Design and Correction of Optical Systems

Lecture 8: Further performance criteria
2015-06-03
Herbert Gross

Summer term 2015
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<td>Diffraction, point spread function, PSF with aberrations, optical transfer function, Fourier imaging model</td>
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<tr>
<td>11</td>
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<td>01.07</td>
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<td>Overview, photographic lenses, microscopic objectives, lithographic systems, eyepieces, scan systems, telescopes, endoscopes</td>
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<td>13</td>
<td>08.07</td>
<td>Special System Examples</td>
<td>Zoom systems, confocal systems</td>
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<td>14</td>
<td>15.07</td>
<td>Further Topics</td>
<td>New system developments, modern aberration theory,...</td>
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</table>
1. PSF: line of sight and apodization
2. Edges and lines
3. Pupil aberrations
4. Sine condition
5. Induced aberrations
6. Vectorial aberrations
7. Fourier imaging formation
8. Caustics
- Deviation of centroid ray from chief ray

- First possibility:
  - asymmetrical apodization
  - coincidence in image plane

- Second possibility:
  - coma phase aberration
  - coincidence in pupil
Line of Sight

- Chief ray: centroid of geometrical pupil area
  \[ y_{c}^{(CR)}(z) = \frac{\iiint y \, dx \, dy}{\iiint dx \, dy} = \frac{1}{A} \cdot \iiint y \, dx \, dy \]

- Centroid ray: centroid of energy
  \[ y_{c}^{(Cen)}(z) = \frac{\iiint y \cdot I(x, y, z) \, dx \, dy}{\iiint I(x, y, z) \, dx \, dy} = \frac{1}{P} \cdot \iiint y \cdot I(x, y, z) \, dx \, dy \]

- Due to wave equation the centroid propagates along a straight line: Line of sight

- Wave aberrations of odd order in the azimuthal term influence the centroid
  - tilt and coma-like aberrations of any order
  - centroid has an offset against the peak of intensity
  - simple calculation possible:

  \[ y_{c}^{(CR)}(z) = \frac{2 \cdot z}{A_{ExP}} \cdot \sum_{n=1,3,5,\ldots}^{\infty} \sqrt{2(n+1)} \cdot c_{n} \]
- Pupil with apodization
e.g. non-homogeneous asymmetrical illumination
- Defocus: centroid moves on a straight line (line of sight)
- Peak of intensity moves on a curve (bananicity)
- **Apodisation of the pupil:**
  1. Homogeneous
  2. Gaussian
  3. Bessel

- **Psf in focus:**
  - different convergence to zero for larger radii

- **Encircled energy:**
  - same behavior

- **Complicated:**
  - Definition of compactness of the central peak:
    1. **FWHM:**
       - Airy more compact as Gauss
       - Bessel more compact as Airy
    2. **Energy 95%:**
       - Gauss more compact as Airy
       - Bessel extremly worse
- Small aperture: Diffraction limited
  Spot size corresponds to Airy diameter
  Spot size depends on wavelength

- Large aperture: Diffraction negligible
  Aberration limited
  Geometrical effects not wavelength dependent
  But: small influence of dispersion

\[
\text{Log } D_{\text{foc}} = 10 \frac{\text{m}}{\lambda} = 1 \frac{\text{m}}{\lambda}
\]

\[
\lambda = 10 \mu m
\]

\[
\lambda = 1 \mu m
\]

\[
\lambda = 550 \text{ nm}
\]
- Normalized axial intensity for uniform pupil amplitude

\[ I(u) = I_0 \cdot \left[ \frac{\sin u}{u} \right]^2 \]

- Decrease of intensity onto 80%:

\[ \frac{1}{2} \Delta z_{\text{diff}} = 0.493 \cdot \frac{\lambda}{n \cdot \sin^2 u} \approx \frac{1}{2} \cdot R_u \]

- Scaling measure: Rayleigh length
  - geometrical optical definition
  depth of focus: \( 1R_E \)

\[ R_u = \frac{\lambda}{n' \sin^2 u'} = \frac{n' \cdot \lambda}{NA^2} \]

- Gaussian beams: similar formula

\[ R_u = \frac{\lambda}{n' \pi \theta_o^2} \]
Fresnel Edge Diffraction

- Diffraction at an edge in Fresnel approximation

- Intensity distribution, Fresnel integrals $C(x)$ and $S(x)$

\[ I(t) = \frac{1}{2} \left[ \left( \frac{1}{2} - C(t) \right)^2 + \left( \frac{1}{2} - S(t) \right)^2 \right] \]

scaled argument

\[ t = \sqrt{\frac{k}{z \cdot \pi}} \quad \text{and} \quad x = \sqrt{\frac{2}{\lambda \cdot z}} \quad \text{and} \quad x = \sqrt{2N_F} \]

- Intensity:
  - at the geometrical shadow edge: 0.25
  - shadow region: smooth profile
  - bright region: oscillations
- ESF with defocussing
- ESF with spherical aberration
- Line image: integral over point spread function
  LSF: line spread function
- Realization: narrow slit convolution of slit width
- But with deconvolution, the PSF can be reconstructed

\[ I_{LSF}(x) = \int I_{PSF}(x, y) dy \]
- Line image:
  Fourier transform of pupil in one dimension

- Line spread function with aberrations
  Here: defocussing

\[
I_{LSF}(x_i) = \frac{\left\{ \int \left| P(x_p, y_p) \cdot e^{\frac{-2\pi i}{\lambda R} x_p x_p} \right|^2 dx_p \right\} dy_p}{\int \left[ \int \left| P(x_p, y_p) \right|^2 dx_p \right] dy_p}
\]

![Graph of Line Spread Function](image)
- Typical behavior of intensity of an edge image for residual aberrations
- The width of the distribution roughly corresponds to the diameter of the PSF
- Derivative of the edge spread function:
  
  edge position at peak location
- Spherical aberration of the chief ray / pupil imaging
- Exit pupil location depends on the field height
Pupil Aberration

- Interlinked imaging of field and pupil
- Distortion of object imaging corresponds to spherical aberration of the pupil imaging
- Corrected spherical pupil aberration: tangent condition

\[ \frac{\tan w'}{\tan w} = \text{const.} \]
- Eyepiece with pupil aberration

- Illumination for decentered pupil: dark zones due to vignetting
Sine Condition

- Lagrange invariante for paraxial angles $U$, $U'$
- sin-condition:
  - extension for finite aperture angle $u$
- Corresponds to energy conservation in the system
- Constant magnification for all aperture zones
- Pupil shape for finite aperture is a sphere
- Definition of violation of the sine condition:
  - OSC (offense against sine condition)
  - OSC = 0 means correction of sagittal coma (aplanatic system)

\[
ny \sin U = n'y' \sin U'
\]
\[
y \sin u = n'y' \sin u'
\]
\[
m = \frac{nU}{n'U'} = \frac{n \sin u}{n' \sin u'}
\]
• Optical path difference for two object points between object and image space
If for example a small field area and a widespread ray bundle is considered, a perfect imaging is possible.

The eikonal with the expression

\[ \delta L = n' \bar{s}' \cdot d\bar{r}' - n \bar{s} \cdot d\bar{r} \]

can be written for \( \delta L = 0 \) as

\[ n \cdot \bar{s} \cdot d\bar{r} = n' \bar{s}' \cdot d\bar{r}' \]

\[ n \cdot dr \cdot \cos \theta = n' \cdot dr' \cdot \cos \theta' \]

\[ n \cdot \cos \theta = n' \cdot \beta \cdot \cos \theta' \]

In the special case of an angle 90° we get with \( \cos(\theta) = \sin(u) \) the Abbe sine condition

\[ m = \frac{n \sin u}{n' \sin u'} \]

with the lateral magnification

\[ m = \frac{d\bar{r}'}{dr} \]
Tangential and Sagittal Coma

- 2 terms of tangential transverse aberration:
  - Sagittal coma depends on $x_p$, describes the asymmetry
  - Tangential coma depends on $y_p$, corresponds to spherical aberration under skew conditions larger by a factor of 3
- Only asymmetry removed with sine condition: sagittal coma vanishes
Skew Spherical aberration

- Decomposition of coma:
  1. Part symmetrical around chief ray: skew spherical aberration

\[ \Delta y^{\text{skewsph}} = \frac{\Delta y^{\text{upcom}} + \Delta y^{\text{lowcom}}}{2} \]

2. Asymmetrical part: tangential coma

\[ \Delta y^{\text{tangcoma}} = \frac{\Delta y^{\text{upcom}} - \Delta y^{\text{lowcom}}}{2} \]

- Skew spherical aberration:
  - Higher order aberration
  - Caustic symmetric around chief ray
Transfer of Energy in Optical Systems

- Conservation of energy
  
  \[ d^2 P = d^2 P' \]

- Invariant local differential flux
  
  \[ d^2 P = L \cdot \sin u \cdot \cos u \cdot dA \cdot du \cdot d\varphi \]

- Assumption: no absorption
  
  \[ T = 1 \]

- Delivers the sine condition
  
  \[ n y \cdot \sin u = n' y' \cdot \sin u' \]
- Sine condition not fulfilled:
  - nonlinear scaling from entrance to exit pupil
  - spatial filtering on warped grid, nonlinear sampling of spatial frequencies
  - pupil size changes
  - apodization due to distortion
  - wave aberration could be calculated wrong
  - quantitative measure of offence against the sine condition (OSC):
    distortion of exit pupil grid
    \[ D_p = \frac{x_{ap}}{f \cdot n \cdot \sin u} - 1 \]
Photometric effect of pupil distortion:
illumination changes at pupil boundary

- Effect induces apodization
- Sign of distortion determines the effect:
  outer zone of pupil brighter / darker
- Additional effect: absolute diameter of pupil changes
General Aplanatic Surface

- General approach of Fermat principle: aplanatic surface

$$n \cdot \sqrt{r^2 + (z-s)^2} + n' \cdot \sqrt{r^2 + (s'-z)^2} = s'n' - ns$$

- Cartesian oval, 4th order

- Special case OPD = 0: $s'n' = ns$

Solution is spherical aplanatic surface

$$n \cdot \sqrt{r^2 + (z-s)^2} + n' \cdot \sqrt{r^2 + (n/n's-z)^2} = 0$$

$$(n/n')^2 \cdot (r^2 + z^2 + s^2 - 2zs) = \left(r^2 + z^2 + (n/n')^2 s^2 - 2zsn/n'\right)$$

$$z^2 \left(n^2 / n'^2 - 1\right) - 2zs \left[(n/n')^2 - n/n'\right] + r^2 \left(n^2 / n'^2 - 1\right) = 0$$

$$\left[z - \frac{sn}{n+n'}\right]^2 + r^2 = \left[\frac{sn}{n+n'}\right]^2$$
Isoplanatism Condition of Staebel-Lihotzky

- Sagittal coma aberration: from the geometry of the figure and Lagrange invariant

\[ \Delta y'_{s} = \frac{y'}{m} \left[ \frac{n \sin u}{n' \sin u'} \cdot \frac{S' - s_p'}{S' - s_p' + \Delta s_{sph}'} - m \right] \]

- Condition of Staebel-Lihotzky

- Problems:  
  - no quantitative measure  
  - only tangential rays are considered  
  - integral criterion

---

**Diagram:**
- Marginal ray
- Chief ray
- Optical axis
- Projection of sagittal coma ray
- Real tangential image plane
- Ideal gaussian image plane
Piecewise Isoplanatism

- Invariance of PSF: to be defined
- Possible options:
  1. relative change of Strehl
  2. correlation of PSF’s
- Examples for microscopic lenses with and without flattening correction
- In medium field size: small isoplanatic patches
- On axis: large isoplanatic area
- Criteria not useful at the edge for low performance

<table>
<thead>
<tr>
<th>System</th>
<th>MO plane 100x1.25 isoplanatic patch size in μm</th>
<th>MO not plane 40x0.85 isoplanatic patch size in μm</th>
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<tr>
<td></td>
<td>Strehl 1%</td>
<td>Psf correlation 0.5%</td>
</tr>
<tr>
<td>on axis</td>
<td>70</td>
<td>72</td>
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<tr>
<td>half field</td>
<td>3.8</td>
<td>3.8</td>
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<tr>
<td>field zone</td>
<td>2.5</td>
<td>2.5</td>
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<tr>
<td>full field</td>
<td>45</td>
<td>3.8</td>
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</table>
Offence Against the Sine Condition

- Conradys OSC (offense against sine condition):
  - measurement of deviation of sagittal coma
  - quantitative validation of the sine condition

\[ \Delta_{OSC} = \frac{y_t' - y_s'}{y_t'} = 1 - \frac{n \sin u}{m \cdot n' \sin u'} \cdot \frac{S' - s'}{s' - s_p'} \]

- Only sagittal coma considered
  - in case of OSC=0 the Staeble-Lihotzky-condition is automatically fulfilled

\[ W_{coma}(y, r_p, 0) = r_p \cdot y_t \cdot \Delta_{OSC} \]

\[ \Delta y_t' = -3 y \cdot \left( m - \frac{n \sin u}{n' \sin u'} \right) \]

- OSC allows for the definition of surface contribution

\[ \Delta_{OSC} = \frac{\sin w_i}{\sin u_i} \cdot \sum_k \frac{(Q_k - Q_k') \cdot n_k i_k^{(CR)}}{h^* k n'_ k u'_ k} \]
## Overview on conditions for aberrations and aplanatism-isoplanatism

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<thead>
<tr>
<th>Nr</th>
<th>Sine cond.</th>
<th>Isoplanat cond.</th>
<th>Isoplanatism condition</th>
<th>Spherical aberration</th>
<th>Sagittal coma</th>
<th>Tangential coma</th>
<th>Imaging system</th>
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<tr>
<td>1</td>
<td>#</td>
<td>#</td>
<td></td>
<td>#</td>
<td>#</td>
<td>#</td>
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<tr>
<td>2a</td>
<td>#</td>
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<td>OSC=0, Conrady</td>
<td>#</td>
<td>0</td>
<td>#</td>
<td>isoplanatic-I</td>
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<tr>
<td>2b</td>
<td>#</td>
<td>✓</td>
<td>Staeble-Lihotzky / Berek</td>
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<td>0</td>
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<td>3a</td>
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<td></td>
<td>0</td>
<td>0</td>
<td></td>
<td>axial aplanatic</td>
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<tr>
<td>3b</td>
<td>✓</td>
<td>✓</td>
<td></td>
<td>0 (skew)</td>
<td>0</td>
<td>0</td>
<td>off-axis aplanatic</td>
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### Table

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<th>sine condition axial Aplanatism</th>
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<td>Tangential coma</td>
<td></td>
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<tr>
<td>Sagittal coma</td>
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<td>0</td>
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<tr>
<td>Spherical aberration</td>
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<td>0</td>
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<tr>
<td>Skew Spherical aberration</td>
<td></td>
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</table>
- Special idea of Seidel to consider the 3rd order as a perturbation of the paraxial ray
- Independent changes/contributions of every surface aberration to the final transverse aberration
- Therefore special reference on paraxial fundamental properties
- Aberration expansion: perturbation theory

- Linear independent contributions only in lowest correction order: Surface contributions of Seidel additive

- Higher order aberrations (5th order,...): nonlinear superposition
  - 3rd order generates different ray heights and angles at next surfaces
  - induces aberration of 5th order
  - together with intrinsic surface contribution: complete error

- Separation of intrinsic and induced aberrations: refraction at every surface in the system
- Example Gabor telescope
  - a lens pre-corrects a spherical mirror to obtain vanishing spherical aberration
  - due to the strong ray deviation at the plate, the ray heights at the mirror change significantly
  - as a result, the mirror has induced chromatical aberration, also the intrinsic part is zero by definition

- Surface contributions and chromatic difference (Aldi, all orders)
Vectorial Aberrations

- Wave aberration field
  \[ W(\vec{H}, \vec{r}_p) = \sum_{j,m,n} W_{klm} \cdot (\vec{H} \cdot \vec{H})^j \cdot (\vec{r}_p \cdot \vec{r}_p)^m \cdot (\vec{r}_p \cdot \vec{r}_p)^n \]

  indices
  \[ k = 2j + m, \quad l = 2n + m \]

- Normalized field vector: \( H \)
  normalized pupil vector: \( r_p \)
  angle between \( H \) and \( r_p \): \( \theta \)

- Expansion according to the invariants
  for circular symmetric components
  \[ \vec{H} \cdot \vec{H} = H^2, \quad r_p \cdot r_p = r_p^2, \quad \vec{H} \cdot \vec{r}_p = H \cdot r_p \cdot \cos \theta \]
Vectorial Aberrations

- Wave aberration field until the 6th order
- Analogue: transverse aberrations with

\[ \Delta \vec{H}' = - \frac{R}{n'} \cdot \nabla_{\vec{r}_p} W \]

<table>
<thead>
<tr>
<th>ord</th>
<th>j</th>
<th>m</th>
<th>n</th>
<th>Term</th>
<th>Name</th>
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<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( W_{000} )</td>
<td>uniform Piston</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>( W_{200} \cdot (\vec{H} \cdot \vec{H}) )</td>
<td>quadratic piston</td>
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<tr>
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<td>0</td>
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<td>0</td>
<td>( W_{111} \cdot (\vec{H} \cdot \vec{r}_p) )</td>
<td>magnification</td>
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<td>0</td>
<td>0</td>
<td>1</td>
<td>( W_{020} \cdot (\vec{r}_p \cdot \vec{r}_p) )</td>
<td>focus</td>
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<tr>
<td>4</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>( W_{040} \cdot (\vec{r}_p \cdot \vec{r}_p)^2 )</td>
<td>spherical aberration</td>
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<td>1</td>
<td>1</td>
<td>( W_{131} \cdot (\vec{r}_p \cdot \vec{r}_p) \cdot (\vec{H} \cdot \vec{r}_p) )</td>
<td>coma</td>
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<tr>
<td></td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>( W_{222} \cdot (\vec{H} \cdot \vec{r}_p)^2 )</td>
<td>astigmatism</td>
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<tr>
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<td>1</td>
<td>0</td>
<td>1</td>
<td>( W_{220} \cdot (\vec{H} \cdot \vec{H}) \cdot (\vec{r}_p \cdot \vec{r}_p) )</td>
<td>field curvature</td>
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<tr>
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<td>1</td>
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<td>( W_{331} \cdot (\vec{H} \cdot \vec{H}) \cdot (\vec{H} \cdot \vec{r}_p) )</td>
<td>distortion</td>
</tr>
<tr>
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<td>2</td>
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<td>0</td>
<td>( W_{400} \cdot (\vec{H} \cdot \vec{H})^2 )</td>
<td>quartic piston</td>
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<tr>
<td>6</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>( W_{240} \cdot (\vec{H} \cdot \vec{H}) \cdot (\vec{r}_p \cdot \vec{r}_p)^2 )</td>
<td>oblique spherical aberration</td>
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<td>1</td>
<td>1</td>
<td>( W_{331} \cdot (\vec{H} \cdot \vec{H}) \cdot (\vec{r}_p \cdot \vec{r}_p) \cdot (\vec{H} \cdot \vec{r}_p) )</td>
<td>coma</td>
</tr>
<tr>
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<td>0</td>
<td>( W_{422} \cdot (\vec{H} \cdot \vec{H}) \cdot (\vec{H} \cdot \vec{r}_p)^2 )</td>
<td>astigmatism</td>
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<tr>
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<td>1</td>
<td>( W_{420} \cdot (\vec{H} \cdot \vec{H}) \cdot (\vec{r}_p \cdot \vec{r}_p) )</td>
<td>field curvature</td>
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<td>( W_{531} \cdot (\vec{H} \cdot \vec{H}) \cdot (\vec{H} \cdot \vec{r}_p) )</td>
<td>distortion</td>
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<tr>
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<td>3</td>
<td>0</td>
<td>0</td>
<td>( W_{660} \cdot (\vec{H} \cdot \vec{H})^3 )</td>
<td>piston</td>
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<td>0</td>
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<td>( W_{060} \cdot (\vec{r}_p \cdot \vec{r}_p)^3 )</td>
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<td>( W_{151} \cdot (\vec{r}_p \cdot \vec{r}_p)^2 \cdot (\vec{H} \cdot \vec{r}_p) )</td>
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<tr>
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<td>2</td>
<td>1</td>
<td>( W_{242} \cdot (\vec{r}_p \cdot \vec{r}_p) \cdot (\vec{H} \cdot \vec{r}_p)^2 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>( W_{333} \cdot (\vec{H} \cdot \vec{r}_p)^3 )</td>
<td></td>
</tr>
</tbody>
</table>
Systems with Non-Axisymmetric Geometry

- Wave aberration

\[ W(\vec{H}, \vec{r}_p) = \sum_q \sum_{j,m,n} W_{kln} \cdot \left[ (\vec{H}_0 - \vec{\sigma}_q) \cdot (\vec{H}_0 - \vec{\sigma}_q)^T \right] \cdot \left[ (\vec{H}_0 - \vec{\sigma}_q) \cdot \vec{r}_p \right]^m \cdot (\vec{r}_p \cdot \vec{r}_p)^n \]

with shift vector

\[ \vec{H}_j = \vec{H}_j^0 - \vec{\sigma}_j \]

- In 3rd order:

1. spherical

\[ W(\vec{H}, \vec{r}_p) = \left[ \sum_q W_{040,q} \right] \cdot (\vec{r}_p \cdot \vec{r}_p)^2 \]

2. coma

\[ W(\vec{H}, \vec{r}_p) = \left[ \sum_q W_{131,q} \right] \cdot \vec{H}_0 - \left[ \sum_q W_{131,q} \cdot \vec{\sigma}_q \right] \cdot (\vec{r}_p \cdot \vec{r}_p) \cdot \vec{r}_p \]

3. astigmatism

\[ W(\vec{H}, \vec{r}_p) = \frac{1}{2} \left( \sum_q W_{220,q} \right) \cdot \vec{H}_0^2 - \left( \sum_q W_{222,q} \cdot \vec{\sigma}_q \right) \cdot \vec{H}_0 + \frac{1}{2} \left( \sum_q W_{222,q} \cdot \vec{\sigma}_q^2 \right) \cdot (\vec{r}_p \cdot \vec{r}_p)^2 \]

4. defocus

\[ W(\vec{H}, \vec{r}_p) = \left( \sum_q W_{220,q} + \frac{1}{2} \sum_q W_{222,q} \right) \cdot \vec{H}_0^2 - \left( 2 \sum_q W_{22,q} \cdot \vec{\sigma}_q + \frac{1}{2} \sum_q W_{222,q} \cdot \vec{\sigma}_q^2 \right) \cdot \vec{H}_0 \]

\[ + \left( \sum_q W_{220,q} + \frac{1}{2} \sum_q W_{222,q} \right) \cdot \vec{\sigma}_q^2 \]

5. distortion

\[ W(\vec{H}, \vec{r}_p) = \left[ \sum_q W_{311,q} \cdot \vec{H}_0^2 \cdot \vec{H}_0 - 2 \sum_q W_{311,q} \cdot (\vec{H}_0 \cdot \vec{\sigma}_q) \cdot \vec{H}_0 + \sum_q W_{311,q} \cdot \vec{\sigma}_q^2 \cdot \vec{H}_0 \right] \]

\[ + 2 \sum_q W_{311,q} \cdot (\vec{H}_0 \cdot \vec{\sigma}_q) \cdot \vec{\sigma}_q - \sum_q W_{311,q} \cdot \vec{\sigma}_q^2 \cdot \vec{\sigma}_q \]
Nodal Theory

- Expanded and rearranged 3rd order expressions:
  - aberrations fields
  - nodal lines/points for vanishing aberration

- Example coma:

  \[
  W_{\text{coma}} = \left( \sum_q W_{131,q} \right) \cdot \left[ \tilde{H}_o - \frac{\sum_q \tilde{\sigma}_q \cdot W_{131,q}}{\sum_q W_{131,q}} \cdot \vec{r}_p \right] \cdot (\vec{r}_p \cdot \vec{r}_p)
  \]

  abbreviation: nodal point location

  \[
  \tilde{a}_{131} = \frac{\sum_q \tilde{\sigma}_q \cdot W_{131,q}}{\sum_j W_{131,q}} = \frac{\sum_q \tilde{\sigma}_q \cdot W_{131,q}}{W^{(c)}_{131}}
  \]

  one nodal point with vanishing coma

  \[
  W_{\text{coma}} = W^{(c)}_{131} \cdot \left[ (\tilde{H}_o - \tilde{a}_{131}) \cdot \vec{r}_p \right] \cdot (\vec{r}_p \cdot \vec{r}_p)
  \]
- Refractive 3D-system
- Free-formed prism
- One coma nodal point
- Two astigmatism nodal points
Fourier Optics – Point Spread Function

- Optical system with magnification \( m \)
  - Pupil function \( P \)
  - Pupil coordinates \( x_p, y_p \)

\[
g_{psf}(x, y, x', y') = N \cdot \int\int P(x_p, y_p) \cdot e^{-\frac{ik}{z} [x_p(x'-mx) + y_p(y'-my)]} \, dx_p \, dy_p
\]

- PSF is Fourier transform of the pupil function (scaled coordinates)

\[
g_{psf}(x, y) = N \cdot \hat{F}[P(x_p, y_p)]
\]
Fourier Theory of Incoherent Image Formation

- Transfer of an extended object distribution \( I(x,y) \)

- In the case of shift invariance (isoplanasy): incoherent convolution

- Intensities are additive

\[
I_{inc}(x',y') = \iint_{-\infty}^{\infty} |g_{psf}(x',x,y,y)|^2 \cdot I(x,y) \, dxdy
\]

\[
I_{inc}(x',y') = \iint_{-\infty}^{\infty} |g_{psf}(x'-x,y'-y)|^2 \cdot I(x,y) \, dxdy
\]

\[
I_{image}(x', y') = I_{psf}(x, y) \ast I_{obj}(x, y)
\]
Fourier Theory of Incoherent Image Formation

Object intensity $I(x,y)$

Squared PSF, intensity-response $I_{psf}(x_p, y_p)$

Result

Object intensity spectrum $I(v_x, v_y)$

Optical transfer function $H_{OTF}(v_x, v_y)$

Result

Image intensity $I'(x', y')$

Image intensity spectrum $I'(v_x', v_y')$
Snellen test chart
Visual Acuity

- Recognition of simple geometrical shapes:
  1. Landolt ring with gap
  2. Letter 'E'
- Blur of image on retina with distance

![Diagram showing visual acuity tests and blur quantification](attachment:diagram.png)
Siemens Star Test Plate
Real Image with Different Chromatical Aberrations

original object  good image  color astigmatism 2 \( \lambda \)

6% lateral color  axial color 4 \( \lambda \)
Caustics

- Early investigations on caustics: Leonardo da Vinci 1508
- Caustics at mirrors and lenses

Ref: J. Nye, Natural focusing
More general:
caustic occurs at every wavefront with concave shape as locus of local curvature

Physically:
- crossing of rays indicates a caustic
- interference with diffraction ripple and ringing is seen

Ref: J. Nye, Natural focusing
Ref: W. Singer
Caustics

- Caustic: envelope of rays
- Locus of local curvature
- Calculation:
  - Caustic: \( \vec{r}_c = (x_c, y_c, z_c) \)
  - Ray direction: \( \vec{s} = (s_x, s_y, s_z) \)
  - Rays: \( \vec{r}_c = \vec{r} + L \cdot \vec{s} \)
  - L distance PC

Variation of point on wavefront:

Solution condition for linear system:

Equation of caustic

\[
\begin{vmatrix}
1 + L \frac{\partial s_x}{\partial x} & L \frac{\partial s_x}{\partial y} & s_x \\
L \frac{\partial s_y}{\partial x} & 1 + L \frac{\partial s_y}{\partial y} & s_y \\
s_x & s_y & 1
\end{vmatrix} = 0
\]

\[
x_c = x + L \cdot s_x \\
y_c = y + L \cdot s_y \\
z_c = L \cdot s_z
\]

\[
\delta x + L \frac{\partial s_x}{\partial x} \cdot \delta x + L \frac{\partial s_x}{\partial y} \cdot \delta y + s_x \cdot \delta L = 0
\]

\[
\delta y + L \frac{\partial s_y}{\partial x} \cdot \delta x + L \frac{\partial s_y}{\partial y} \cdot \delta y + s_y \cdot \delta L = 0
\]

\[
L \frac{\partial s_z}{\partial x} \cdot \delta x + L \frac{\partial s_z}{\partial y} \cdot \delta y + s_z \cdot \delta L = 0
\]
Special case of one dimension $x-z$

Example: spherical aberration for focussing through plane interface

Ray direction

$$s_x = \frac{\partial W}{\partial x}$$

Variation

$$(1 + L \cdot \frac{\partial s_x}{\partial x}) \cdot \delta x + s_x \cdot \delta L = 0$$

$$L \cdot \frac{\partial s_x}{\partial x} \cdot \delta x + s_z \cdot \delta L = 0$$

Geometry and law of refraction

$$s_x = \frac{\partial W}{\partial x} = -n \cdot \frac{x}{a} = -n \cdot \frac{x}{\sqrt{q^2 + x^2}}$$

$$L = -\frac{s_x^2}{\frac{\partial s_x}{\partial x}} = \frac{a}{n} \left[ 1 - (n^2 - 1) \cdot \frac{x^2}{q^2} \right]$$

Approximation of small $x$:
caustic curve

$$z_c = \frac{q}{n} - \frac{3}{2n} \cdot \left[(n^2 - 1) \cdot q \right]^{1/3} \cdot x_c^{2/3}$$