Institute of Applied Physics
Friedrich-Schiller-Universität Jena

Design and Correction of Optical Systems

Lecture 2: Materials and components

2015-04-22

Herbert Gross

Preliminary Schedule

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1	15.04.	Basics	Law of refraction, Fresnel formulas, optical system model, raytrace, calculation approaches
2	22.04.	Materials and Components	Dispersion, anormal dispersion, glass map, liquids and plastics, lenses, mirrors, aspheres, diffractive elements
3	29.04.	Paraxial Optics	Paraxial approximation, basic notations, imaging equation, multi-component systems, matrix calculation, Lagrange invariant, phase space visualization
4	06.05.	Optical Systems	Pupil, ray sets and sampling, aperture and vignetting, telecentricity, symmetry, photometry
5	13.05.	Geometrical Aberrations	Longitudinal and transverse aberrations, spot diagram, polynomial expansion, primary aberrations, chromatical aberrations, Seidels surface contributions
6	20.05.	Wave Aberrations	Fermat principle and Eikonal, wave aberrations, expansion and higher orders, Zernike polynomials, measurement of system quality
7	27.05.	PSF and Transfer function	Diffraction, point spread function, PSF with aberrations, optical transfer function, Fourier imaging model
8	03.06.	Further Performance Criteria	Rayleigh and Marechal criteria, Strehl definition, 2-point resolution, MTF-based criteria, further options
9	10.06.	Optimization and Correction	Principles of optimization, initial setups, constraints, sensitivity, optimization of optical systems, global approaches
10	17.06.	Correction Principles I	Symmetry, lens bending, lens splitting, special options for spherical aberration, astigmatism, coma and distortion, aspheres
11	24.06.	Correction Principles II	Field flattening and Petzval theorem, chromatical correction, achromate, apochromate, sensitivity analysis, diffractive elements
12	01.07.	Optical System Classification	Overview, photographic lenses, microscopic objectives, lithographic systems, eyepieces, scan systems, telescopes, endoscopes
13	08.07.	Special System Examples	Zoom systems, confocal systems
14	15.07.	Further Topics	New system developments, modern aberration theory,...

Contents

Important Test Wavelengths

λ in [nm]	Name	Color	Element
248.3		UV	Hg
280.4		UV	Hg
296.7278		UV	Hg
312.5663		UV	Hg
334.1478		UV	Hg
365.0146	i	UV	Hg
404.6561	h	violett	Hg
435.8343	g	blau	Hg
479.9914	F'	blau	Cd
486.1327	F	blau	H
546.0740	e	grün	Hg
587.5618	d	gelb	He
589.2938	D	gelb	Na
632.8			HeNe-Laser
643.8469	C'	rot	Cd
656.2725	C	rot	H
706.5188	r	rot	He
852.11	s	IR	Cä
1013.98	t	IR	Hg
1060.0			Nd:YAG-Laser

Chromatical Evaluation of Optical Systems

- Chromatical performance evaluation of optical systems:
  Usage of one main (central) wavelength and two secondary wavelenghts

Main wavelength			1st secondary wavelength			2nd secondary wavelength		
e	546.07	green	F'	480.0	bue	C'	643.8	red
d	587.56	yellow	F	486.1	blue	C	656.3	red

- Additional definition of wvalengths at the boundaries of the used spectral range, e.g.
  - one further wavelength near to the UV edge (g, i)
  - one further wavelength near to the IR-edge (s,t)

Atomic Model of Dispersion

- Atomic model for the refractive index: oscillator approach of atomic field interaction

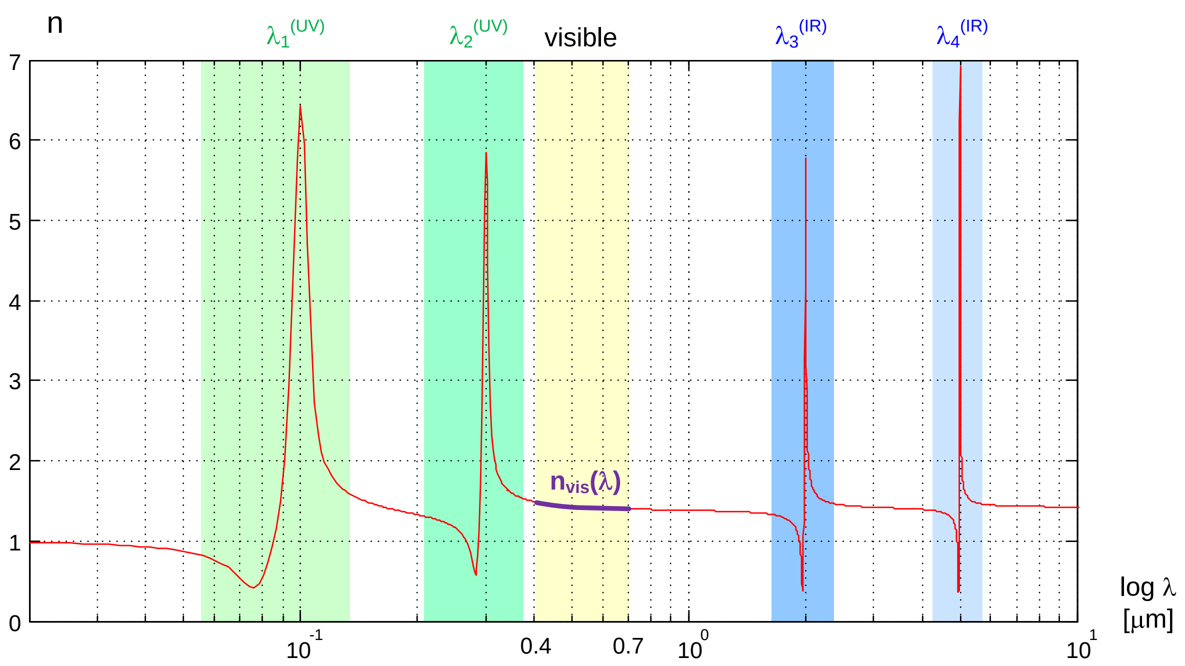
$$\left(n_r + i \cdot n_i\right)^2 = \frac{Ne^2}{2\pi \cdot c\varepsilon_0 m} \cdot \sum_j \frac{f_j \lambda^2 \lambda_j^2}{2\pi \cdot c \cdot \left(\lambda^2 - \lambda_j^2\right) + i\gamma_j \lambda \lambda_j^2}$$

- Sellmeier dispersion formula: corresponding function

$$n^2 = A + \sum_j \frac{B_j \cdot \lambda^2}{\lambda^2 - C_j}$$

- Special case of coupled resonances: example quartz, degenerated oscillators

$$n^2 = A + \frac{B_0 \cdot \lambda^4}{\left(\lambda^2 - \lambda_o^2\right)^2} + \sum_{j=1} \frac{B_j \cdot \lambda^2}{\lambda^2 - C_j}$$



Dispersion

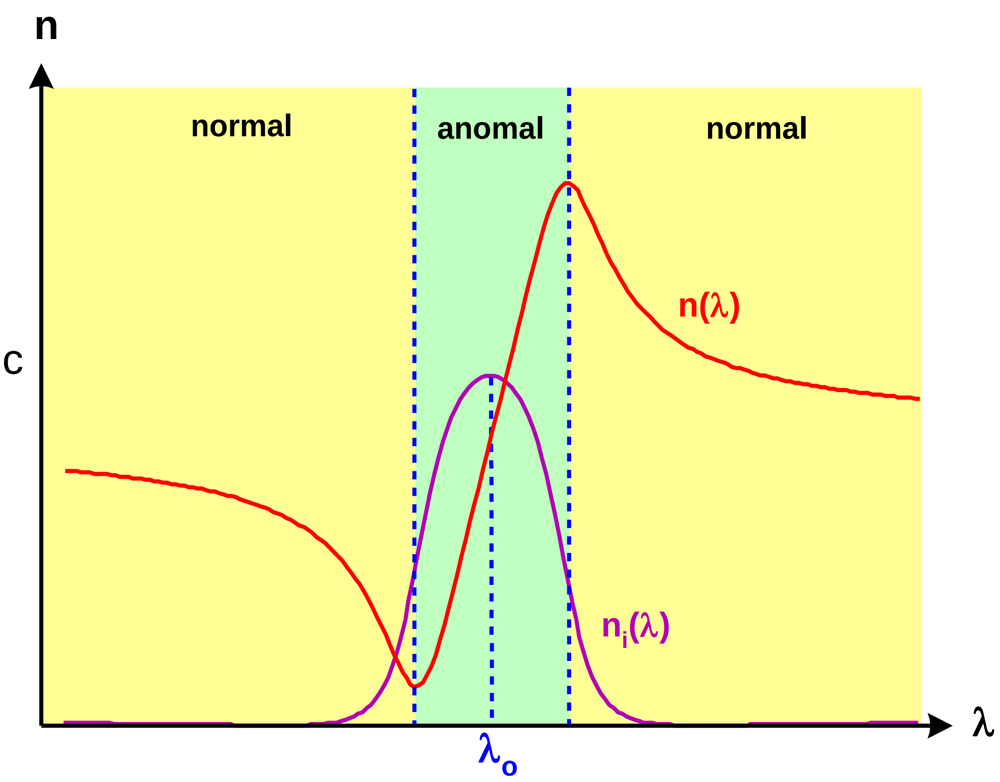
- Dispersion:
  Refractive index changes with wavelength

$$\Delta n = n(\lambda_1) - n(\lambda_2)$$

- Normale dispersion: larger index n for shorter wavelengths,
  Ray bending of blue rays stonger than red

$$\frac{dn}{d\lambda} < 0$$

- Notice:
  Diffraction dispersion is anomalous
  with dn/dλ > 0
  The different sign allows for chromatic
  correction in diffractive elements.



Dispersion formulas

- Schott formula
  empirical

$$n=\sqrt{a_o+a_1\lambda^2+a_2\lambda^{-2}+a_3\lambda^{-4}+a_4\lambda^{-6}+a_5\lambda^{-8}}$$

- Sellmeier
  Based on oscillator model

$$n(\lambda)=\sqrt{A+B\frac{\lambda^2}{\lambda^2-\lambda_1^2}+C\frac{\lambda^2}{\lambda^2-\lambda_2^2}}$$

- Bausch-Lomb
  empirical

$$n(\lambda)=\sqrt{A+B\lambda^2+C\lambda^4+\frac{D}{\lambda^2}+\frac{E\lambda^2}{(\lambda^2-\lambda_o^2)+\frac{F\lambda^2}{\lambda^2-\lambda_o^2}}}$$

- Herzberger
  Based on oscillator model

$$n(\lambda)=a_o+a_1\lambda^2+\frac{a_2}{\lambda^2-\lambda_o^2}+\frac{a_3}{\left(\lambda^2-\lambda_o^2\right)^2}$$

$$mit \quad \lambda_o=0.168\,\mu m$$

- Hartmann
  Based on oscillator model

$$n(\lambda)=a_o+\frac{a_1}{a_3-\lambda}+\frac{a_4}{a_5-\lambda}$$

Dispersion and Abbe number

- Description of dispersion:

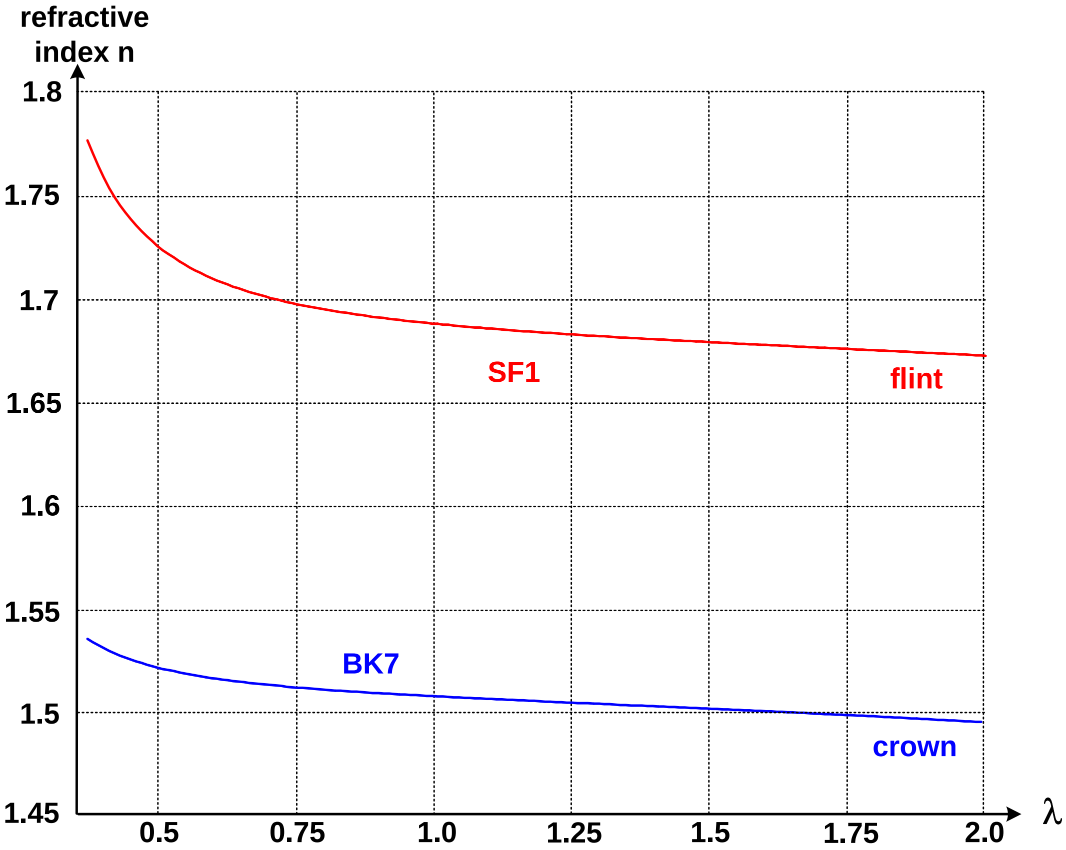
  Abbe number $$\nu(\lambda) = \frac{n(\lambda) - 1}{n_{F'} - n_{C'}}$$

- Visual range of wavelengths:
  typically d,F,C or e,F',C' used

  $$\nu_e = \frac{n_e - 1}{n_{F'} - n_{C'}}$$

- Typical range of glasses
  $\nu_e = 20 ...100$

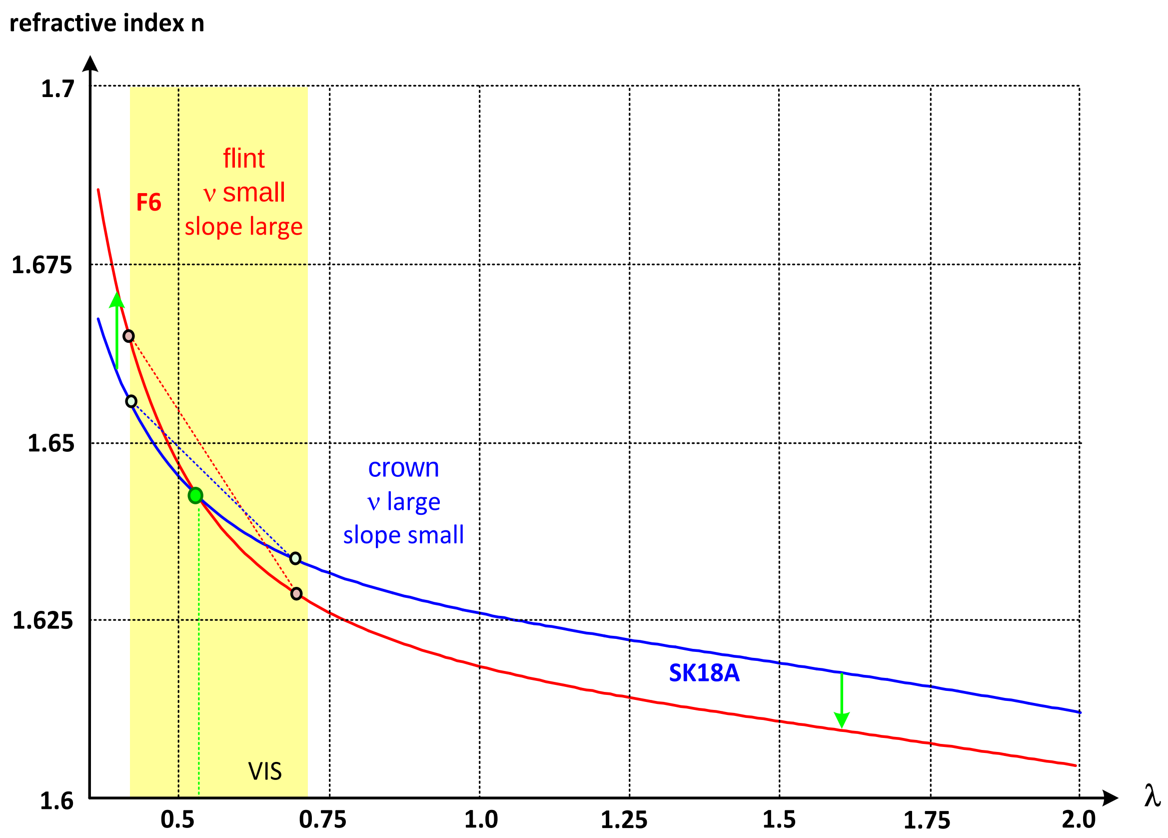
- Two fundamental types of glass:
  Crown glasses:
  n small, ν large, dispersion low
  Flint glasses:
  n large, ν small, dispersion high



Dispersion

Material with different dispersion values:

- Different slope and curvature of the dispersion curve
- Stronger change of index over wavelength for large dispersion
- Inversion of index sequence at the boundaries of the spectrum possible



Abbe Number and Achromatization

- Curvatures c_j of the radii of a lens

$$c_1 = \frac{1}{r_1}\ ,\ c_2 = \frac{1}{r_2}$$

- Focal power at the center wavelength e for a thin lens

$$F_e = (n_e - 1)(c_1 - c_2) = (n_e - 1)\cdot \Delta c$$

- Difference in focal powers for outer wavelengths F', C'

$$\Delta F = F_{F'} - F_{C'} = (n_{F'} - n_{C'})\cdot \Delta c = \frac{n_{F'} - n_{C'}}{n_e - 1}\cdot (n_e - 1)\Delta c = \frac{F_e}{\nu_e}$$

with the Abbe number

$$\nu_e = \frac{n_e - 1}{n_{F'} - n_{C'}}$$

- Focal length at the center wavelength

$$f_e = \frac{1}{F_e} = \frac{1}{(n_e - 1)\Delta c}$$

- Difference of the focal lengths for outer wavelengths

$$\Delta f = f_{F'} - f_{C'} = \frac{n_{C'} - n_{F'}}{(n_{F'} - 1)(n_{C'} - 1)\Delta c} \approx \frac{n_{C'} - n_{F'}}{(n_e - 1)^2 \Delta c} = -\frac{f_e}{\nu_e}$$

- Achromatization condition for two thin lenses close together

$$\Delta F = \frac{F_1}{\nu_1} + \frac{F_2}{\nu_2} = \frac{1}{f_1 \nu_1} + \frac{1}{f_2 \nu_2} = 0$$

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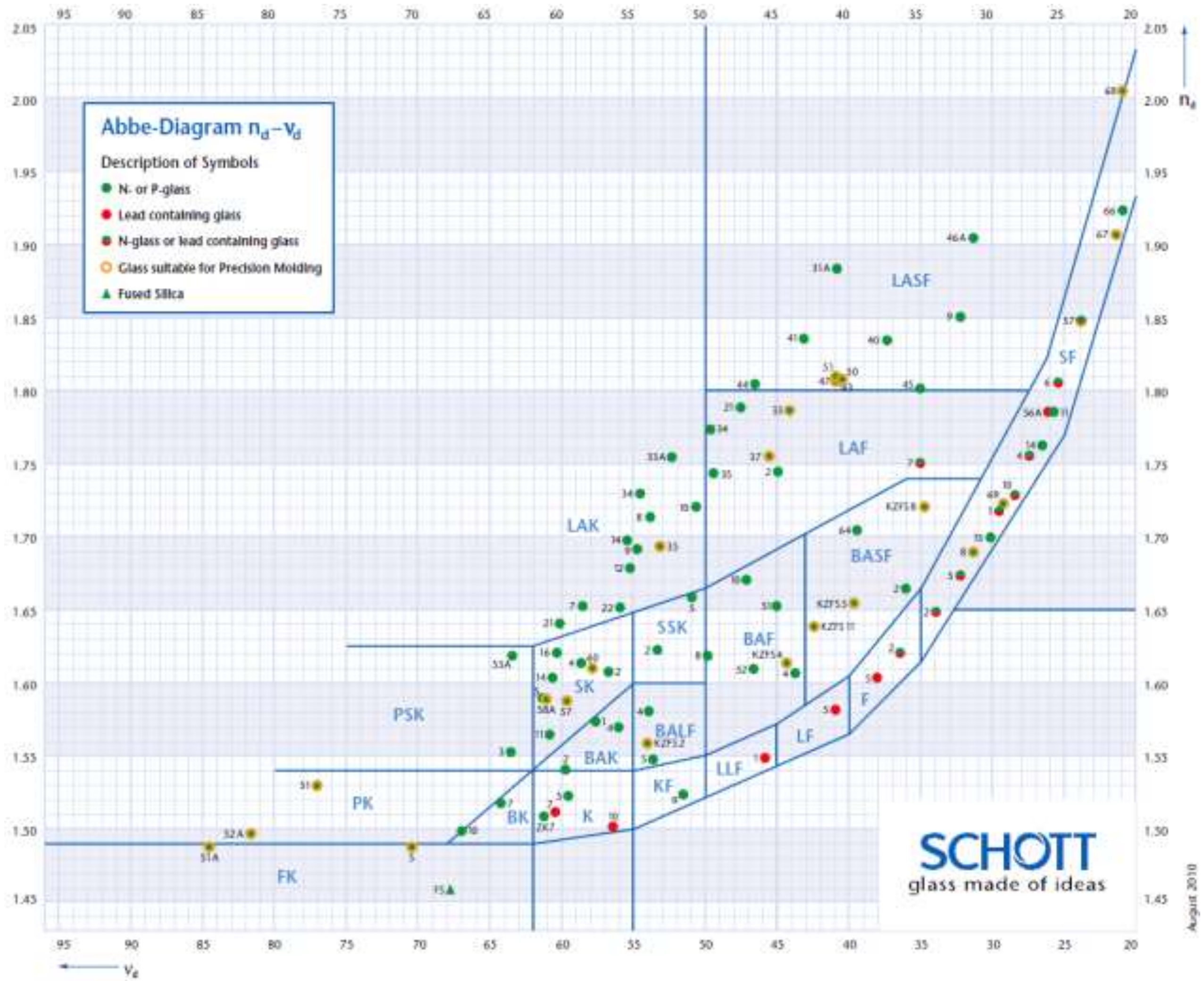
Glass Diagram

- Usual representation of glasses:
  diagram of refractive index vs dispersion n(ν)

- Left to right:
  Increasing dispersion
  decreasing Abbe number



Relative Partial Dispersion

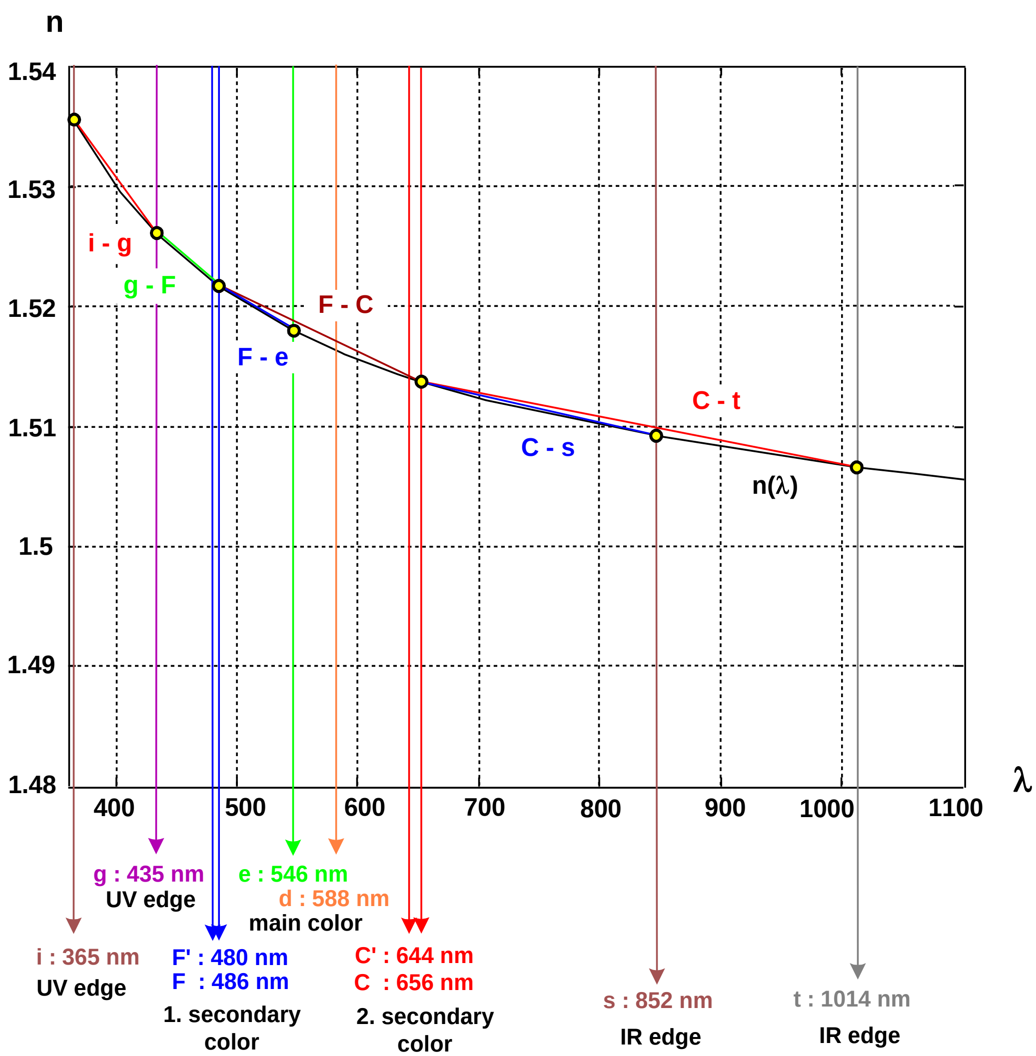
- Relative partial dispersion :
  Change of dispersion slope with λ
  Different curvature of dispersion curve

- Definition of local slope for selected wavelengths relative to secondary colors

$$P_{\lambda_1\lambda_2} = \frac{n(\lambda_1) - n(\lambda_2)}{n_{F'} - n_{C'}}$$

- Special λ-selections for characteristic ranges of the visible spectrum

λ = 656 / 1014 nm far IR
λ = 656 / 852 nm near IR
λ = 486 / 546 nm blue edge of VIS
λ = 435 / 486 nm near UV
λ = 365 / 435 nm far UV



Partial Dispersion and Normal Line

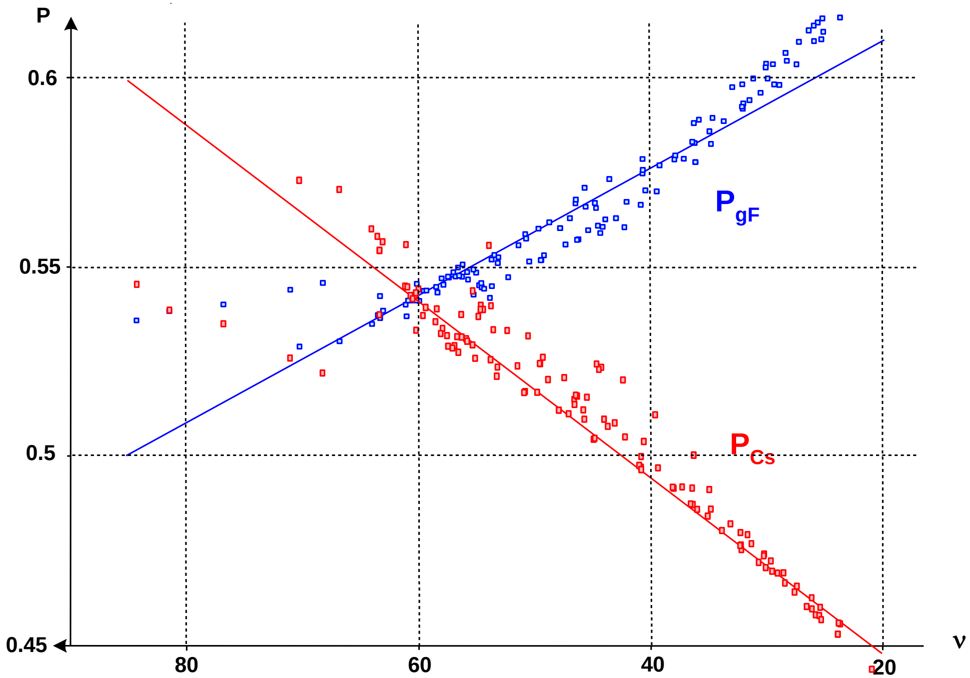
- The relative partial dispersion changes approximately linear with the dispersion for glasses

$$P_{\lambda 1,\lambda 2} = a_{\lambda 1,\lambda 2} \cdot \nu_d + b_{\lambda 1,\lambda 2}$$

- Nearly all glasses are located on the normal line in a P-ν-diagram
- The slope of the normal line depends on the selection of wavelengths
- Glasses apart from the normal line shows anomalous partial dispersion ΔP

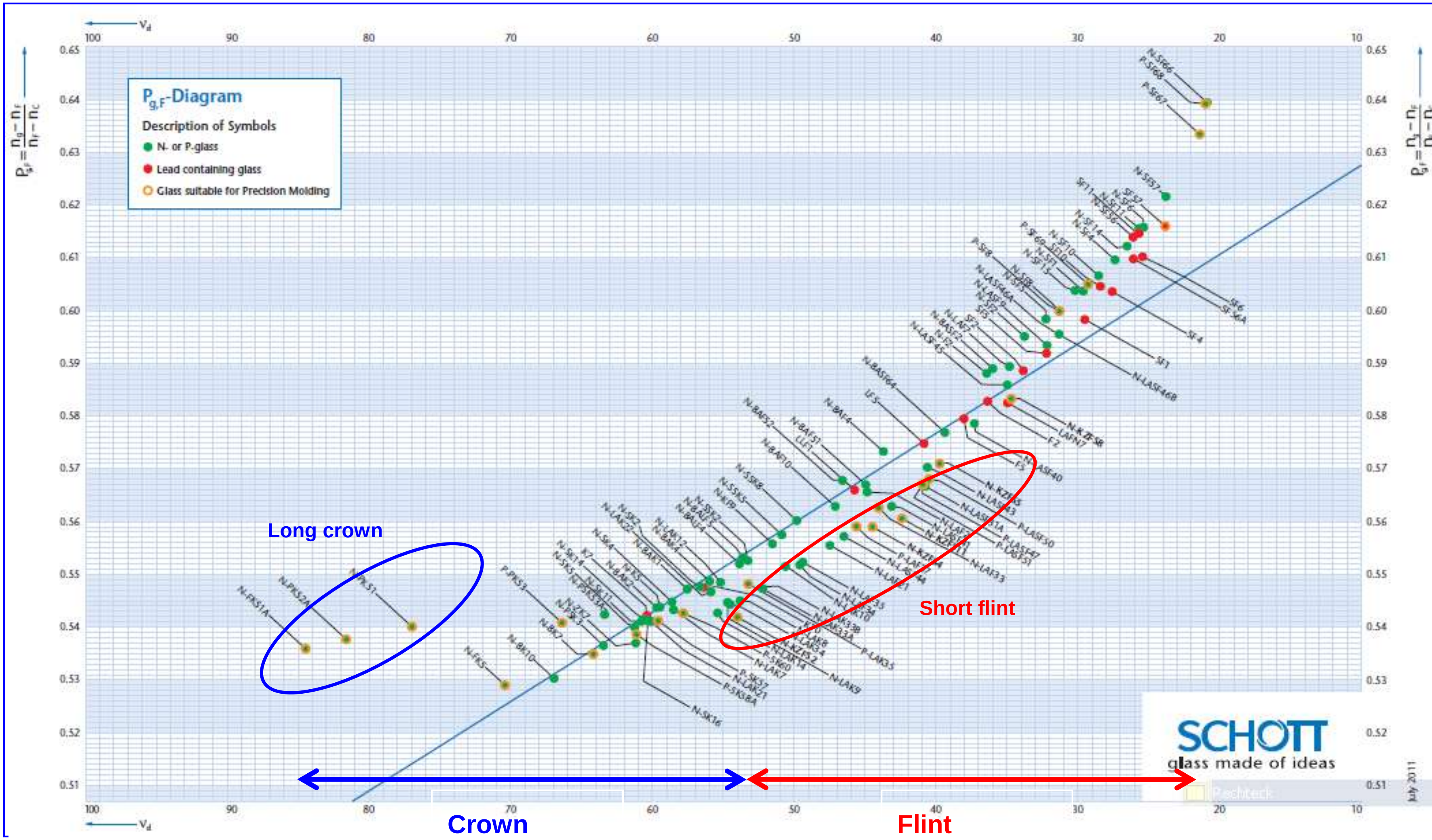
$$P_{\lambda 1 \lambda 2} = a_{\lambda 1 \lambda 2} \cdot \nu_d + b_{\lambda 1 \lambda 2} + \Delta P_{\lambda 1 \lambda 2}$$

these material are important for chromatical correction of higher order



Relative Partial Dispersion

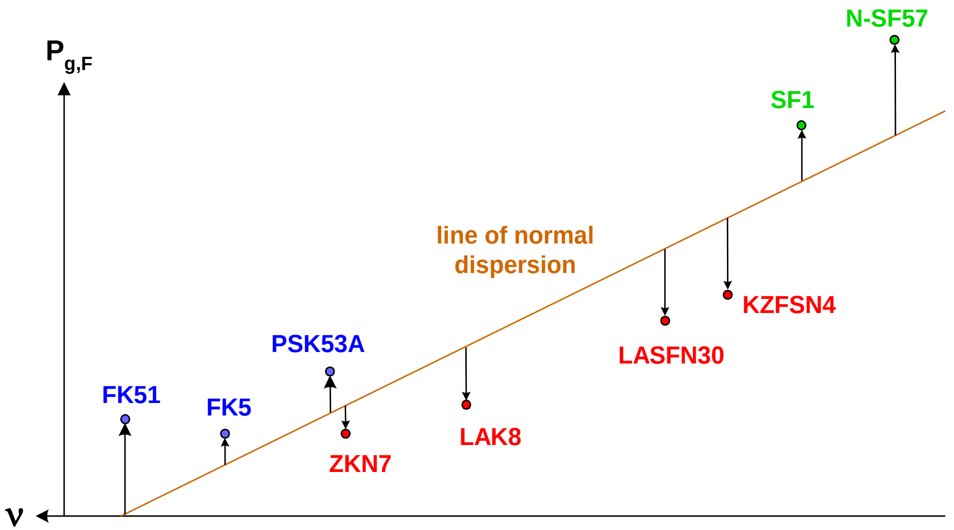
- Long crown and short flint as special realizations of large P

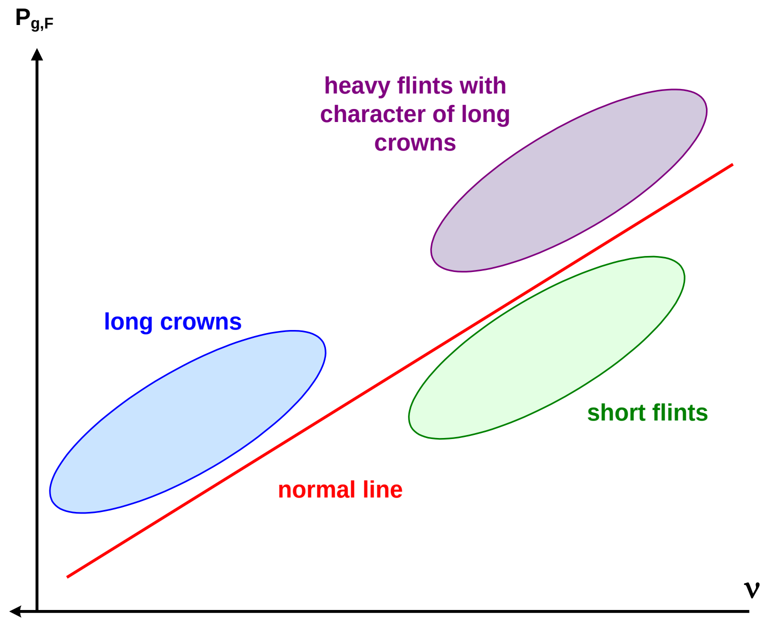


Ref.:H. Zuegge

Anomalous Partial Dispersion

- There are some special glasses with a large deviation from the normal line
- Of special interest:
  long crowns and short flints





Plastics

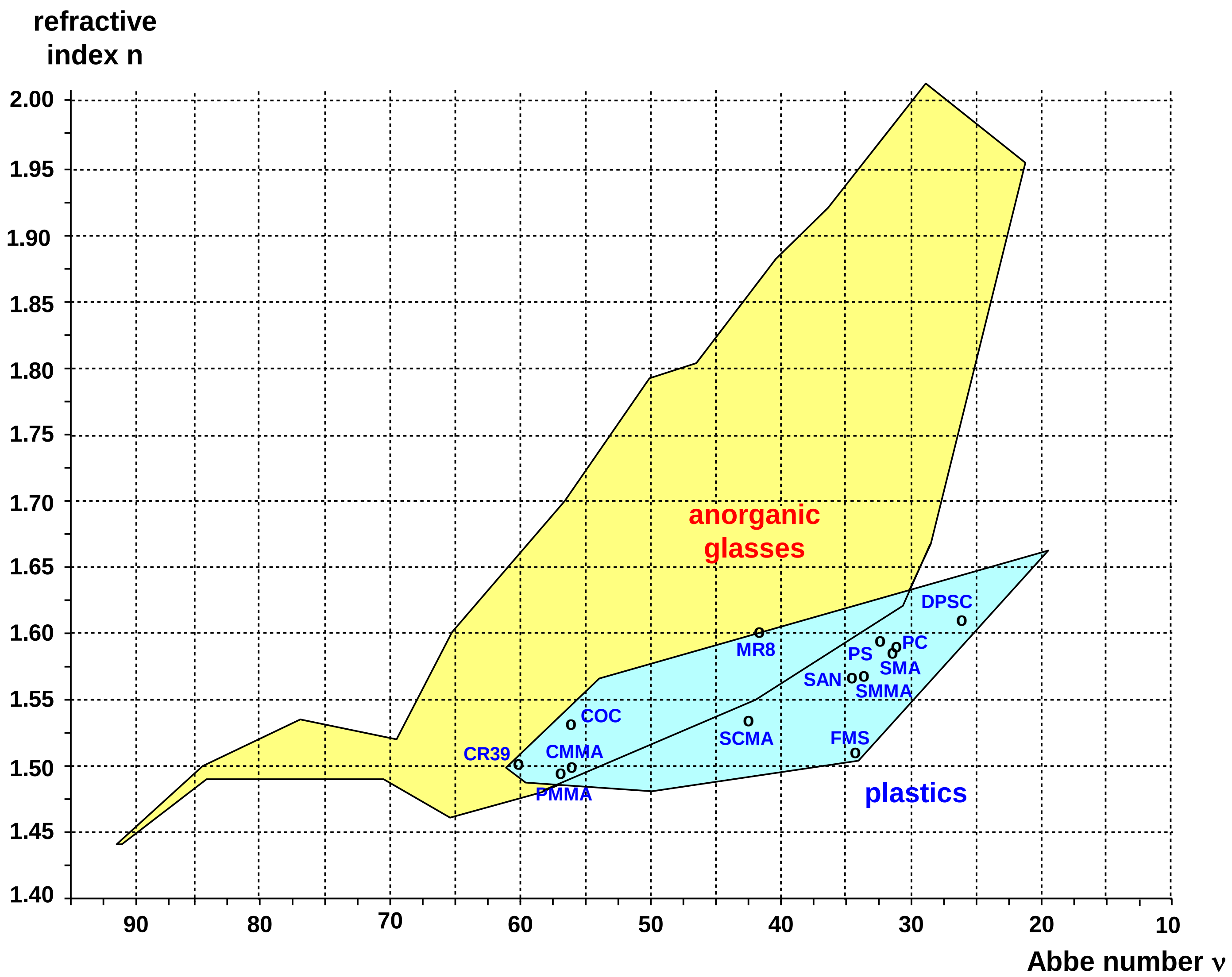
Material	Index at 546 nm	Abbe numb er	Max Temp	Therm expan 10^{-6} K^{-1}	Scatt er in %	Trans. 3mm,	Density g/cm^3
PMMA - Polymethyl-Methacrylat	1.49280	57	90	65	2	92	1.19
PC - Polycarbonat - Makrolon, Lexan	1.59037	30	120	69	4	87	1.20
CR39 - DEGBAC - Gießharz	1.5011	57.8	100	120	1		1.32
PS - Polystyrol	1.590	30.8	80	70	3	89	1.06
DPSC - Diphenyl-sulfidcarbonat	1.612	26.0					
CMMA	1.50	56.0					
Styrol, SAN	1.566	34.7	95	65	4	90	1.09
SMA	1.585	31.3					
SMMA	1.568	33.5					
FMS	1.508	34.0					
SCMA	1.535	42.5					
COC	1.533	56.0					
MR8	1.60	42.0					

Properties of Plastic Materials

1. Stress induced birefringence during processing
2. Generation of local inhomogenieties of the refractive index in die casting
3. Water intake (swelling) : change of shape (up to 4%) and decrease in the refractive index
4. Electro-static charge
5. Aging due to cold forming, polymerization, opalescence, yellowing
6. Strong thermal variation of the refractive index
7. Limiting temperature (above the transition temperature the material is destroyed) 100 ... 120 °C
8. For an increased abrasive hardness and for the prevention from charging and swelling ,special coatings may have to be applied.
9. During the cooling process significant changes occur in the volume caused by shrinking. There are two different types of plastics
   a. thermosets, shrinking 0.4%...0.7%
   b. thermoplasts, shrinking 4%...14%

Plastic Materials

Plastics in the
n - ν - diagram



Usage of Plastics in Optical Systems

- Most attractive use of plastics: Consumer optics
  - benefit of light weight
  - critical cost
  - high number of pieces
- Advantages for special components due to manufacturing technique:
  - complex surface shapes, arrays, aspheres
  - for injection moulding cost of complex shape only for master piece
- Typical products with plastics components:
  - Eye glasses
  - binoculars
  - photographic lenses
  - pic-up objective lenses
  - illumination systems

Plastics vs Glass Materials

Comparison plastics with glasses

property	unit	range plastics	range glass
refractive index n		1.49 ...1.61	1.44...1.95
dispersion ν		25...57	20...90
uniformity of the refractive index		10^{-3}...10^{-4}	10^{-4}...10^{-6}
temperature dependence of the refractive index	$10^{-6}*K^{-1}$	-100...-160	-10...+10
Vickers hardness	N/mm^2	120...190	3000...7000
thermal expansion	$10^{-6}*grd^{-1}$	70...100	5...10
thermal conductivity	$Wm^{-1grd-1}$	0.15...0.23	0.5...1.4
internal transmission in the green range		0.97...0.993	0.999
stress - optical coefficient	$10^{-12}Pa^{-1}$	40	3
stress- birefringence		$5*10^{-5}$...10^{-3}	0
density	g/cm^3	1.05...1.32	2.3...6.2
water intake	%	0.1...0.8	0

UV- and IR - Materials

material	refractive index	λ-range (μm)	UV	IR	ν	P
MgF_2	1.389	0.12 - 9.0	•	•		
ZnS	2.25	0.4 - 14.5		•	12.831	0.271
CaF_2, calcium fluoride	1.42	0.12 - 11.5	•	•	47.53	0.397
ZnSe	2.44	0.5 - 22.0		•	34.20	0.291
MgO	1.69...1.737	0.28 - 9.5		•	53.4	
CdTe	2.70	0.9 - 31.0		•		
diamond	2.3757	0.25...3.7, 6.0...	•	(•)	2387	0.469
germanium	4.003	2.0...15		•	701.42	0.556
silicon	3.433	1.2...15		•	420.16	0.523
BaF_2	1.474	0.18...12	•	•	81.7	0.651
SiO_2 , quartz	1.544	0.15...4.0	•	(•)	69.9	0.653
Al_2O_3 , sapphire	1.769	0.17...5.5	•	(•)	6.626	0.650

Lens shape

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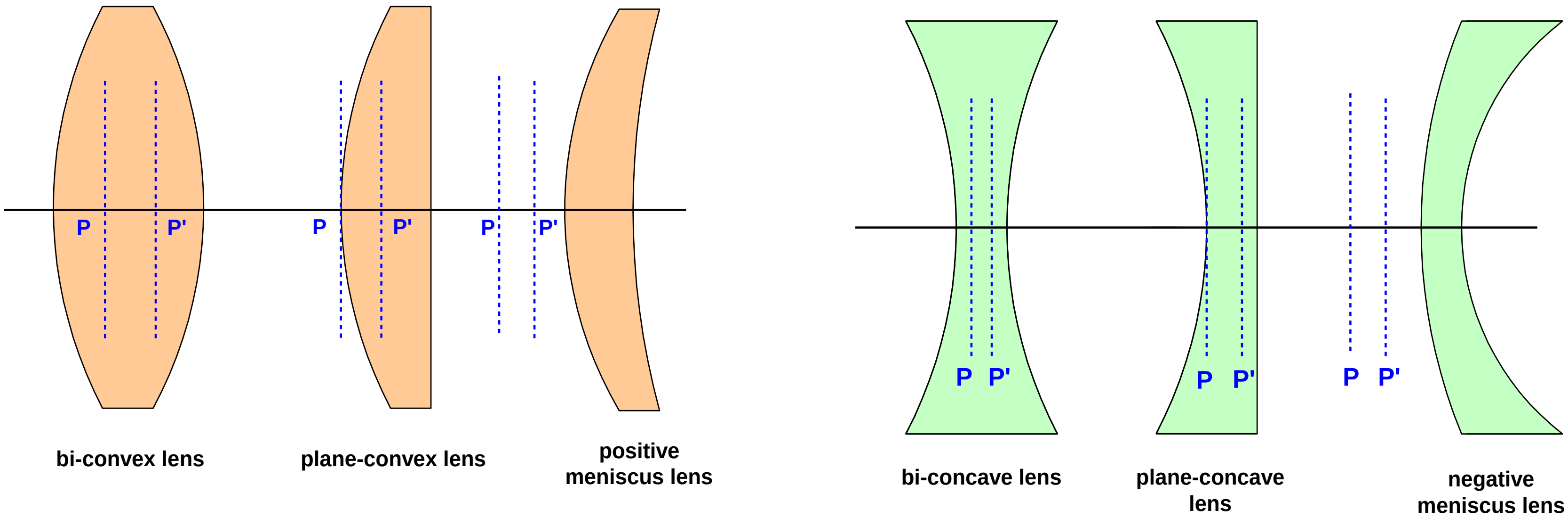
- Different shapes of singlet lenses:
  1. bi-, symmetric
  2. plane convex / concave, one surface plane
  3. Meniscus, both surface radii with the same sign

- Convex: bending outside
  Concave: hollow surface

- Principal planes P, P‘: outside for mesicus shaped lenses



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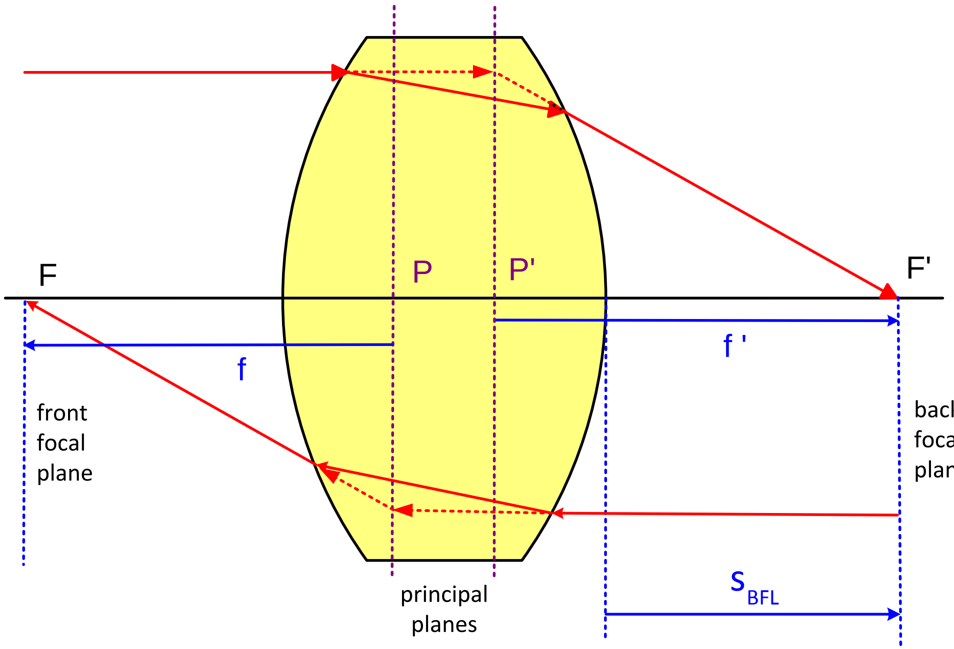
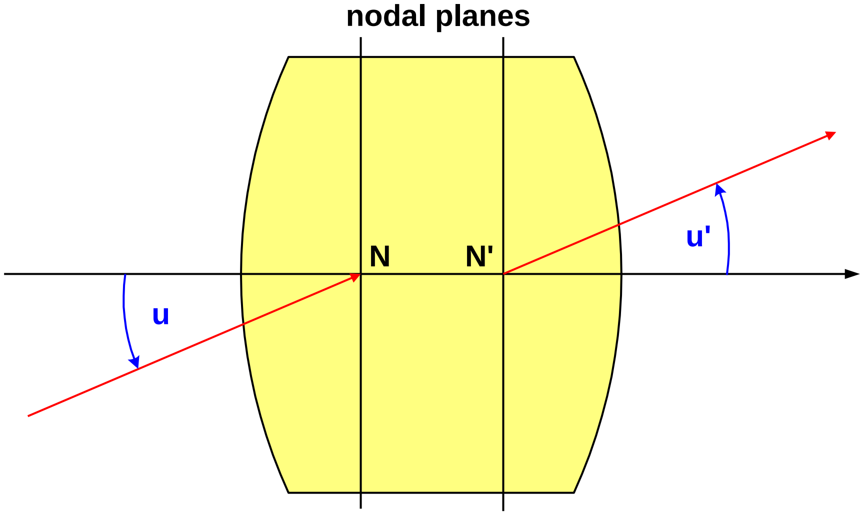
Cardinal Elements of a Lens

- Focal points:
  1. incoming ray parallel to the axis intersects the axis in F‘
  2. ray through F is leaves the lens parallel to the axis

  The focal lengths are referenced on the principal planes



- Nodal points:
  Ray through N goes through N‘ and preserves the direction



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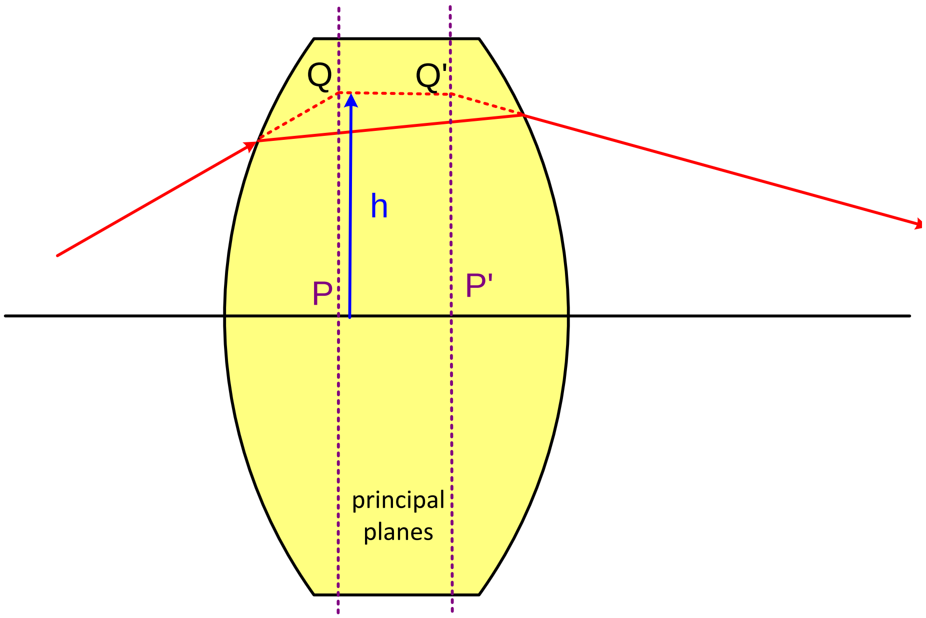
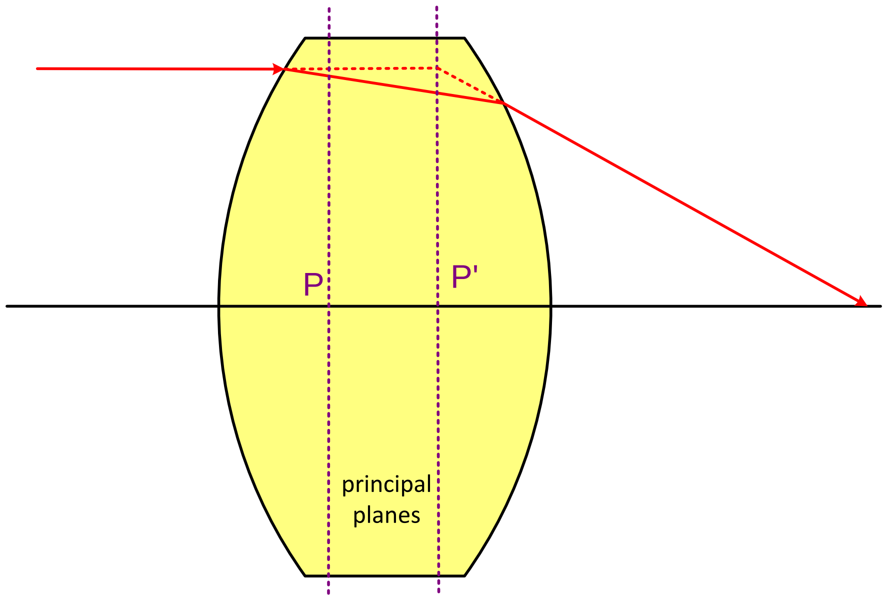
Cardinal Elements of a Lens

- Principal plane P:
  incoming ray hits intersection point with P is transferred with the same height h to P‘



- Special case of incident ray parallel to the axis:
  principal plane P‘:
  location of apparent ray bending



Main properties of a lens

- Main notations and properties of a lens:

  - radii of curvature r_1 , r_2
    curvatures c
    sign: r > 0 : center of curvature
    is located on the right side
  - thickness d along the axis
  - diameter D
  - index of refraction of lens material n

$$c_1 = \frac{1}{r_1} \qquad c_2 = \frac{1}{r_2}$$

- Focal length (paraxial)

$$f = \frac{y_F'}{\tan u} \quad , \quad f' = \frac{y}{\tan u'}$$

- Optical power

$$F = -\frac{n}{f} = \frac{n'}{f'}$$

- Back focal length
  intersection length,
  measured from the vertex point

$$s_{F'} = f' + s_{P'}$$

Notations of a lens

P principal point

S vertex of the surface

F focal point

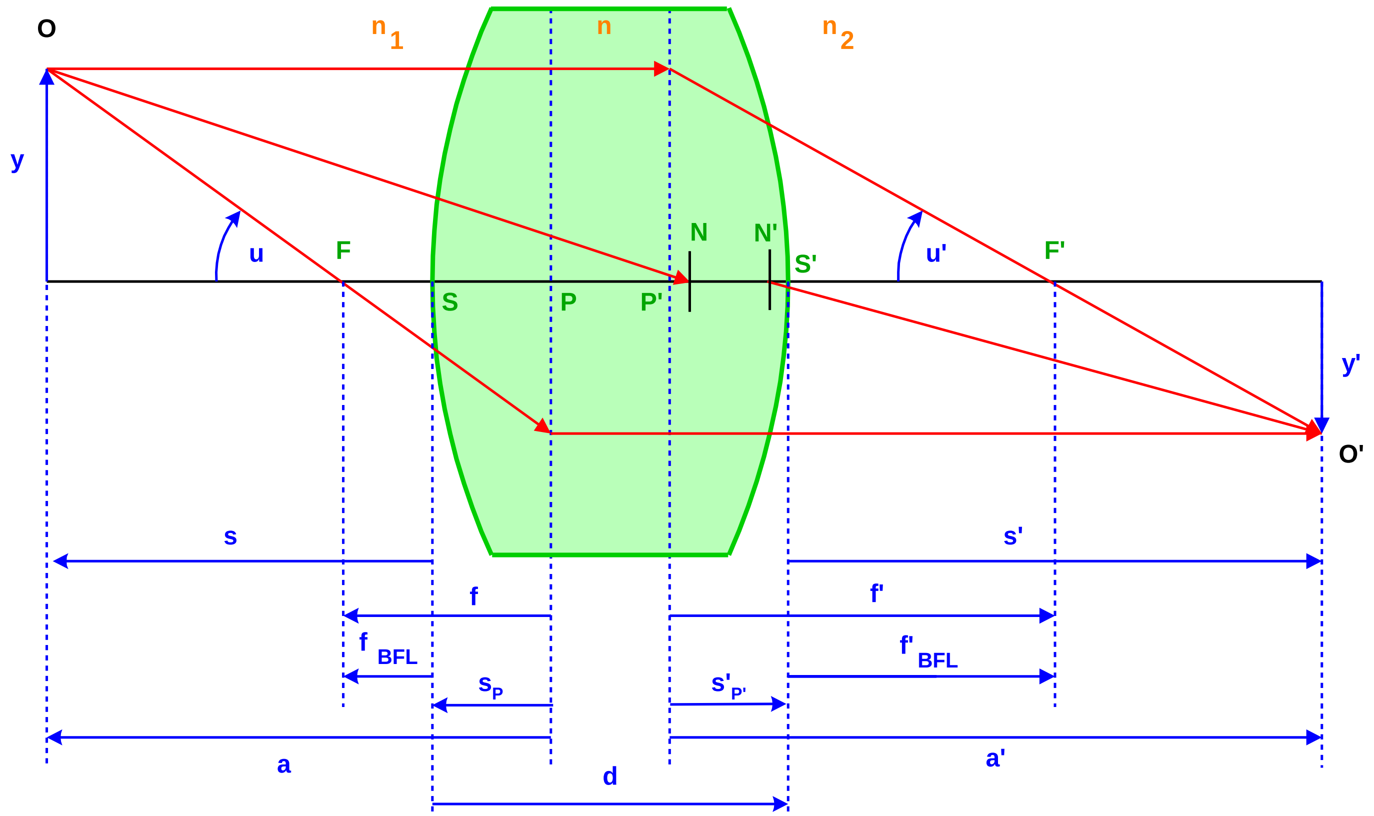
s intersection point of a ray with axis

f focal length PF

r radius of surface curvature

d thickness SS‘

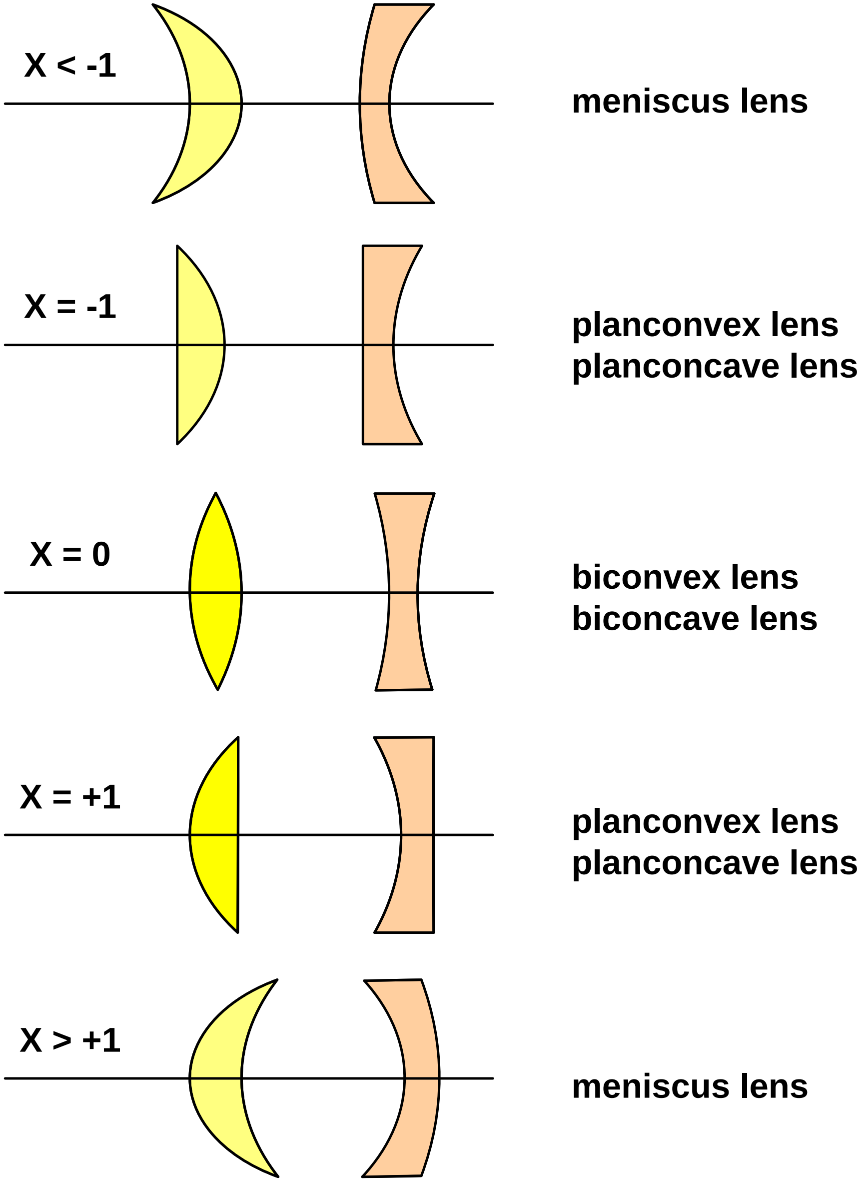
n refrative index



Bending of a Lens

- Bending: change of shape for invariant focal length
- Parameter of bending

$$X = \frac{R_1 + R_2}{R_2 - R_1}$$

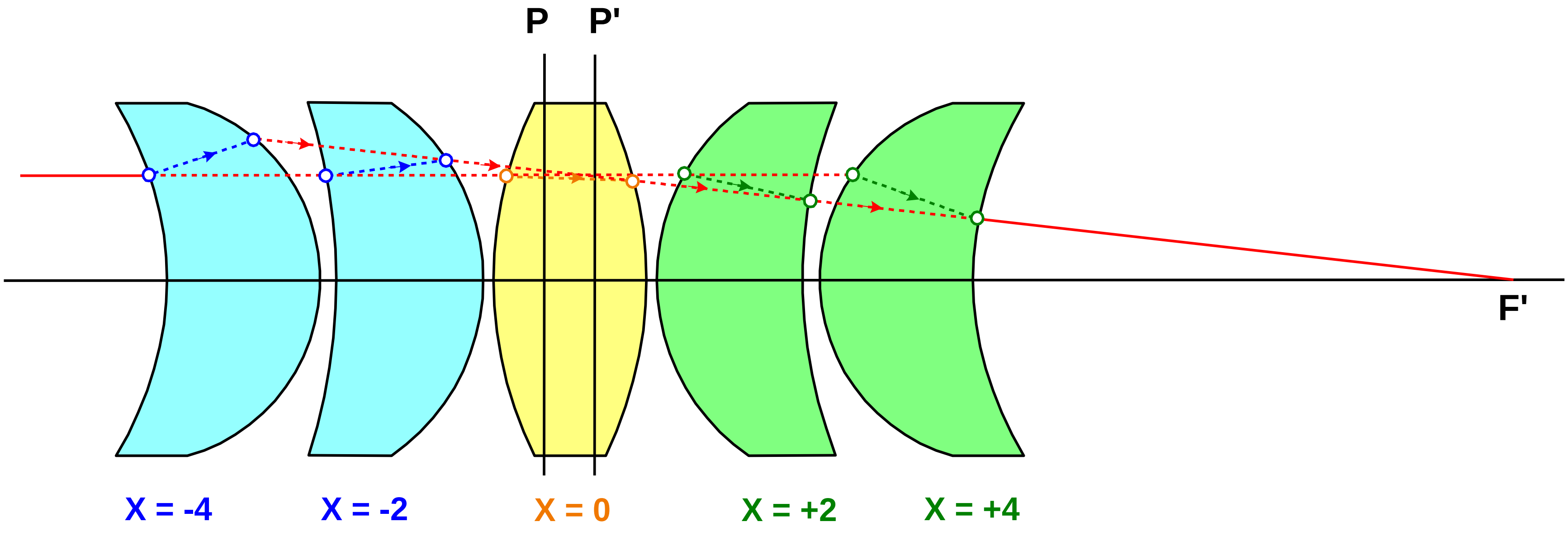


Lens bending und shift of principal plane

- Ray path at a lens of constant focal length and different bending
- Quantitative parameter of description X:

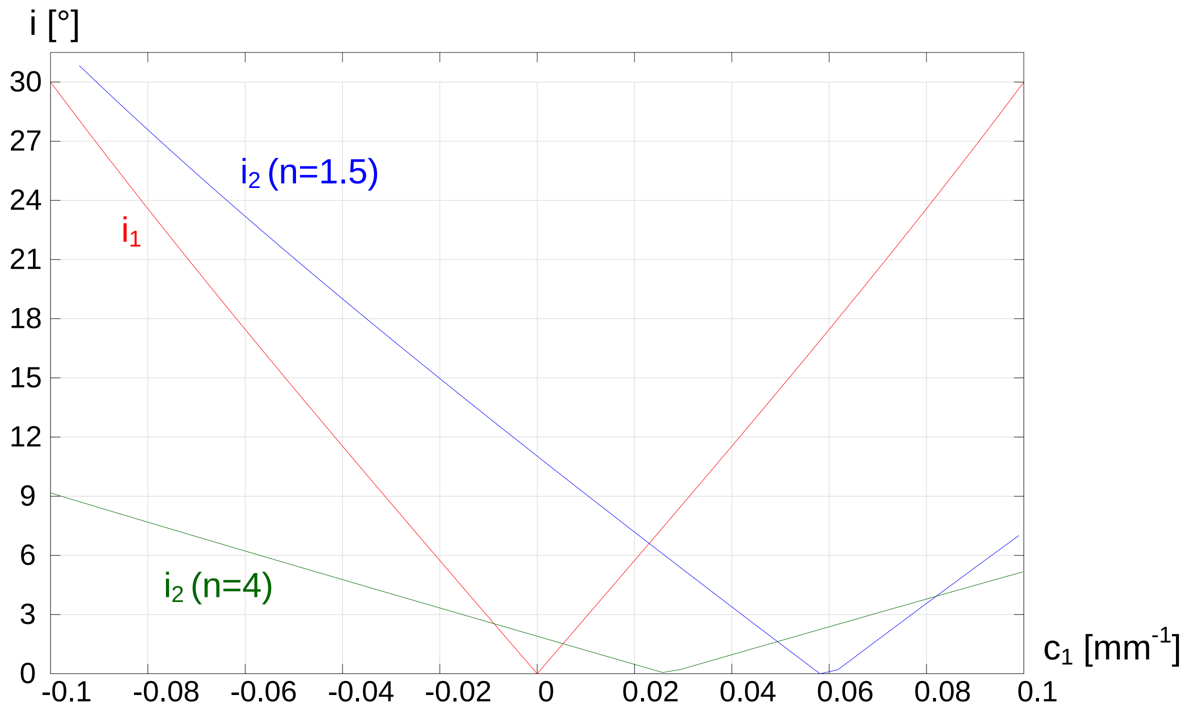
$$X = \frac{R_1 + R_2}{R_2 - R_1}$$

- The ray angle inside the lens changes
- The ray incidence angles at the surfaces changes strongly
- The principal planes move
  For invariant location of P, P‘ the position of the lens moves



Incidence of Bended Lens

- Changes of the incidence angles at the front and the rear surface of a bended lens
- Figure without sign of incidence angle
- Angle at the second surface depends on the refractive index



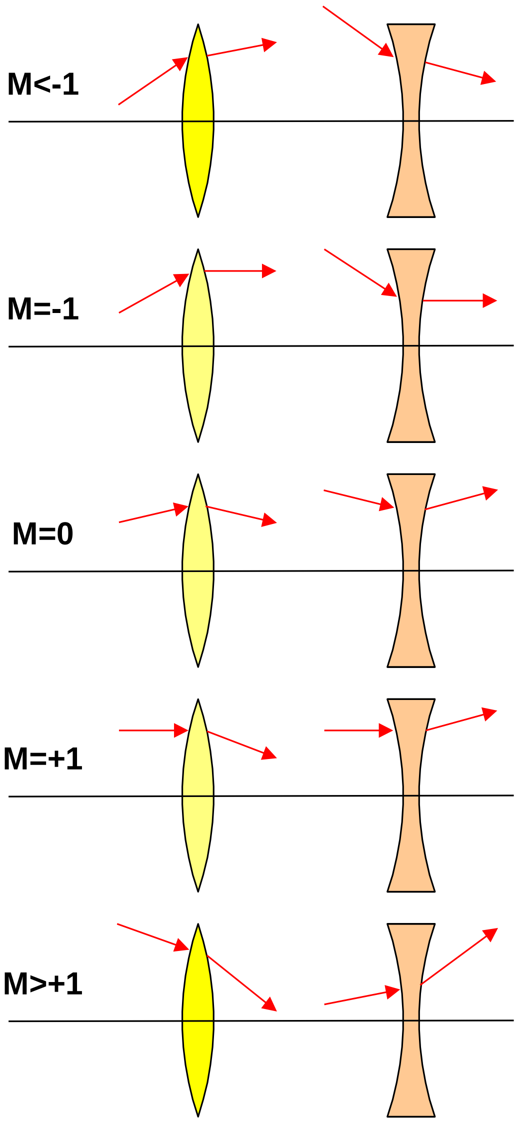
Magnification Parameter

- Magnification parameter M:
  defines ray path through the lens

$$M = \frac{U'+U}{U'-U} = \frac{1+m}{1-m} = \frac{2f}{s} + 1 = \frac{2f}{s'} - 1$$

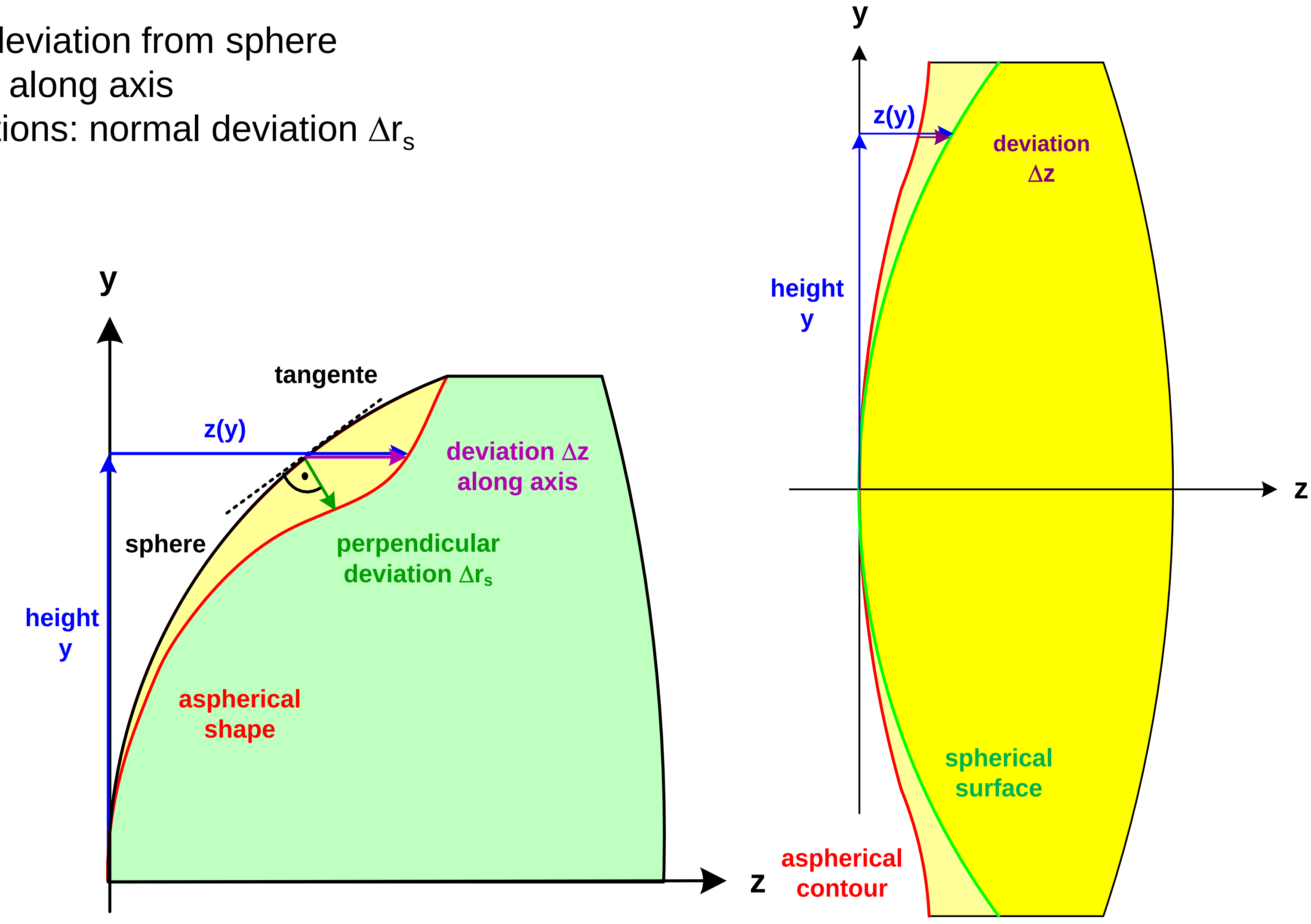
- Special cases:
  1. M = 0 : symmetrical 4f-imaging setup
  2. M = -1: object in front focal plane
  3. M = +1: object in infinity

- The parameter M strongly influences the aberrations



Aspheres - Geometry

- Reference: deviation from sphere
- Deviation Δz along axis
- Better conditions: normal deviation Δr_s



Conic Sections

- Explicite surface equation, resolved to z

  Parameters: curvature c = 1 / R

  conic parameter κ

$$z = \frac{c\left(x^2+y^2\right)}{1+\sqrt{1-(1+\kappa)c^2\left(x^2+y^2\right)}}$$

- Influence of κ on the surface shape

Parameter	Surface shape
κ = - 1	paraboloid
κ < - 1	hyperboloid
κ = 0	sphere
κ > 0	oblate ellipsoid (disc)
0 > κ > - 1	prolate ellipsoid (cigar)

- Relations with axis lengths a,b of conic sections

$$\kappa = \left(\frac{a}{b}\right)^2 - 1 \qquad c = \frac{b}{a^2} \qquad b = \frac{1}{\left|c(1+\kappa)\right|} \qquad a = \frac{1}{\left|c\sqrt{\left|1+\kappa\right|}\right|}$$

Simple Asphere – Parabolic Mirror

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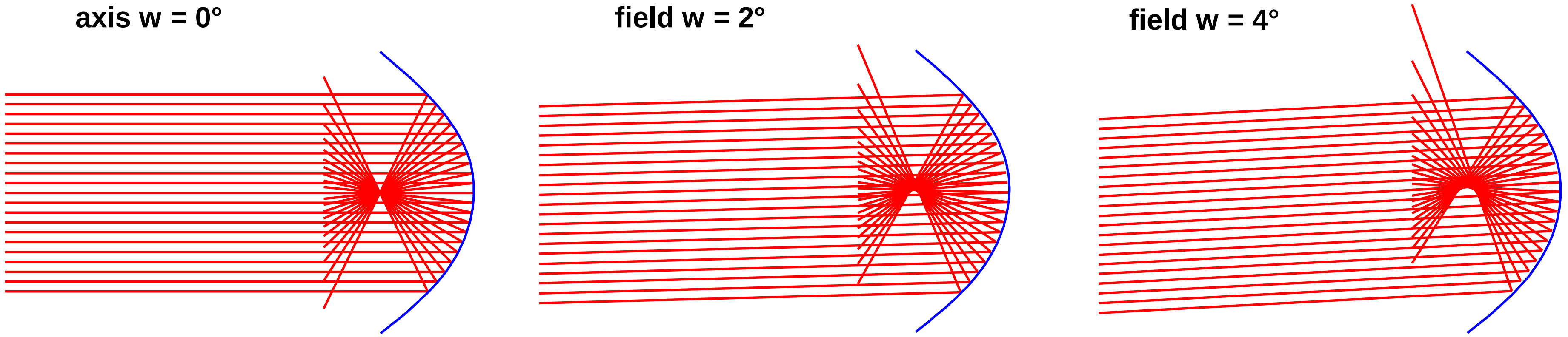
- Equation $$z = \frac{y^2}{2R_s}$$
- Radius of curvature in vertex: R_s
- Perfect imaging on axis for object at infinity
- Strong coma aberration for finite field angles
- Applications:
  1. Astronomical telescopes
  2. Collector in illumination systems

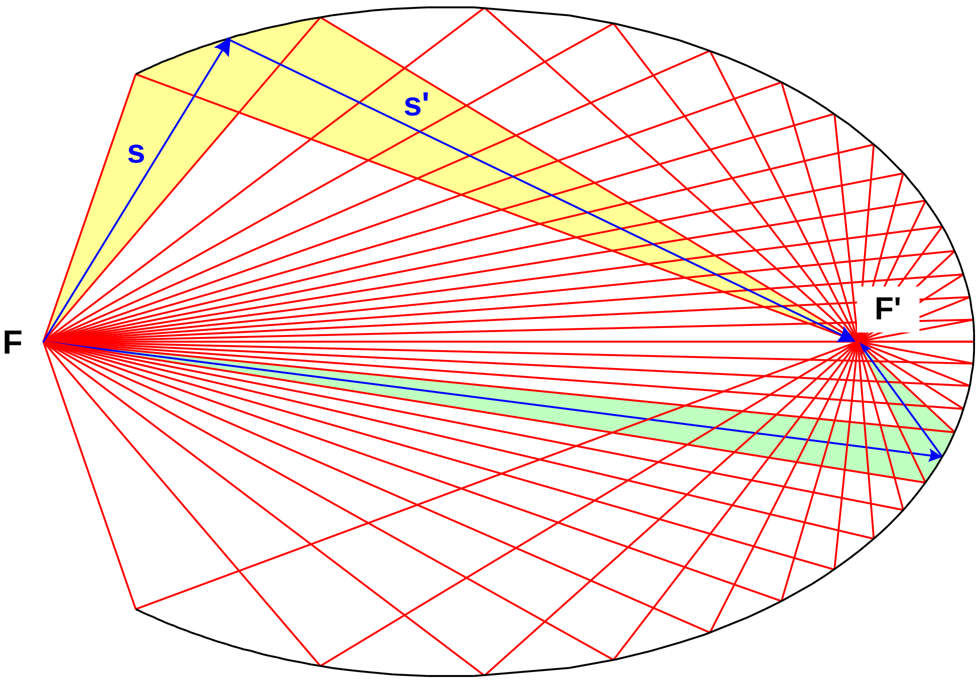


Simple Asphere – Elliptical Mirror

- Equation

$$z = \frac{cy^2}{1+\sqrt{1-(1+\kappa)y^2c^2}}$$

- Radius of curvature r in vertex, curvature c
  eccentricity κ
- Two different shapes: oblate / prolate
- Perfect imaging on axis for finite object and image loaction
- Different magnifications depending on
  used part of the mirror
- Applications:
  Illumination systems



General Aspherical Surface

- Conic surface as basic shape

- Additional correction of the sag by a Taylor expansion
  Only even powers: no kink at r=0

$$z(x,y) = \frac{c\left(x^2+y^2\right)}{1+\sqrt{1-(1+\kappa)\cdot c^2\left(x^2+y^2\right)}} + \sum_{k=2}^{k_{max}} a_k \cdot \left(x^2+y^2\right)^k$$

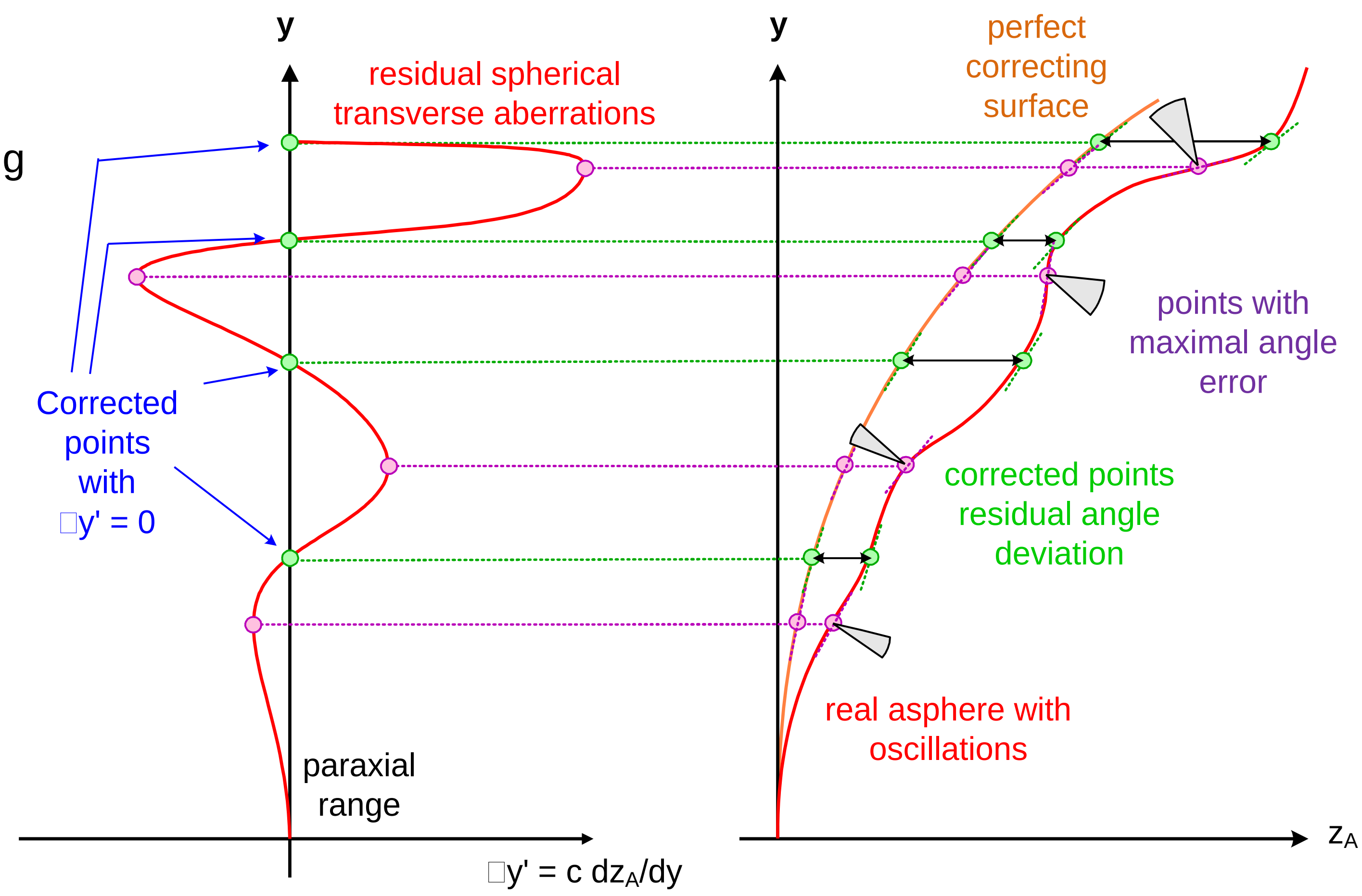
- Mostly rotational symmetric shape considered

$$z(r) = \frac{c \cdot r^2}{1+\sqrt{1-(1+\kappa)\cdot c^2 r^2}} + \sum_{k=2}^{k_{max}} a_k \cdot r^{2k}$$

- Problems with this representation:
  1. added contributions not orthogonal, bad performance during optimization
  2. non-normalized representation, coefficients depend on absolute size of the diameter (very small high order coefficients for large diameters)
  3. Oscillatory bahavior, large residual slope error can occur
  4. in optics slope and not sag is relevant
  5. the coefficients can not be measured/are hard to control, tolerancing is critical and comlicated
  6. the added sag is along z , more important is a correction perpendicular to the surface for strong aspheres

Aspheres: Correction of Higher Order

- Correction at discrete sampling
- Large deviations between sampling points
- Larger oscillations for higher orders
- Better description: slope, defines ray bending



Aspherical Expansion Order

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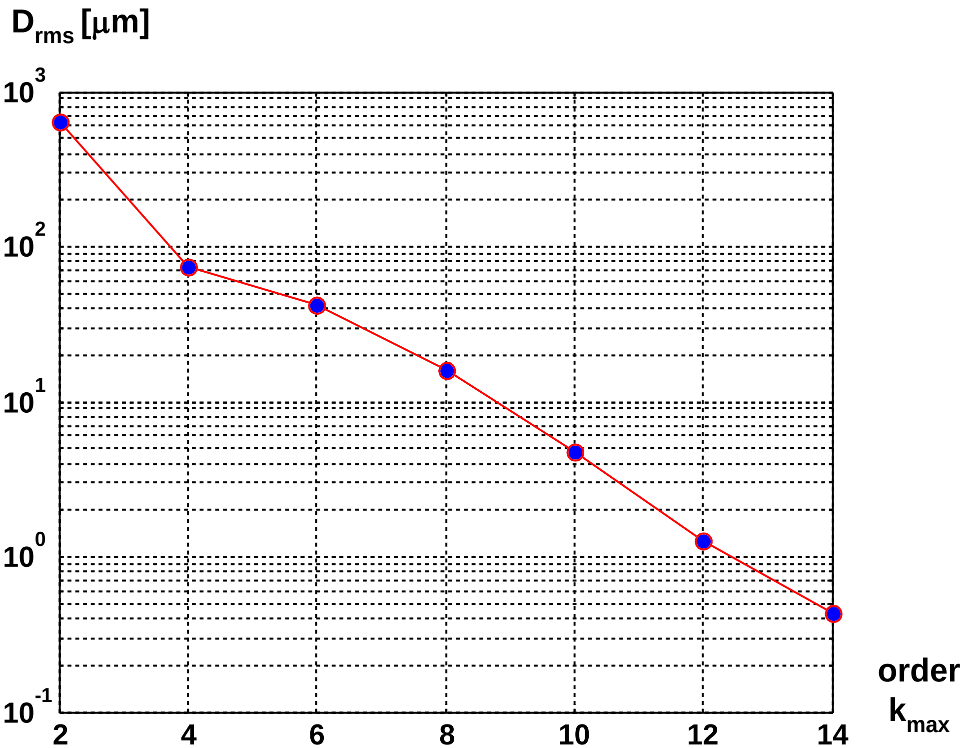
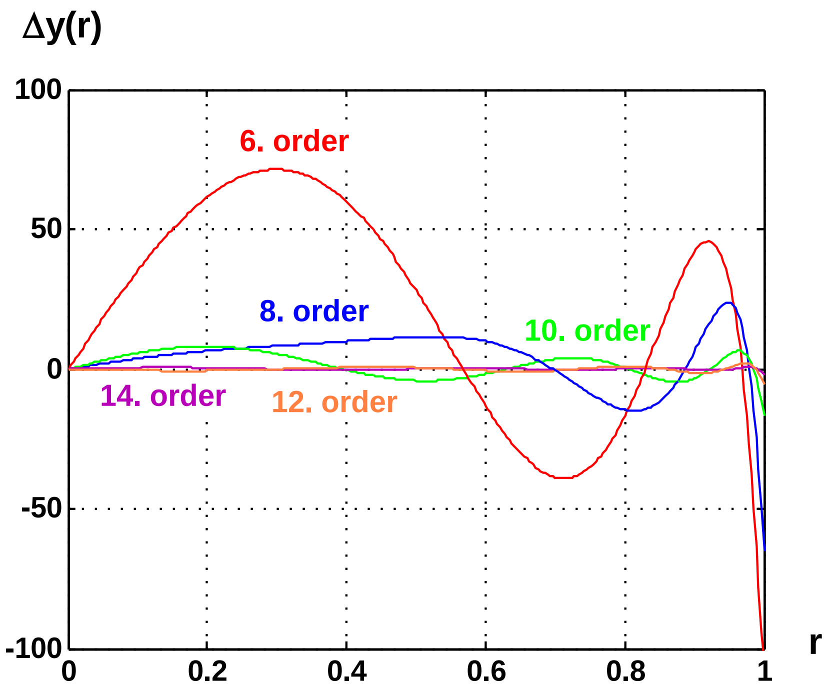
- Improvement by higher orders
- Generation of high gradients





Deviation of Light

Mechanisms of light deviation and ray bending

- Refraction

$$n \cdot \sin\theta = n' \cdot \sin\theta'$$

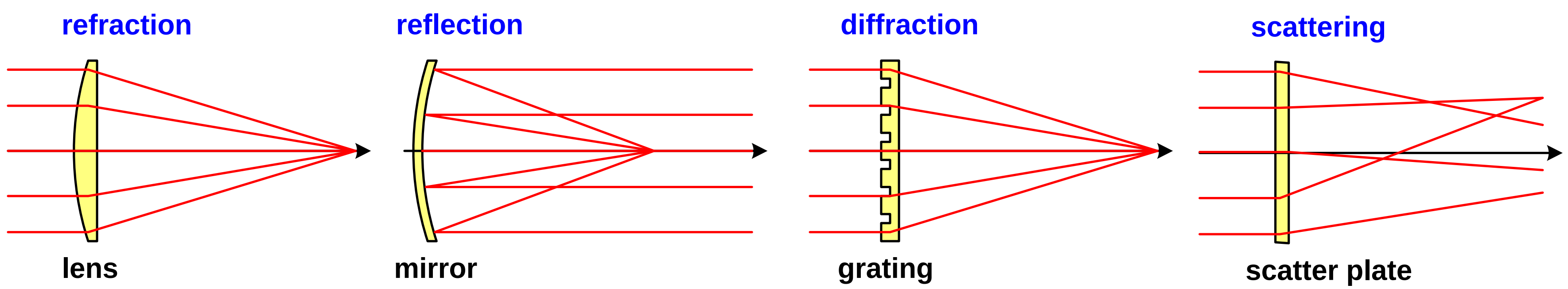
- Reflection

$$\theta = -\theta'$$

- Diffraction according to the grating equation

$$g \cdot \left(\sin\theta - \sin\theta_o\right) = m \cdot \lambda$$

- Scattering (non-deterministic)

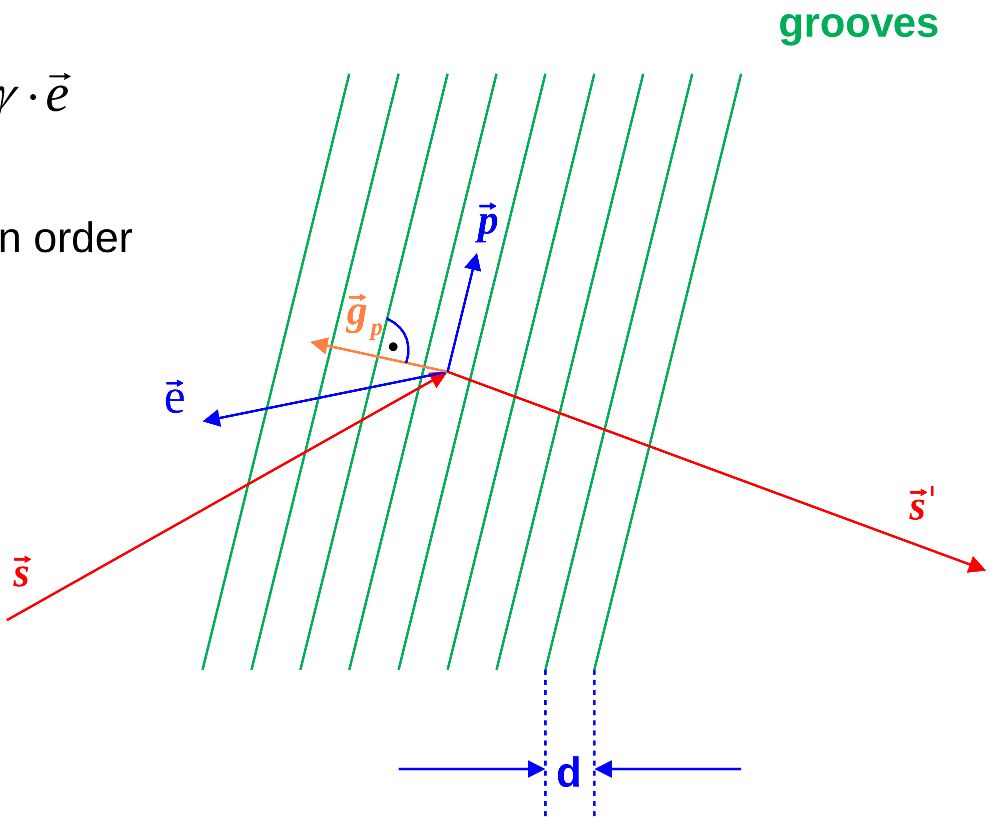


Diffracting Surfaces

- Surface with grating structure:
  new ray direction follows the grating equation
- Local approximation in the case of space-varying grating width

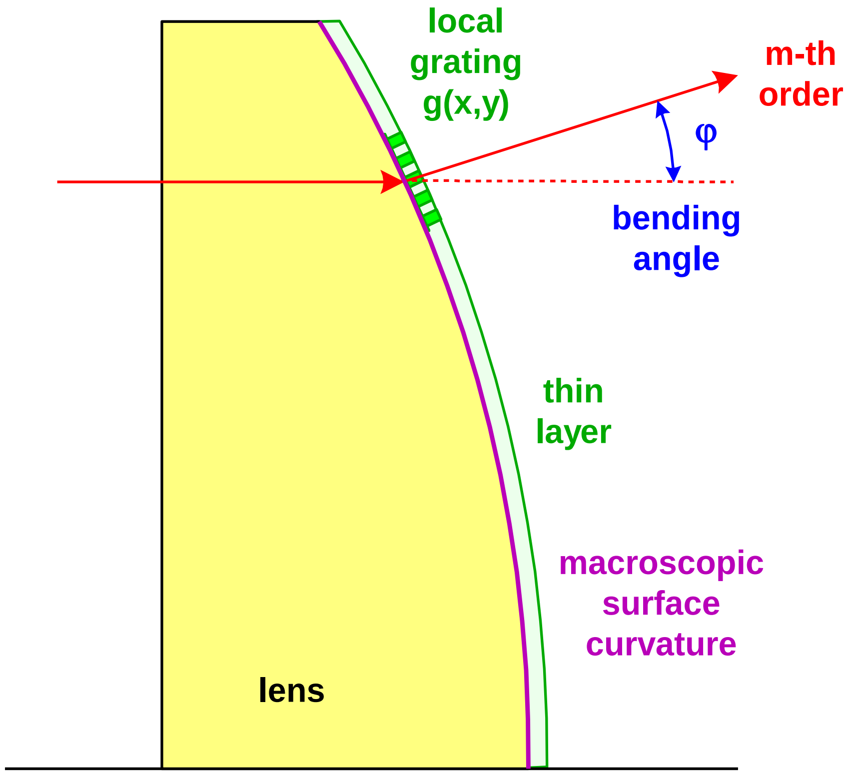
$$\vec{s}\,' = \frac{n}{n'} \cdot \vec{s} + \frac{m \lambda g}{n' d} \cdot \hat{\vec{g}} + \gamma \cdot \vec{e}$$

- Raytrace only into one desired diffraction order
- Notations:
  g : unit vector perpendicular to grooves
  d : local grating width
  m : diffraction order
  e : unit normal vector of surface
- Applications:
  - diffractive elements
  - line gratings
  - holographic components



Diffractive Optics:

- Local micro-structured surface
- Location of ray bending :
  macroscopic carrier surface
- Direction of ray bending :
  local grating micro-structure



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Summary of Important Topics

- Mathematical description of dispersion: Sellmeier formulas, corresponds to modelling adjacent absorption lines, typically 2-3 terms considered
- Characterization of optical materials: refractive index n, Abbe number ν (dispersion)
- Abbe number: mean slope of dispersion curve, small ν means large dispersion
- Choice of special wavelengths to describe the dispersion behavior (e.g.: e, F, C)
- Partial dispersion P: more refined description of dispersion: curvature
- Normal glass line: most glasses fulfill P proportional to ν
- Glass diagram: n-ν-chart, distinction of
  1. crown glasses, n low, ν high, dispersion low
  2. flint glasses; n high, ν low, dispersion high
- Plastics materials: for high volume systems, bad performance in comparison to glass (thermal, index low, straylight, transmission)
- Lenses: simple elements, focal length as main parameter
- Lenses: importance of principal plane, plane of arteficial ray bending
- Lens bending: constant focal length but varying shape: incidence changed
- Simple aspheres: conic sections, mainly important for mirrors
- Aspherical lenses: one more degree of freedom
- Expansion aspheres: problem of oscillating sag
- Diffractive elements: local ray bending governed by grating equation