) \]

- Result

- Vanishing contribution:
  1. first bracket: vertex ray
  2. second bracket: concentric
  3. bracket: aplanatic surface

- Discussion with the Delano formula

\[
S_j = \omega_j^4 Q_j^2 \left( \frac{1}{n_j s'_j} - \frac{1}{n_j s_j} \right)
\]

\[
S_j = \left( \frac{h_j}{h_1} \right)^4 \cdot n_j^2 \cdot \left( \frac{1}{R_j} - \frac{1}{s_j} \right)^2 \cdot \left( \frac{1}{n_j s'_j} - \frac{1}{n_j s_j} \right)
\]

\[ h_j = 0 \]

\[ R_j = s_j \]

\[ n'_j s'_j = n_j s_j \]

- Discussion with the Delano formula

\[
\Delta s'_{SPH} = \Delta s'_{SPH} \cdot \frac{n_i U_1 \sin u_1}{n'_k U'_k \sin u'_k} + \sum_j n_j \cdot h \cdot \sin \frac{i'-i}{2} \cdot \frac{2i \cdot \sin \frac{i'-u}{2}}{U_j \sin u'_j}
\]

2. concentric corresponds to \( i' = i \)
3. aplanatic condition corresponds to \( i' = u \)
Aplanatic Surfaces with Vanishing Spherical Aberration

- Aplanatic surfaces: zero spherical aberration:
  1. Ray through vertex \( s' = s = 0 \)
  2. concentric \( s' = s \) \( \text{und} \) \( u = u' \)
  3. Aplanatic \( ns = n' s' \)

- Condition for aplanatic surface:
  \[ r = \frac{ns}{n+n'} = \frac{n' s'}{n+n'} = \frac{ss'}{s+s'} \]

- Virtual image location

- Applications:
  1. Microscopic objective lens
  2. Interferometer objective lens
- Aplanatic lenses
- Combination of one concentric and one aplanatic surface: zero contribution of the whole lens to spherical aberration
- Not useful:
  1. aplanatic-aplanatic
  2. concentric-concentric bended plane parallel plate, nearly vanishing effect on rays
Single Lens free of Spherical aberration

- Object location at infinity
- Refraction with only one surface: exact analytical solution

a) Rear surface
   exact hyperbola
   front surface plane
   \( \kappa_2 = -2.35 \)

b) Front surface
   exact ellipsoid
   rear surface
   concentric
   \( \kappa_1 = -0.434 \)
Aplanatic Surface

- Spherical aberration vanishes for all orders

- Aplanatic lens at high NA:
  effective real NA is higher than paraxial

- Further possible aplanatic lenses
  1. less practical importance
  2. used in microscopic objective front lens
Bending of a Lens

- Bending: change of shape for invariant focal length
- Parameter of bending

\[ X = \frac{r_1 + r_2}{r_2 - r_1} \]

Diagram showing different types of lenses based on the parameter \( X \):
- \( X < -1 \): meniscus lens
- \( X = -1 \): planconvex lens, planconcave lens
- \( X = 0 \): biconvex lens, biconcave lens
- \( X = 1 \): planconvex lens, planconcave lens
- \( X > 1 \): meniscus lens
Magnification Parameter

- Magnification parameter M:
  defines ray path through the lens

\[ M = \frac{U' + U}{U' - U} = \frac{1 + m}{1 - m} = \frac{2f}{s} + 1 = \frac{2f}{s'} - 1 \]

- Special cases:
  1. \( M = 0 \): symmetrical 4f-imaging setup
  2. \( M = -1 \): object in front focal plane
  3. \( M = +1 \): object in infinity

- The parameter M strongly influences the aberrations
Lens Contributions of Seidel

- Spherical aberration

\[
S_{\text{lens}} = \frac{1}{32n(n-1)f^3} \left[ \frac{n^3}{n-1} + \frac{n+2}{n-1} \left( X - \frac{2(n^2-1)}{n+2} \cdot M \right)^2 - \frac{n^2(n-1)}{n+2} \cdot M^2 \right]
\]

- Special impact on correction:
  1. Special quadratic dependence on bending \(X\)
     Minimum at
     \[X_{\text{sph min}} = -\frac{2(n^2-1)}{n+2} M\]
  2. No correction for small \(n\) and \(M\)
  3. Correction for large
     \(n\): infrared materials
     \(M\): virtual imaging
     Limiting value
     \[M_{s=0}^2 = \frac{n(n+2)}{(n-1)^2}\]
Spherical Aberration of a bended Lens

- Separation of the spherical aberration of a thin lens into the contributions of the two surfaces
- Effect of bending, represented by the curvature $c_1$ of the front surface

\[ i_1 = 0 \quad \text{first surface} \]
\[ i_2 = 0 \quad \text{second surface} \]

\[ \text{lens total} \]
Spherical Aberration: Lens Bending

- Spherical aberration and focal spot diameter as a function of the lens bending (for n=1.5)
- Optimal bending for incidence averaged incidence angles
- Minimum larger than zero: usually no complete correction possible

![Graph showing spherical aberration as a function of lens bending](image)
Lens of Best Shape

- Optimal bending of a focusing lens for collimated input: $M = +1$
  \[
  X_{sph\min} = -\frac{2(n^2 - 1)}{n + 2} M
  \]

- Plan convex lens for $n = 1.686$

- Higher indices: meniscus shape

<table>
<thead>
<tr>
<th>index</th>
<th>$n$</th>
<th>bending $X$</th>
<th>shape</th>
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<tbody>
<tr>
<td>1.4</td>
<td></td>
<td>-0.565</td>
<td>bi convex</td>
</tr>
<tr>
<td>1.686</td>
<td></td>
<td>-1</td>
<td>plan convex</td>
</tr>
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<td>2</td>
<td></td>
<td>-1.5</td>
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<td>-3.2</td>
<td>meniscus</td>
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<td>4</td>
<td></td>
<td>-5</td>
<td>meniscus</td>
</tr>
<tr>
<td>10</td>
<td></td>
<td>-16.5</td>
<td>meniscus</td>
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</tbody>
</table>
G-Sum Formula

- Alternative formula for the 3rd order spherical aberration of a thin lens:

  G-sum formula of Conrady

  \[
  \Delta s'_{sph} = -\frac{h^4}{n'u'} \cdot \sum G_i c_i^3 - G_2 c^2 c_1 + G_3 c^2 v + G_4 c c_1^2 - G_5 c c_1 v + G_6 c v^2
  \]

- Vergence / object distance
  \[ v = \frac{1}{s} \]

- Curvatures
  \[ c = \frac{n - 1}{f'}, \quad c_1 = \frac{1}{r_1} \]

- G-factors for refractive index
  \[
  G_1 = \frac{1}{2} \cdot n^2 \cdot (n - 1), \quad G_2 = \frac{1}{2} \cdot (2n + 1) \cdot (n - 1)
  \]
  \[
  G_3 = \frac{1}{2} \cdot (3n + 1) \cdot (n - 1), \quad G_4 = \frac{1}{n} \cdot (n + 2) \cdot (n - 1)
  \]
  \[
  G_5 = \frac{2}{n} \cdot (n^2 - 1), \quad G_6 = \frac{1}{2n} \cdot (3n + 2) \cdot (n - 1)
  \]
Spherical Aberration

- Lens bending for $n = 1.5$
Spherical Aberration

- Lens Bending for $n = 1.9$
• Änderung der Eingangsschnittweite für:
  1. Plankonvexlinse
  2. Bikonvexlinse
  3. Achromat
• Variation der sphärischen Aberration

Spherical Aberration: Object Distance

- $\lambda = 550 \text{ nm}$
- Linse : BK7
- $f = 100 \text{ mm}$
- $\sin \theta = 0.12$
Spherical Corrected Singlets

- Exact surface without spherical aberration
- Approach with Fermat principle
  \[ n \cdot z + n' \cdot \sqrt{(f - z)^2 + y^2} = f \cdot n' \]
- Result: depending on ratio of refractive indices
  \[ \left[ \frac{z - \frac{n' f}{n + n'}}{n' f} \right]^2 - \frac{n + n'}{n - n'} \cdot y^2 = 1 \]
- Ray bending at one surface only: very sensitive component
Spherical Aberration of an Achromate

- Achromatic condition does not contain the curvature:
- Bending can be used to correct for spherical aberration at the edge for the center wavelength
- Index difference must be small enough to allow for spherical correction
- Complex dependence on indices and Abbe number ratio
Spherical Aberration: Zone Error

- Wave aberration of 6th order
- Zonal coefficient $A_z$
- Spherical aberration with correction at the edge $A_z = -A_s$
- Smallest wave aberration at zone
- Smallest spot diameter for $A_d = -3/2A_s$ at

\[ \Delta s'_{\text{min}} = -\frac{3A_s}{\sin^2 u'} = \frac{3}{4} \Delta s'_{R}\]

with diameter

\[ D_{\text{min}} = \frac{2A_s}{\sin u'} = -\frac{1}{2} \Delta s'_{R} \sin u' \]

\[ = \frac{1}{4} D_{\text{parax}} \]

\[ W = A_d r_p^2 + A_s r_p^4 + A_z r_p^6 \]

\[ \Delta s'_{R} = 0 \]

\[ r_p = 1/\sqrt{2} = 0.707 \]
Correction of Spherical Aberration

- Comparison of lenses for the same aperture:
  1. single lens with optimal bending
  2. Dublett with optimal bending
  3. Triplet with optimal bending
  4. Triplet with aplanatic-concentric meniscus lenses
  5. Triplet with compensating negative lens
  6. Four lenses
## Correction of Spherical Aberration

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<thead>
<tr>
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<tbody>
<tr>
<td>1</td>
<td>Plankonvexlinse</td>
<td>366</td>
<td>206</td>
<td>5.21</td>
<td>1.7</td>
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<td>2</td>
<td>Dublett aus Plankonvexlinsen</td>
<td>136</td>
<td>76.8</td>
<td>1.91</td>
<td>5.2</td>
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<td>3</td>
<td>Zwiebelschalen-Dublett</td>
<td>63.9</td>
<td>36.2</td>
<td>0.903</td>
<td>12.8</td>
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<td>4</td>
<td>Dublett, randkorrigiert</td>
<td>26.1</td>
<td>13.5</td>
<td>0.221</td>
<td>23.1</td>
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<td>Achromat, verkittet</td>
<td>21.4</td>
<td>11.1</td>
<td>0.168</td>
<td>35.8</td>
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<td>6</td>
<td>Achromat, aufgerissen</td>
<td>5.1</td>
<td>2.54</td>
<td>0.024</td>
<td>97.3</td>
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<tr>
<td>7</td>
<td>Achromat mit Meniskus</td>
<td>2.94</td>
<td>1.48</td>
<td>0.0167</td>
<td>98.8</td>
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<tr>
<td>8</td>
<td>Zwiebelschalen-4-Linser</td>
<td>0.008</td>
<td>0.005</td>
<td>0.0001</td>
<td>100.0</td>
</tr>
</tbody>
</table>
Spherical Aberration Correction

- Correction of spherical aberration by splitting the ray bending
- Optimal bending of lenses
- Splitting of lenses
- Smooth reducing of spherical aberration or marginal correction

\[ W_{\text{rms}} = 5.21 \lambda \]
\[ W_{\text{rms}} = 1.91 \lambda \]
\[ W_{\text{rms}} = 0.91 \lambda \]
\[ W_{\text{rms}} = 0.221 \lambda \]
\[ W_{\text{rms}} = 0.168 \lambda \]
\[ W_{\text{rms}} = 0.026 \lambda \]
\[ W_{\text{rms}} = 0.0159 \lambda \]
\[ W_{\text{rms}} = 0.0001 \lambda \]
- Small difference in refractive index
- Growing higher order contributions
Higher Order Circular Symmetric Zernikes

- Zernike function with circular symmetry with growing order
- Normalized to +1 in centre and at the edge
- Growing spatial frequencies
- Highest slope at the edge of the pupil
Spherical Correction / Higher Orders

- Partial correction of residual spherical aberration by 5th order or 5th and 7th order
- Different (alternating) sign of coefficients
- Residual total error significantly smaller
- Large gradients at the edge
- 3rd and 5th order compensation: residual zonal error at \( \frac{1}{\sqrt{2}} = 0.707 \) of the pupil radius

a) 2 orders

b) 3 orders
Conic sections

- Explicite surface equation, resolved to $z$
  Parameters: curvature $c = 1 / R$
  conic parameter $\kappa$
- Influence of $\kappa$ on the surface shape

\[
z = \frac{c(x^2 + y^2)}{1 + \sqrt{1 - (1 + \kappa)c^2(x^2 + y^2)}}
\]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Surface shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa = -1$</td>
<td>paraboloid</td>
</tr>
<tr>
<td>$\kappa &lt; -1$</td>
<td>hyperboloid</td>
</tr>
<tr>
<td>$\kappa = 0$</td>
<td>sphere</td>
</tr>
<tr>
<td>$\kappa &gt; 0$</td>
<td>oblate ellipsoid (disc)</td>
</tr>
<tr>
<td>$0 &gt; \kappa &gt; -1$</td>
<td>prolate ellipsoid (cigar )</td>
</tr>
</tbody>
</table>

- Relations with axis lengths $a,b$ of conic sections

\[
\kappa = \left(\frac{a}{b}\right)^2 - 1 \\
c = \frac{b}{a^2} \\
b = \frac{1}{c(1 + \kappa)} \\
a = \frac{1}{c\sqrt{1 + \kappa}}
\]
Aspherical shape of conic sections

- Conic aspherical surface
- Variation of the conical parameter $\kappa$

$$z = \frac{cy^2}{1 + \sqrt{1 - (1 + \kappa) \cdot c^2 y^2}}$$
General Aspherical Surface

- Conic surface as basic shape
- Additional correction of the sag by a Taylor expansion
  Only even powers: no kink at r=0
- Mostly rotational symmetric shape considered
- Problems with this representation:
  1. added contributions not orthogonal, bad performance during optimization
  2. non-normalized representation, coefficients depend on absolute size of the diameter (very small high order coefficients for large diameters)
  3. Oscillatory behavior, large residual slope error can occur
  4. in optics slope and not sag is relevant
  5. the coefficients can not be measured/are hard to control, tolerancing is critical and complicated
  6. the added sag is along z, more important is a correction perpendicular to the surface for strong aspheres

\[ z(x, y) = \frac{c(x^2 + y^2)}{1 + \sqrt{1 - (1 + \kappa) \cdot c^2 (x^2 + y^2)}} + \sum_{k=2}^{k_{\text{max}}} a_k \cdot (x^2 + y^2)^k \]

\[ z(r) = \frac{c \cdot r^2}{1 + \sqrt{1 - (1 + \kappa) \cdot c^2 r^2}} + \sum_{k=2}^{k_{\text{max}}} a_k \cdot r^{2k} \]
Aspheres - Geometry

- Reference: deviation from sphere
- Deviation $\Delta z$ along axis
- Better conditions: normal deviation $\Delta r_s$
Aspherical Surfaces

- Additional degrees of freedom for correction
- Exact correction of spherical aberration for a finite number of aperture rays
- Strong asphere: many coefficients with high orders, large oscillative residual deviations in zones
- Location of aspherical surfaces:
  1. spherical aberration: near pupil
  2. distortion and astigmatism: near image plane
- Use of more than 1 asphere: critical, interaction and correlation of higher orders
Aspheres: Correction of Higher Order

- Correction at discrete sampling
- Large deviations between sampling points
- Larger oscillations for higher orders
- Better description: slope, defines ray bending

Corrected points with \( y' = 0 \), paraxial range

\[ y' = c \frac{dz_A}{dy} \]

residual spherical transverse aberrations

perfect correcting surface

points with maximal angle error

corrected points residual angle deviation

real asphere with oscillations
- Improvement by higher orders
- Generation of high gradients
Forbes Aspheres

- New representation of aspherical expansions according to Forbes (2007)

\[ z(r) = \frac{c \cdot r^2}{1 + \sqrt{1 - (1 + \kappa) \cdot c^2 r^2}} + \sum_{k=2}^{k_{\text{max}}} a_k \cdot Q_k(r^2) \]

- Special polynomials \( Q_k(r^2) \):
  1. Slope contributions are orthogonal in space and slope
  2. tolerancing is easily measurable
  3. optimization has better performance
  4. usually fewer coefficients are necessary
  5. use of normalized radial coordinate makes coefficients independent on diameter

- Two different versions possible:
  a) strong aspheres: deviation defined along \( z \)
  b) mild aspheres: deviation defined perpendicular to the surface
Forbes Aspheres

- New representations of Forbes
  Typical shape of contributions of the 6 lowest correction terms

a) strong  
b) mild
- Correction on axis and field point
- Field correction: two aspheres

Aspherical Single Lens

\[
\begin{align*}
\text{spherical} & \quad 250 \, \mu m & \quad 250 \, \mu m & \quad 250 \, \mu m \\
\text{one aspherical} & \quad 250 \, \mu m & \quad 250 \, \mu m & \quad 250 \, \mu m \\
\text{double aspherical} & \quad 250 \, \mu m & \quad 250 \, \mu m & \quad 250 \, \mu m
\end{align*}
\]
Lithographic Projection: Improvement by Aspheres

- Considerable reduction of length and diameter by aspherical surfaces
- Performance equivalent
- 2 lenses removable

<table>
<thead>
<tr>
<th>Type</th>
<th>NA</th>
<th>Lenses</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Spherical</td>
<td>0.8</td>
<td>31</td>
</tr>
<tr>
<td>b) Aspherical</td>
<td>0.8 , 8</td>
<td>29</td>
</tr>
</tbody>
</table>

Ref: W. Ulrich
- Location depending on correction target:
  - spherical: pupil plane
  - coma and astigmatism: field plane
- No effect on Petzval curvature
Aspherical Sensitivity

- Example:
  - Lithographic lens
- Sensitivities for aspherical correction
Aplanatic Aspherical Systems

- Aplanatic Telescope with two aspheres

- Point-by-point determination of aplanatic imaging conditions
Special correcting free shaped aspheres:
- Inversion of incoming wave front
- Application: final correction of lithographic systems