Institute of Applied Physics
Friedrich-Schiller-Universität Jena

Design and Correction of Optical Systems

Lecture 1: Basics

2015-04-15

Herbert Gross

Institute of Applied Physics
Friedrich-Schiller-Universität Jena

Preliminary Schedule

1	15.04.	Basics	Law of refraction, Fresnel formulas, optical system model, raytrace, calculation approaches
2	22.04.	Materials and Components	Dispersion, anormal dispersion, glass map, liquids and plastics, lenses, mirrors, aspheres, diffractive elements
3	29.04.	Paraxial Optics	Paraxial approximation, basic notations, imaging equation, multi-component systems, matrix calculation, Lagrange invariant, phase space visualization
4	06.05.	Optical Systems	Pupil, ray sets and sampling, aperture and vignetting, telecentricity, symmetry, photometry
5	13.05.	Geometrical Aberrations	Longitudinal and transverse aberrations, spot diagram, polynomial expansion, primary aberrations, chromatical aberrations, Seidels surface contributions
6	20.05.	Wave Aberrations	Fermat principle and Eikonal, wave aberrations, expansion and higher orders, Zernike polynomials, measurement of system quality
7	27.05.	PSF and Transfer function	Diffraction, point spread function, PSF with aberrations, optical transfer function, Fourier imaging model
8	03.06.	Further Performance Criteria	Rayleigh and Marechal criteria, Strehl definition, 2-point resolution, MTF-based criteria, further options
9	10.06.	Optimization and Correction	Principles of optimization, initial setups, constraints, sensitivity, optimization of optical systems, global approaches
10	17.06.	Correction Principles I	Symmetry, lens bending, lens splitting, special options for spherical aberration, astigmatism, coma and distortion, aspheres
11	24.06.	Correction Principles II	Field flattening and Petzval theorem, chromatical correction, achromate, apochromate, sensitivity analysis, diffractive elements
12	01.07.	Optical System Classification	Overview, photographic lenses, microscopic objectives, lithographic systems, eyepieces, scan systems, telescopes, endoscopes
13	08.07.	Special System Examples	Zoom systems, confocal systems
14	15.07.	Further Topics	New system developments, modern aberration theory,...

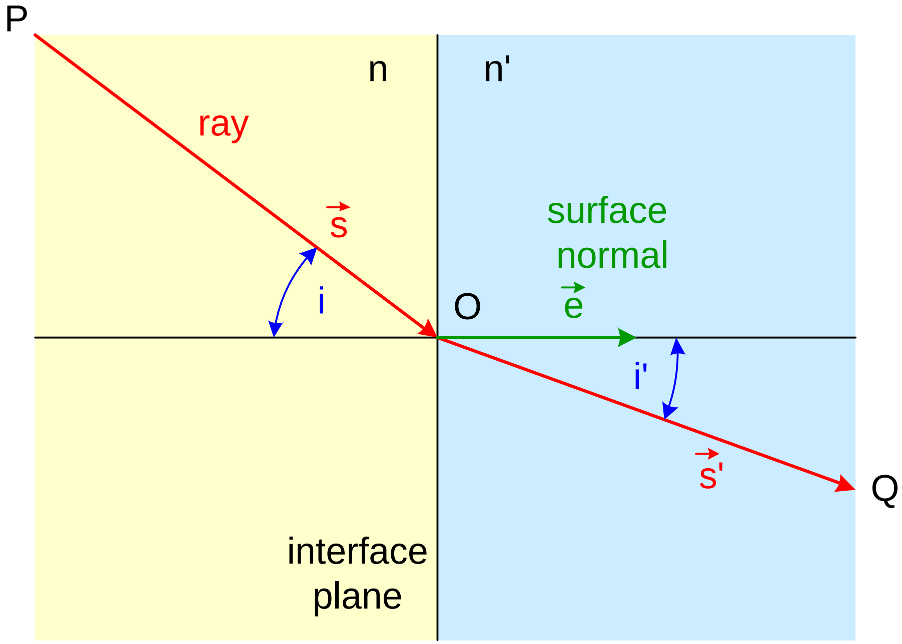
Contents

1. Refraction
2. Fresnel formulas
3. Optical systems
4. Raytrace
5. Calculation approaches

Law of Refraction

Institute of Applied Physics
Friedrich-Schiller-Universität Jena

- Angle deviation at skew incidence
- Change of magnification at curved surfaces, lensing effect



Law of Refraction

- Scalar law of refraction (Snells law) $$n \cdot \sin i = n' \cdot \sin i'$$

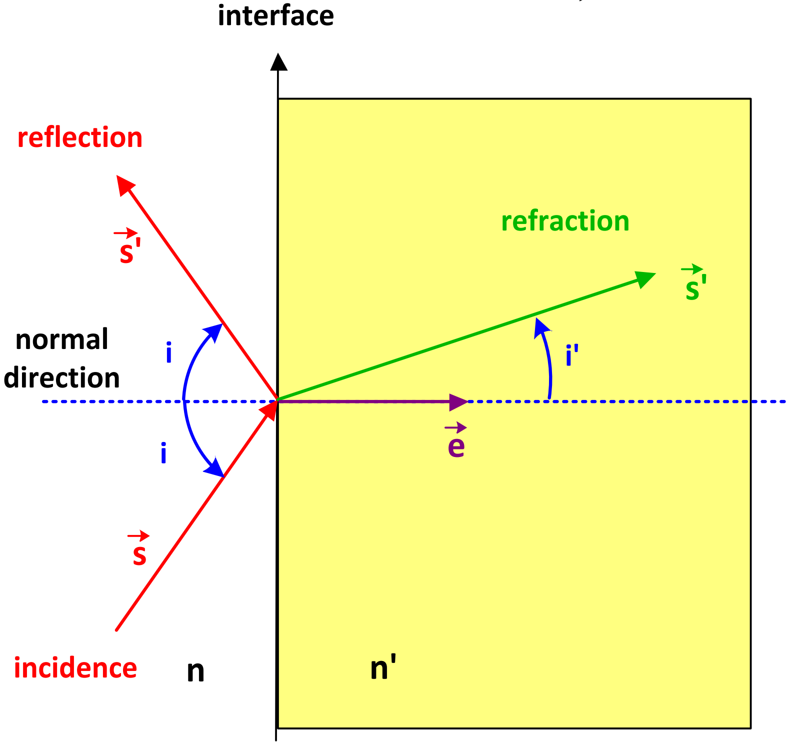
- Vectorial form $$\vec{s}' = \frac{n}{n'} \cdot \vec{s} + \left(-\frac{n}{n'} \cdot \vec{e} \cdot \vec{s} + \sqrt{1 - \left(\frac{n}{n'} \right)^2 \cdot \left[1 - \left(\vec{e} \cdot \vec{s} \right)^2 \right]} \right) \cdot \vec{e}$$

- Special case reflection

$$\vec{s}' = \vec{s} - 2 \cdot \vec{e} \cdot \left(\vec{e} \cdot \vec{s} \right)$$

- All vectors in the plane of incidence

- Fundamental basis:
  Principle of Fermat
  Invariance of field components



Law of Refraction

- Simple derivation of the law of refraction:
  constant optical path length for two rays,
  optical path length:
  product of index of refraction times geometrical path length

$$n \cdot a = n' \cdot a'$$

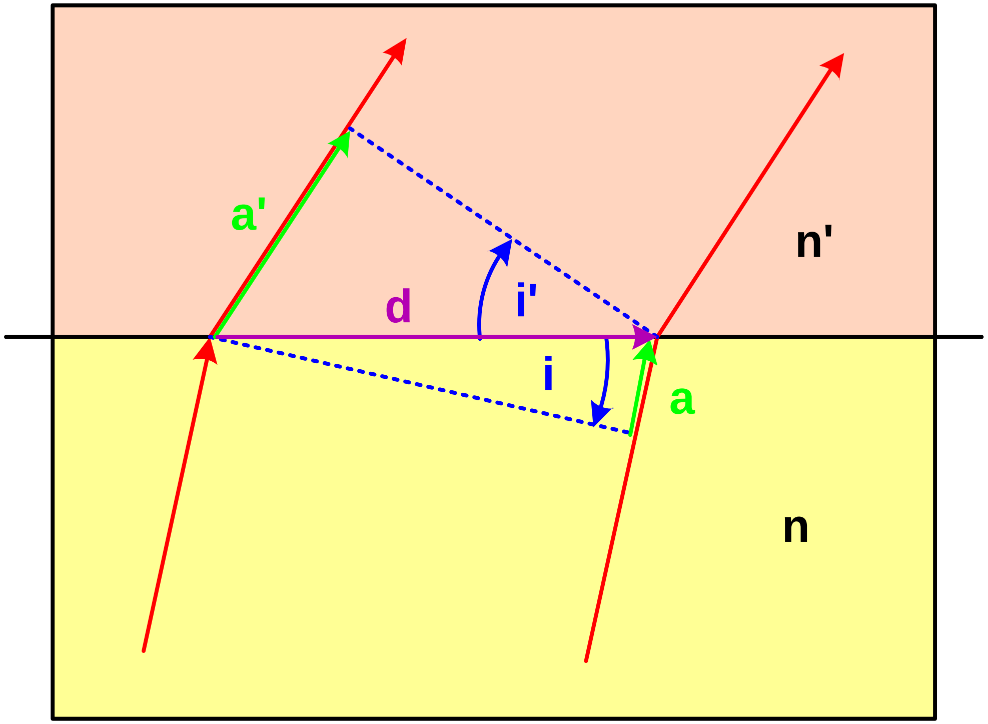
- Geometrical condition of triangles

$$a = d \cdot \sin i$$

$$a' = d \cdot \sin i'$$

- Insertion delivers the law of
  refraction

$$n \cdot \sin i = n' \cdot \sin i'$$



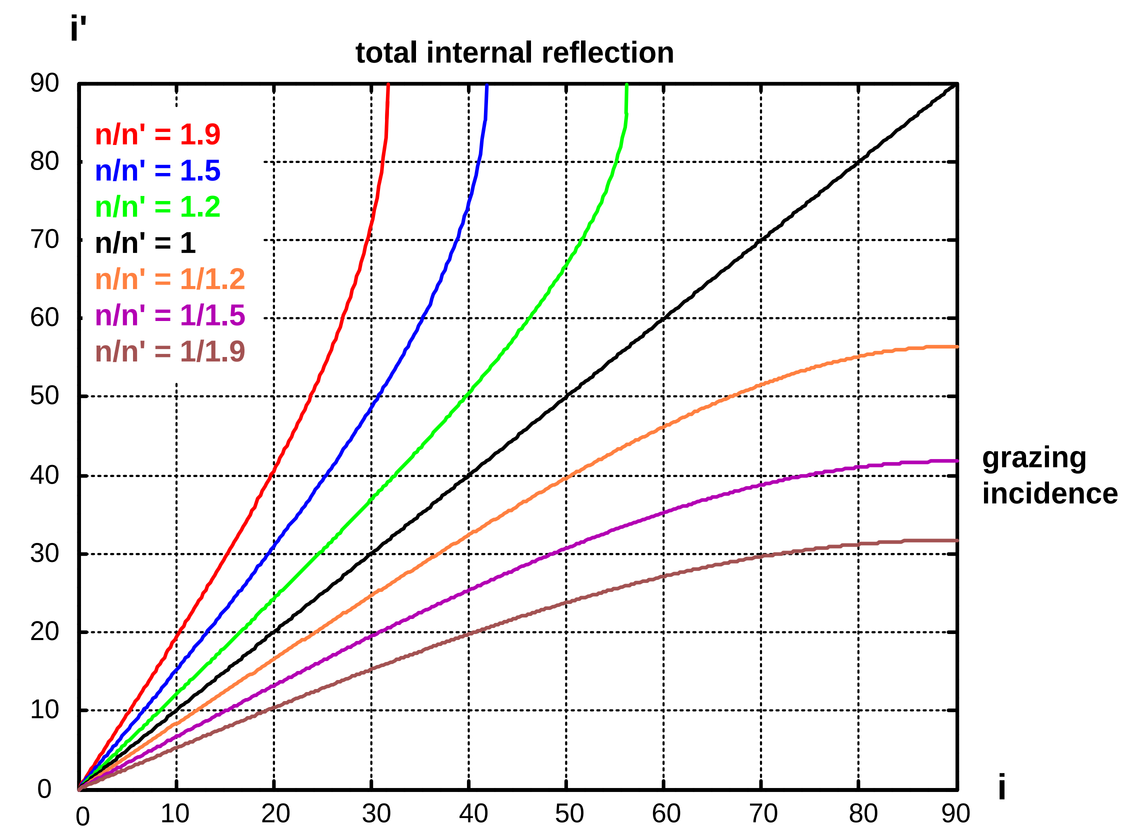
- Scalar law of refraction $n \cdot \sin i = n' \cdot \sin i'$

- Sine is limited by -1…+1:

  1. gracing incidence at i=90°

  2. total internal reflection with i‘ = 90°

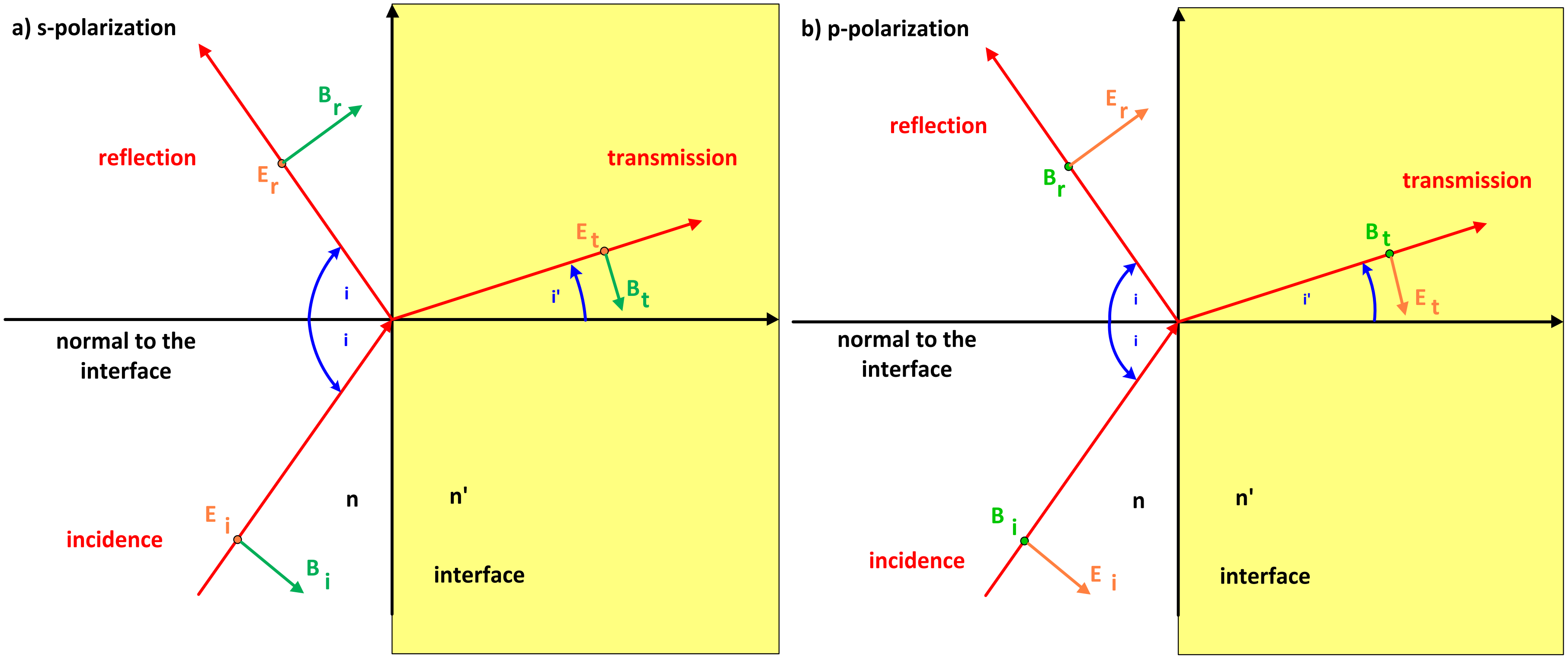
     for $\sin i = n' / n$



Fresnel Formulas

Institute of Applied Physics
Friedrich-Schiller-Universität Jena

- Schematical illustration of the ray refraction (reflection at an interface
- The cases of s- and p-polarization must be distinguished



Fresnel Formulas

- Electrical transverse polarization
  TE, s- or σ-polarization, E perpendicular to incidence plane $\perp$

- Magnetical transverse polarization
  TM, p- or p-polarization, E in incidence plane $\parallel$

- Boundary condition of Maxwell equations
  at a dielectric interface:
  continuous tangential component of E-field

$$\varepsilon_1 \cdot E_{1n} = \varepsilon_2 \cdot E_{2n}$$

$$E_{1t} = E_{2t}$$

- Amplitude coefficients for
  reflected field

$$r_{TE} = \left.\frac{E_r}{E_e}\right|_{TE} \qquad r_{TM} = \left.\frac{E_r}{E_e}\right|_{TM}$$

  transmitted field

$$t_{TE} = \left.\frac{E_t}{E_e}\right|_{TE} = r_{TE} + 1 \qquad t_{TM} = \frac{n}{n'} \cdot \left(r_{TM} + 1\right)$$

- Reflectivity and transmission
  of light power

$$R = \frac{P_r}{P_e} = \left|r^2\right| \qquad T = \frac{P_t}{P_e} = \frac{n' \cdot \cos i'}{n \cdot \cos i} \cdot \left|t^2\right|$$

Fresnel Formulas: Stokes Relations

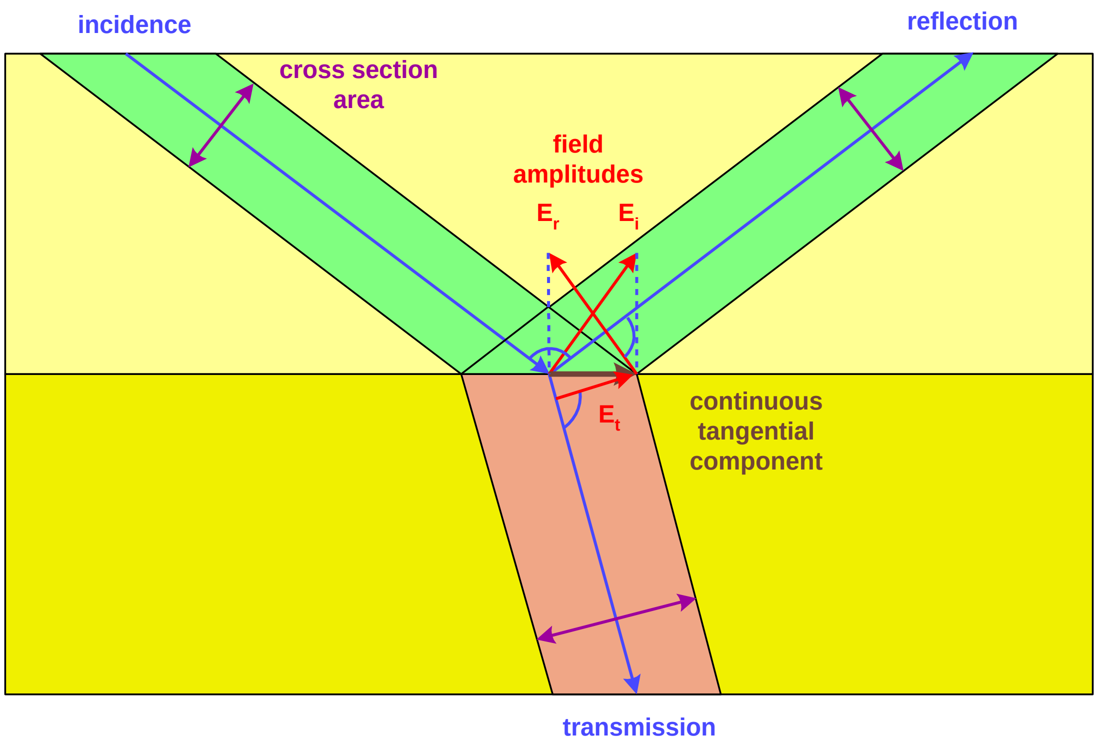
- Relation between the amplitude coefficients for reflection/transmission:

1. s-components:
   field components additive
   minus sign due to phase jump

$$t_\perp - r_\perp = 1$$

2. p-components:
   energy preservation but change of area size due to projection, correction factor, no additivity of intensities

$$t_\parallel \cdot \frac{\cos i'}{\cos i} + r_\parallel = 1$$



Fresnel Formulas

- Coefficients of amplitude for reflected rays, s and p

$$r_{E\perp} = -\frac{\sin(i-i')}{\sin(i+i')} = \frac{n\cdot\cos i - \sqrt{n'^2 - n^2\cdot\sin^2 i}}{n\cdot\cos i + \sqrt{n'^2 - n^2\cdot\sin^2 i}} = \frac{n\cdot\cos i - n'\cdot\cos i'}{n\cdot\cos i + n'\cdot\cos i'} = \frac{k_{ez} - k_{tz}}{k_{ez} + k_{tz}}$$

$$r_{E\parallel} = \frac{\tan(i-i')}{\tan(i+i')} = \frac{n'^2\cdot\cos i - n\cdot\sqrt{n'^2 - n^2\cdot\sin^2 i}}{n'^2\cdot\cos i + n\cdot\sqrt{n'^2 - n^2\cdot\sin^2 i}} = \frac{n'\cdot\cos i - n\cdot\cos i'}{n'\cdot\cos i + n\cdot\cos i'} = \frac{n'^2\cdot k_{ez} - n^2\cdot k_{tz}}{n'^2\cdot k_{ez} + n^2\cdot k_{tz}}$$

- Coefficients of amplitude for transmitted rays, s and p

$$t_{E\perp} = \frac{2n\cdot\cos i}{n\cdot\cos i + n'\cdot\cos i'} = \frac{2n\cdot\cos i}{n\cdot\cos i + \sqrt{n'^2 - n^2\cdot\sin^2 i}} = \frac{2n\cdot\cos i}{n\cdot\cos i + n'\cdot\cos i'} = \frac{2k_{ez}}{k_{ez} + k_{tz}}$$

$$t_{E\parallel} = \frac{2n\cdot\cos i}{n'\cdot\cos i + n\cdot\cos i'} = \frac{2n'\cdot n\cdot\cos i}{n'^2\cdot\cos i + n\cdot\sqrt{n'^2 - n^2\cdot\sin^2 i}} = \frac{2n\cdot\cos i}{n'\cdot\cos i + n\cdot\cos i'} = \frac{2n'^2\cdot k_{ez}}{n'^2\cdot k_{ez} + n^2\cdot k_{tz}}$$

Fresnel Formulas

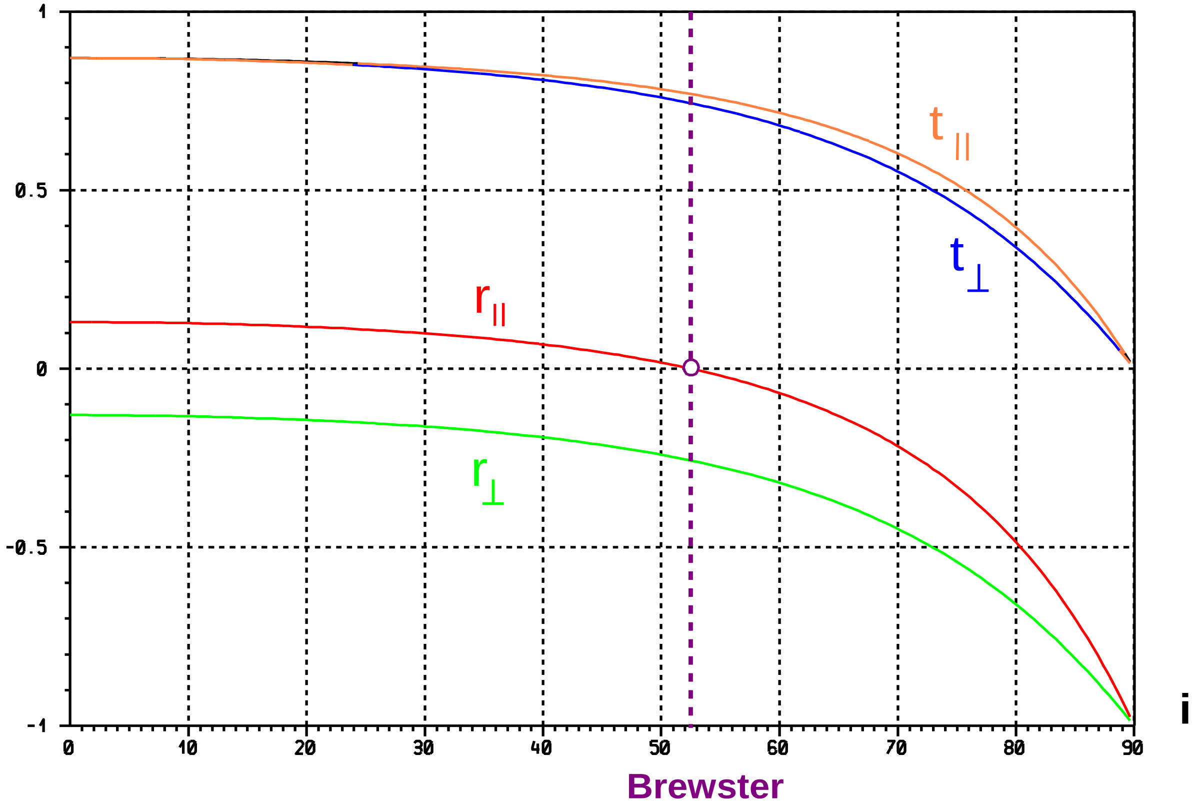
Institute of Applied Physics
Friedrich-Schiller-Universität Jena

- Typical behavior of the Fresnel amplitude coefficients as a function of the incidence angle for a fixed combination of refractive indices

- i = 0
  Transmission independent on polarization
  Reflected p-rays without phase jump
  Reflected s-rays with phase jump of π (corresponds to r<0)

- i = 90°
  No transmission possible
  Reflected light independent on polarization

- Brewster angle:
  completely s-polarized reflected light



Fresnel Formulas: Energy vs. Intensity

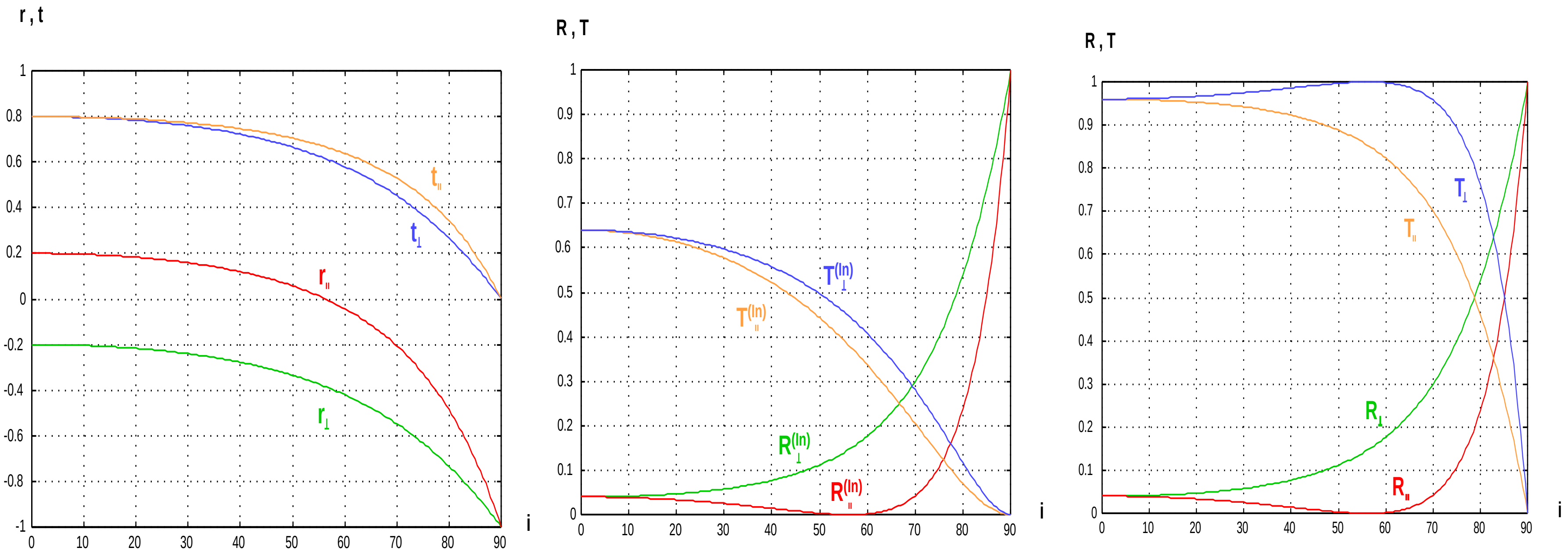
Institute of Applied Physics
Friedrich-Schiller-Universität Jena

Fresnel formulas, different representations:

1. Amplitude coefficients, with sign

2. Intensity cefficients: no additivity due to area projection

3. Power coefficients: additivity due to energy preservation



Fresnel Formulas

- Reflectivity and transmittivity of power

$$R_\perp = \frac{\sin^2(i-i')}{\sin^2(i+i')} \qquad R_\parallel = \frac{\tan^2(i-i')}{\tan^2(i+i')} \qquad T_\perp = \frac{\sin 2i' \cdot \cos 2i}{\sin^2(i+i')} \qquad T_\parallel = \frac{\cos 2i \cdot \sin 2i'}{\sin^2(i+i') \cdot \cos^2(i-i')}$$

- Arbitrary azimuthal angle τ of polarization: decompositioin of components

$$R = R_\parallel \cdot \cos^2 \tau_e + R_\perp \cdot \sin^2 \tau_e \qquad T = T_\parallel \cdot \cos^2 \tau_e + T_\perp \cdot \sin^2 \tau_e$$

- In case of vanishing absorption:
  Energy preservation

$$R + T = 1$$

- Special case of normal incidence

$$R_\perp = \left(\frac{n-n'}{n+n'}\right)^2 \qquad T_\perp = \frac{4n \cdot n'}{(n+n')^2}$$

- Typical values for some glasses and optical materials in air

n	R
1.4	2.778 %
1.5	4.0 %
1.8	8.16 %
2.4	16.96 %

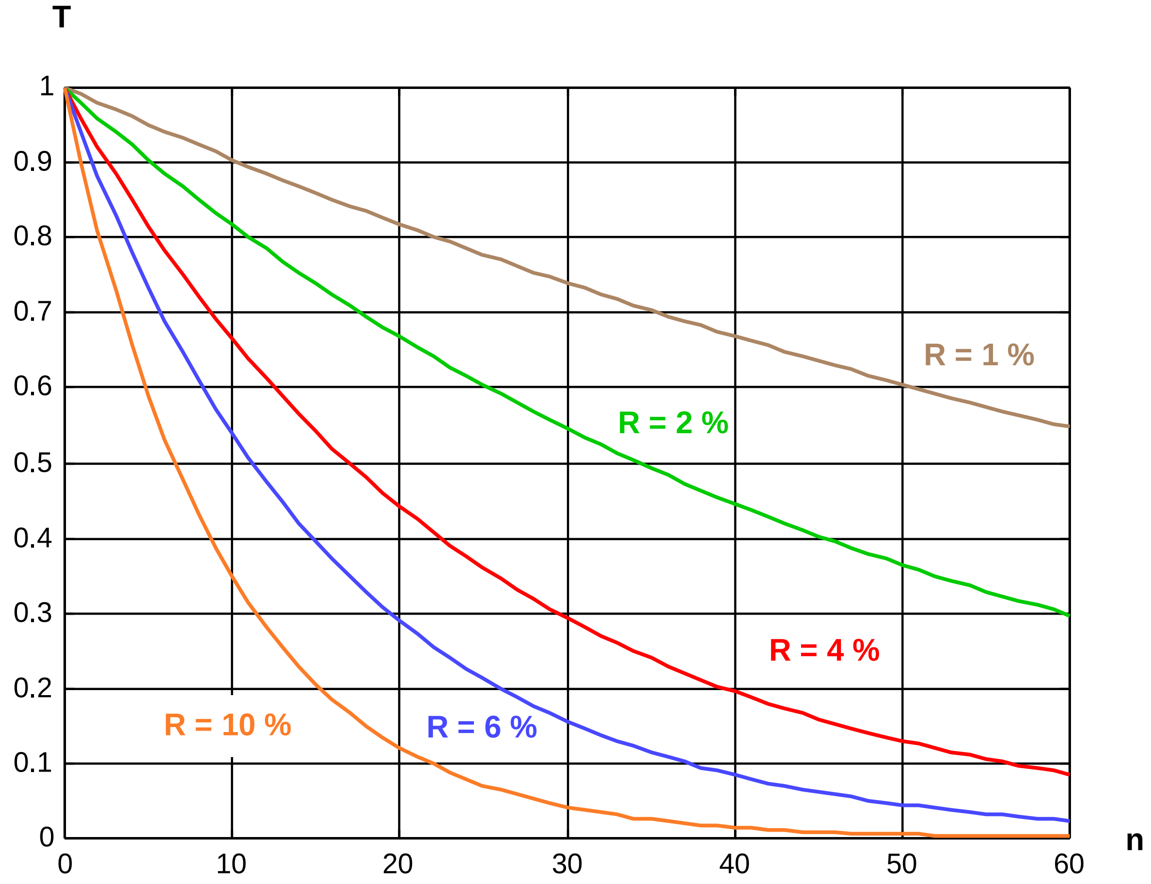
Transmission in Optical Systems

- Residual reflectivity of the (identical) surfaces in an optical system with n surfaces:
  Overall transmission of energy:

$$T_{ges} = (1 - R)^n$$

- Transmission decreases nonlinear

- Practical consequences:
  1. loss of signal energy
  2. contrast reduction in case of imaging
  3. occurence of ghost images



Brewster Angle

Institute of Applied Physics
Friedrich-Schiller-Universität Jena

- Brewster case of reflection:
  reflected and transmitted ray are perpendicular

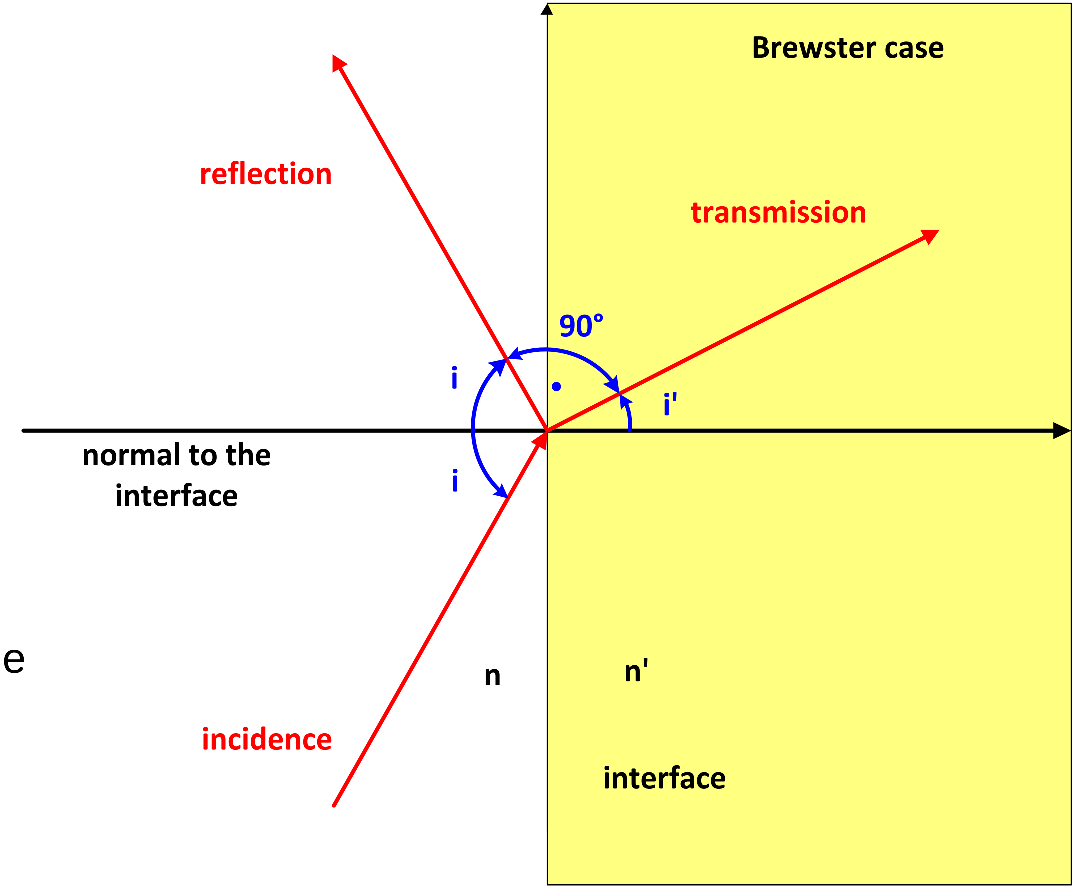
$$i + i' = 90°$$

- Condition of Brewster angle

$$\tan i_B = \frac{n'}{n}$$

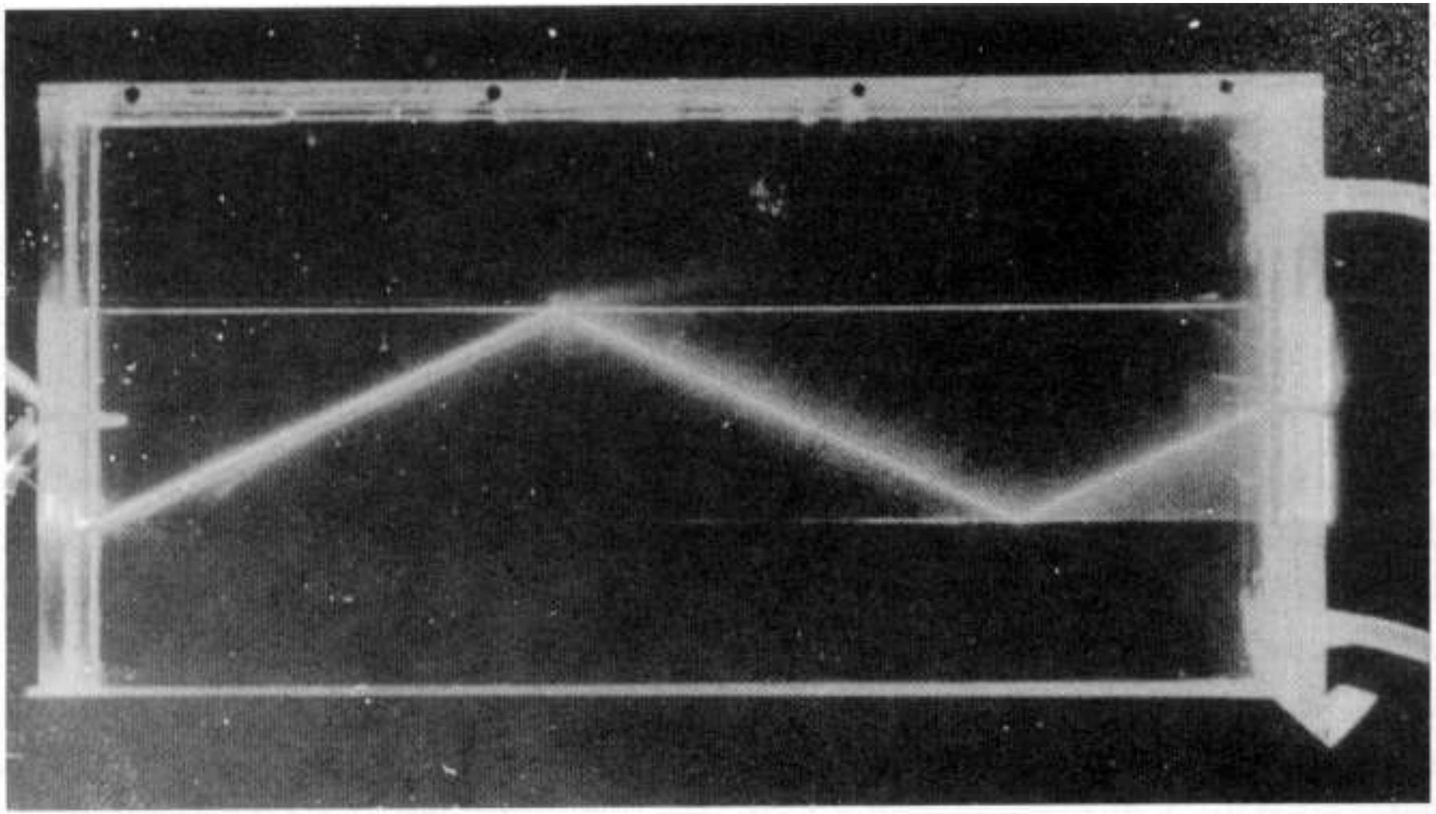
- The reflected light is completely s polarized

- Application:
  stack of plates under Brewster angle as polarizer



Total Internal Reflection in Fibers

- Total internal reflection between core and cladding in a step index fiber



Ref: M. Kaschke

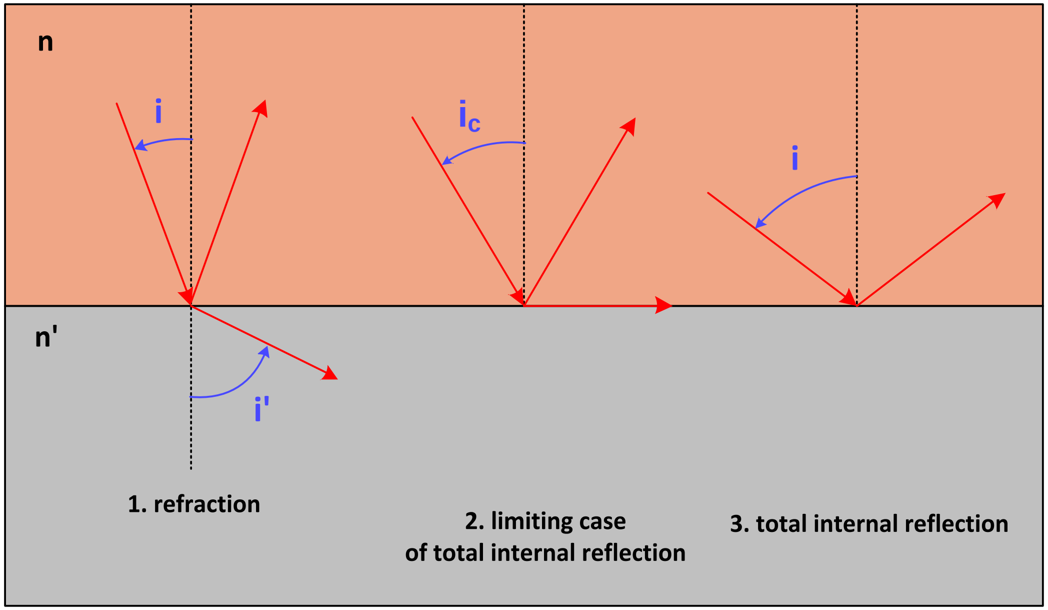
Total Internal Reflection

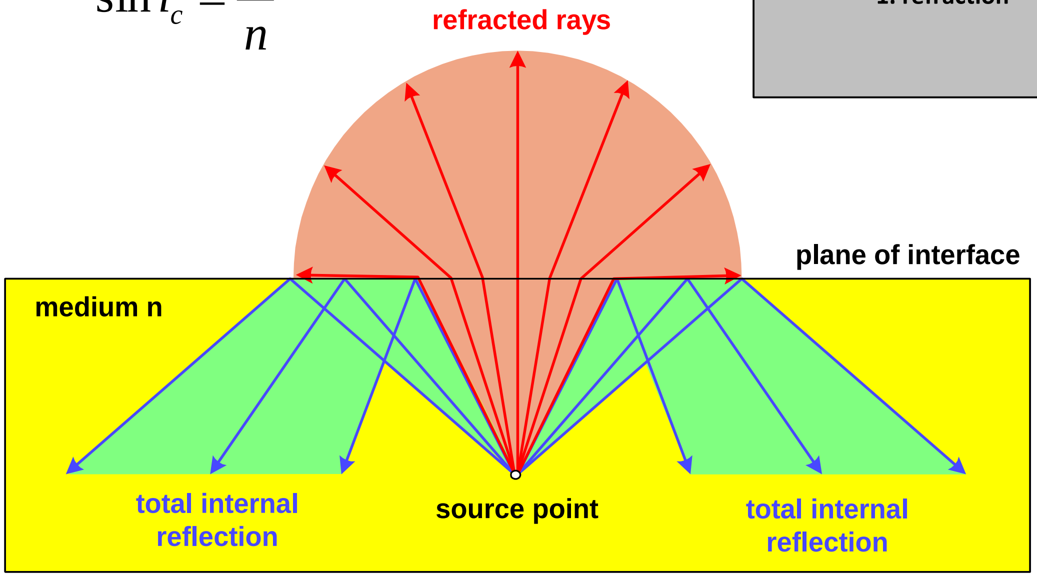
- Limiting angle i_c of total internal reflection:
  no light leaves the medium with the higher index

$$R_{\perp} = R_{\parallel} = 1$$

- Condition:

$$\sin i_c = \frac{n'}{n}$$



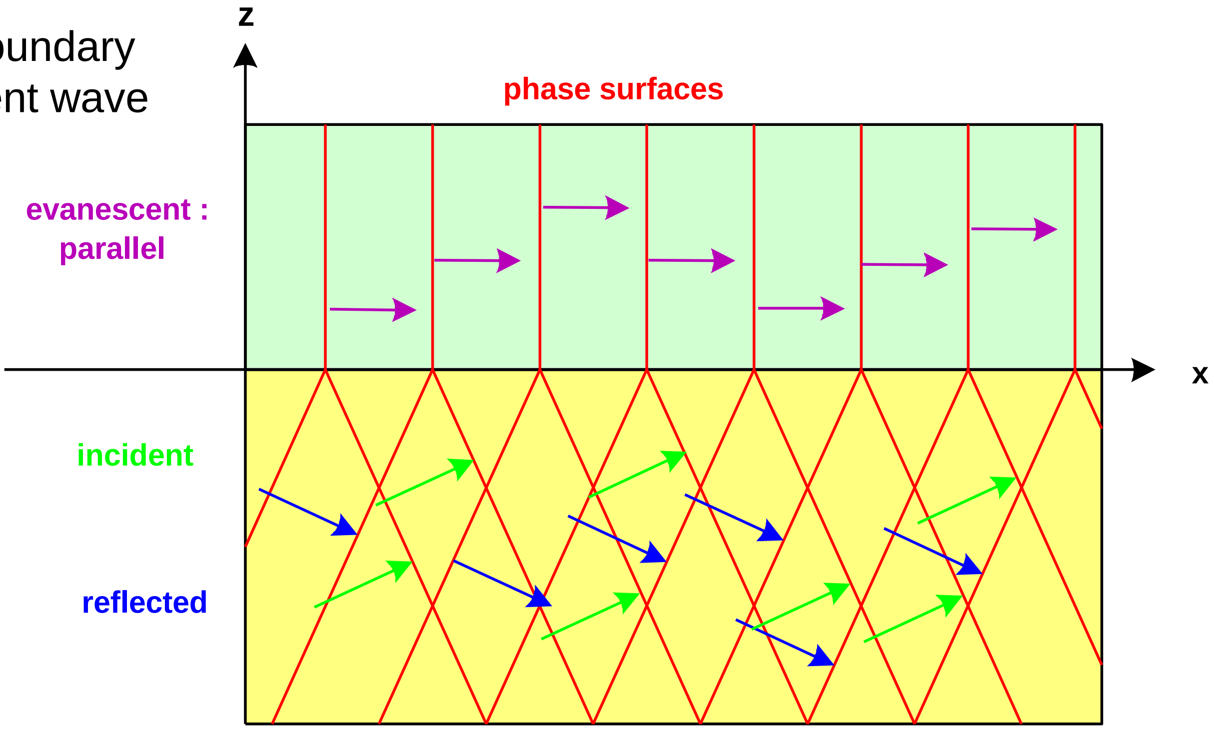


Total Internal Reflection

- In case of total internal reflection, the Fresnel formulae reads

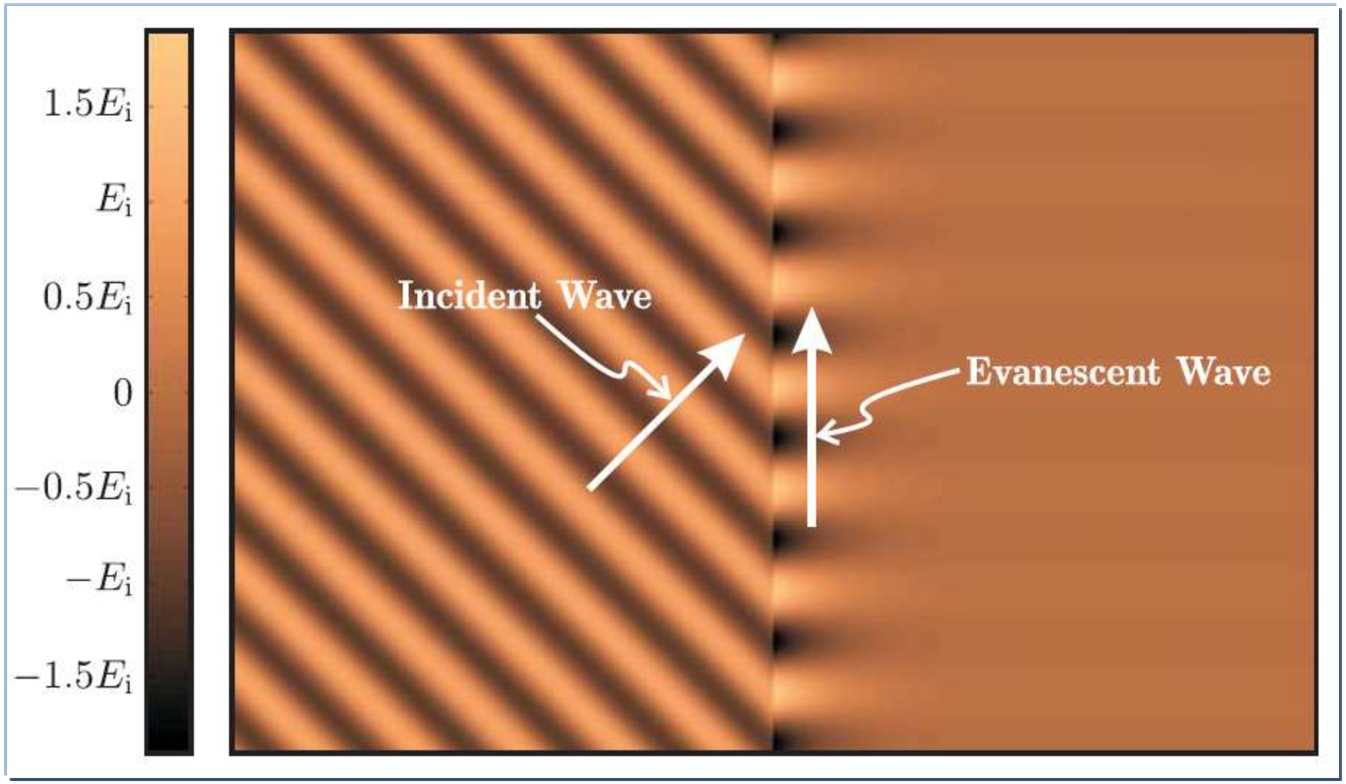
$$r_\perp = \frac{n\cos i - \sqrt{-1}\cdot\sqrt{n^2\sin^2 i - n'^2}}{n\cos i + \sqrt{-1}\cdot\sqrt{n^2\sin^2 i - n'^2}} \qquad r_\parallel = \frac{n^3\cos i - n'^2\cdot\sqrt{-1}\cdot\sqrt{n^2\sin^2 i - n'^2}}{n^3\cos i + n'^2\cdot\sqrt{-1}\cdot\sqrt{n^2\sin^2 i - n'^2}}$$

- The wave penetrates the boundary and generates an evanescent wave which propagates along the boundary



Evanescent Field

- Visualization of the evanescent wave
- Evanescent field: finite penetration depth propagation parallel to the interface plane
- Data: n = 1.5 , n' = 1.0 ; $\theta = 45^\circ$



Ref: Peatros

Description of Optical Systems

- Interface surfaces
  - mathematical modelled surfaces
  - planes, spheres, aspheres, conics, free shaped surfaces,…

- Size of components
  - thickness and distances along the axis
  - transversal size,circular diameter, complicated contours

- Geometry of the setup
  - special case: rotational symmetry
  - general case: 3D, tilt angles, offsets and decentrations, needs vectorial approach

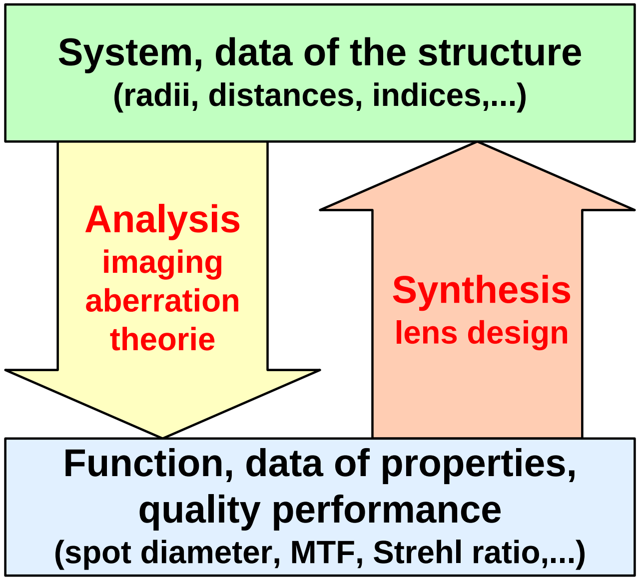
- Materials
  - refractive indices for all used wavelengths
  - other properties: absorption, birefringence, nonlinear coefficients, index gradients,…

- Special surfaces
  - gratings, diffractive elements
  - arrays, scattering surfaces

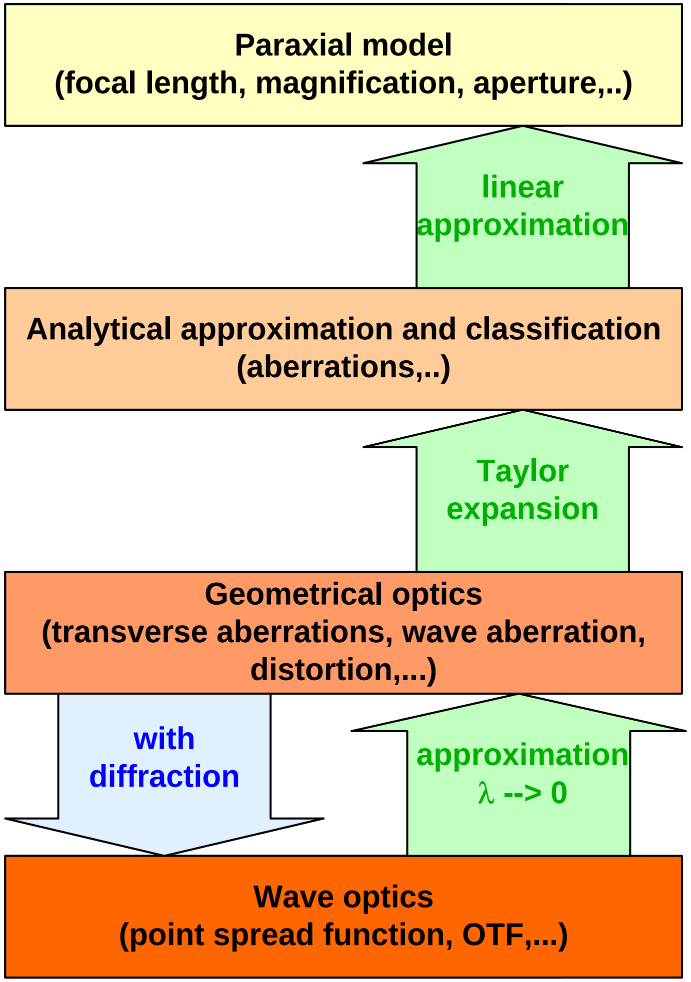
Modelling of Optical Systems

Institute of Applied Physics
Friedrich-Schiller-Universität Jena

- Principal purpose of calculations:



- Imaging model with levels of refinement



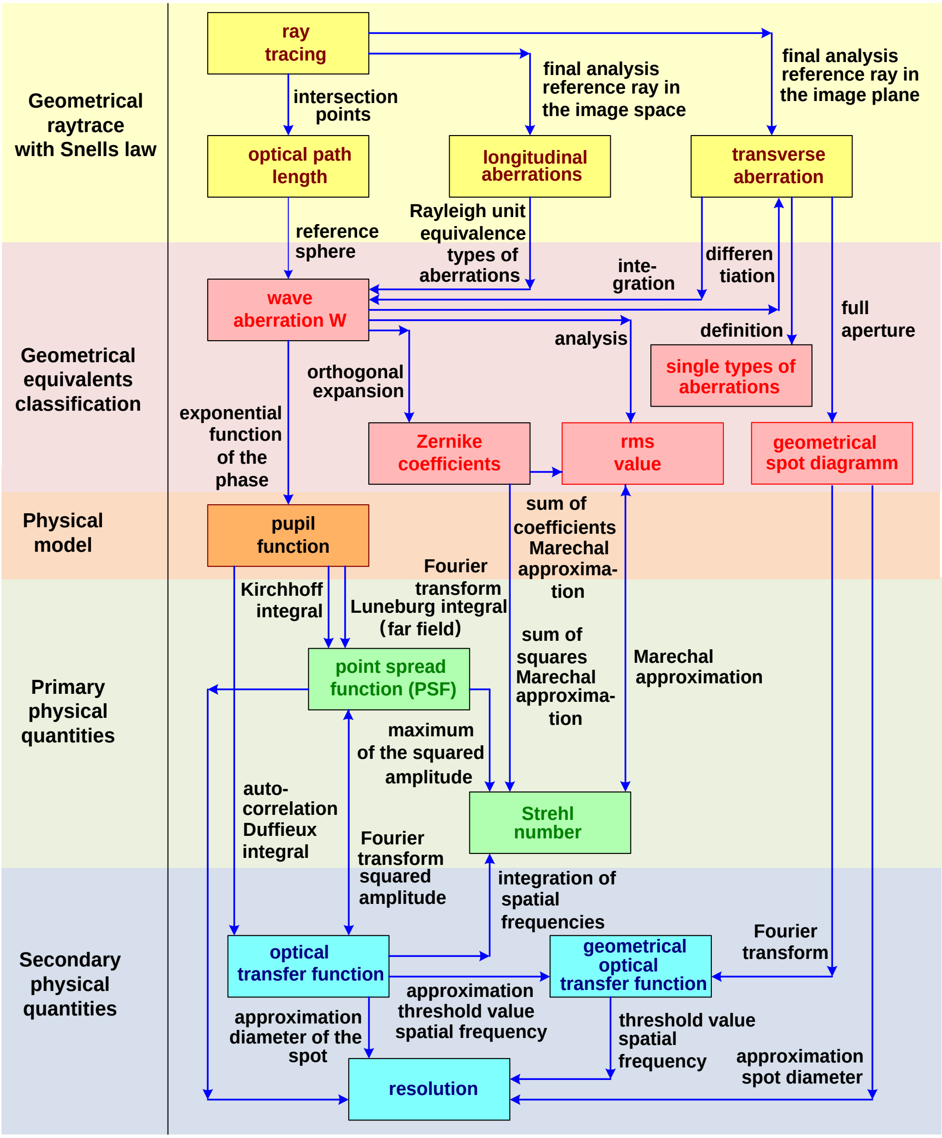
Ref: W. Richter

Modelling of Optical Systems

- Five levels of modelling:

1. Geometrical raytrace with analysis
2. Equivalent geometrical quantities, classification
3. Physical model: complex pupil function
4. Primary physical quantities
5. Secondary physical quantities

- Blue arrows: conversion of quantities



Institute of Applied Physics
Friedrich-Schiller-Universität Jena

Scheme of Raytrace

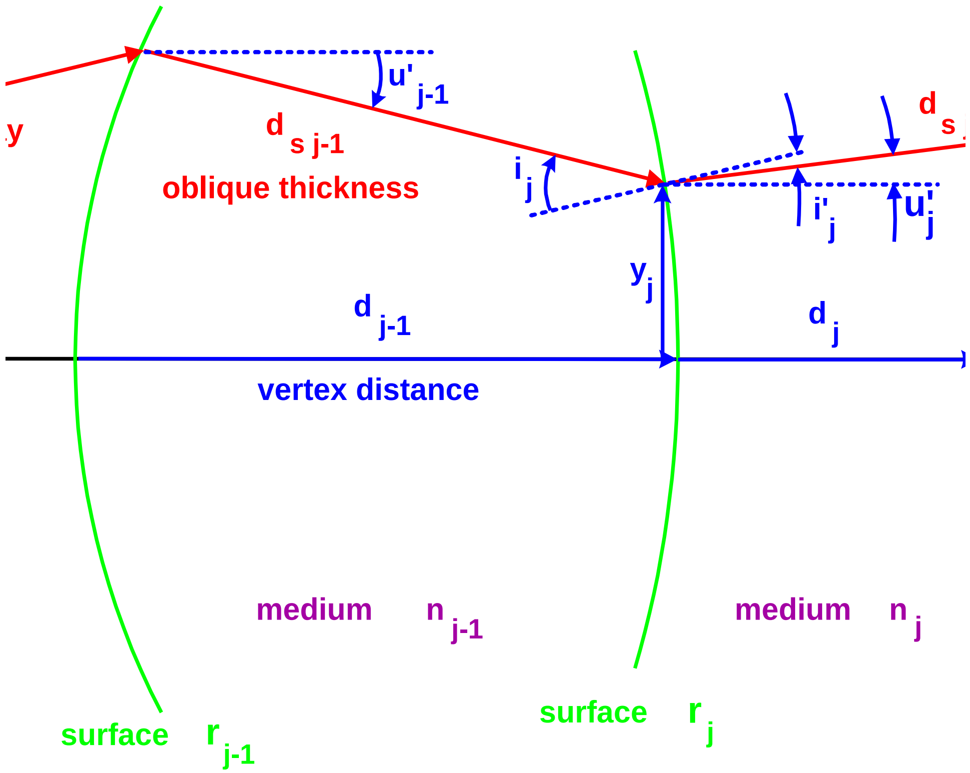
- Ray: straight line between two intersection points
- System: sequence of spherical surfaces
- Data: - radii, curvature c=1/r

  - vertex distances

  - refractive indices

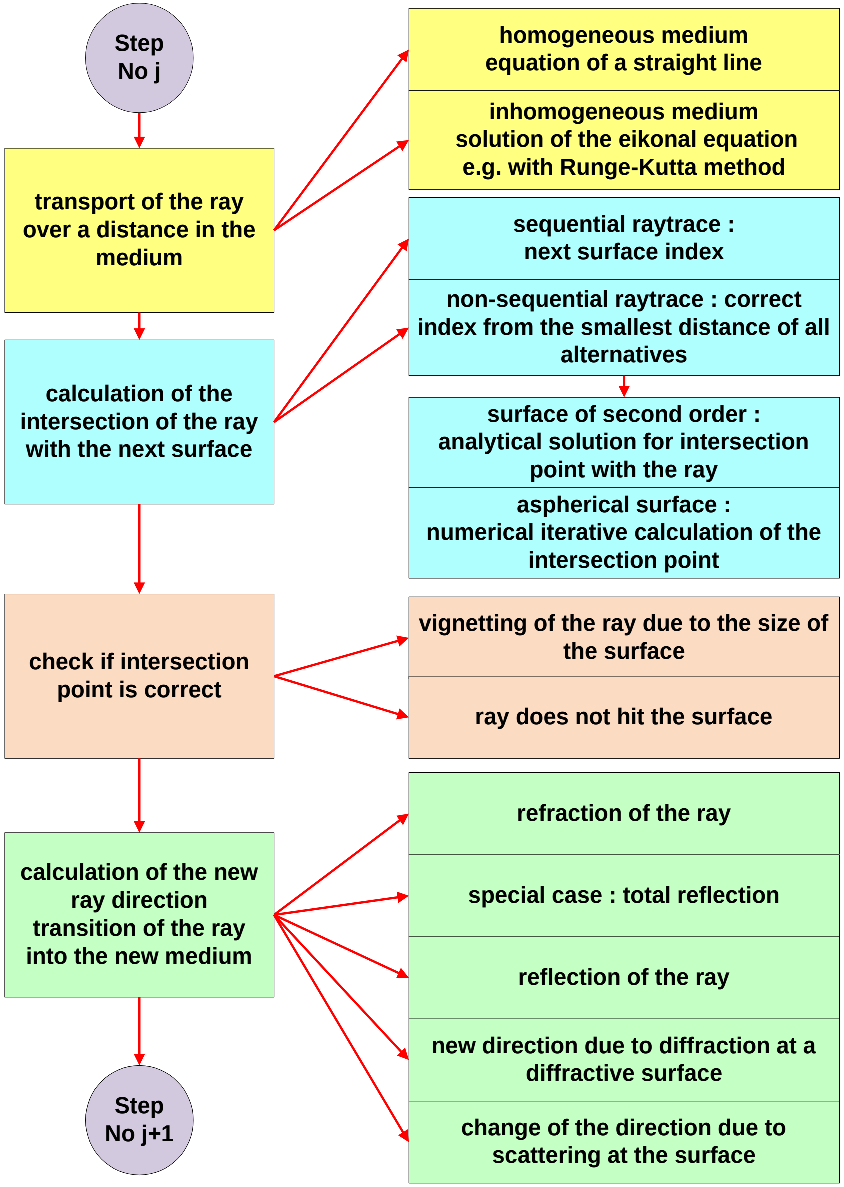
  - transverse diameter
- Surfaces of 2nd order:

  Calculation of intersection points analytically possible: fast computation



Workflow Raytrace

- Two step process:
  1. Transition to next surface
     intersection point,
     test of plausibility
  2. Dielectric interface
     refraction and new direction
     test of plausibility



Raytrace Formulas

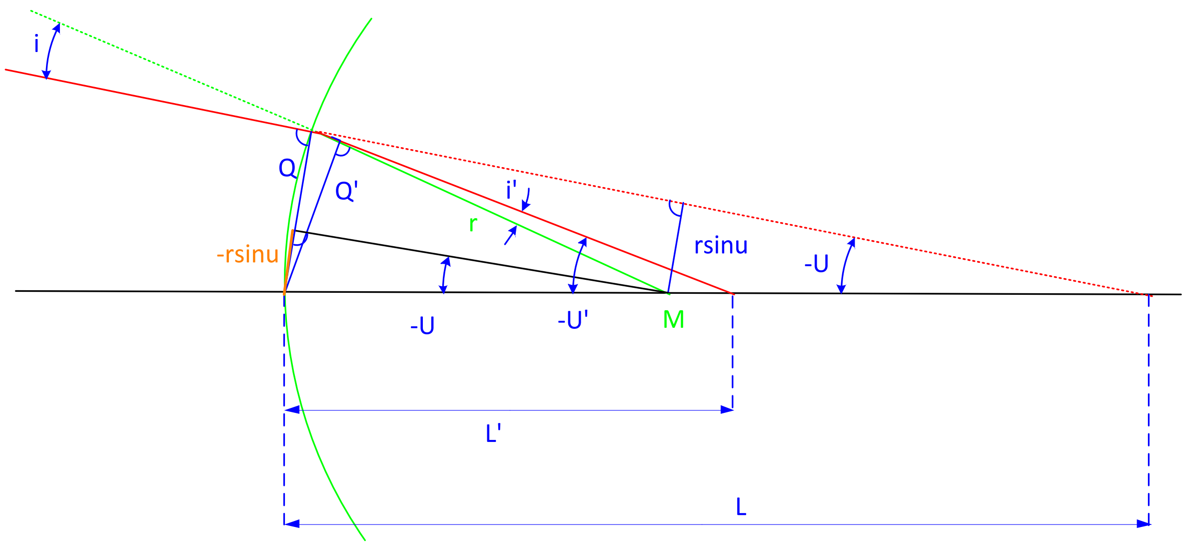
- Different sets of formulas:
  1. Paraxial formulas, for reference of ideal imaging
  2. Meridional formulas, for circular symmetry
  3. Vectorial formulas, 3D systems, state of the art today
  4. Differential formulas, transfer of neighbourhood, ray density, astigmatism

- Special aspects:
  1. Aspherical surfaces, numerical iterative calculation of intersection points
  2. Gradient media, eikonal differentail equation, Runge-Kutta numerical stepwise
  3. Diffractive elements, local grating equation
  4. Non-sequential raytrace, illumination and straylight
  5. Scattering surfaces, Monte-Carlo decision for new direction
  6. Photometric correct raytracing, transfer of relative weighting factor
  7. Polarization raytrace, transfer of Jones vector on a ray
  8. Geometrical approximated edge diffraction, ray deviation depends on edge distance

Institute of Applied Physics
Friedrich-Schiller-Universität Jena

Paraxial y-U-Method

- Paraxial ray trace formulas



- Parameters of ray description:
  1. ray height y
  2. ray angle U

- Transfer to next surface index j -→ j+1

  height

  angle of incidence

$$y_j = y_{j-1} - d_{j-1} \cdot U_{j-1}$$

  refraction

$$i_j = c_j \cdot y_j - U_{j-1}$$

  new direction angle

$$i_j' = \frac{n_j}{n_j'} i_j$$

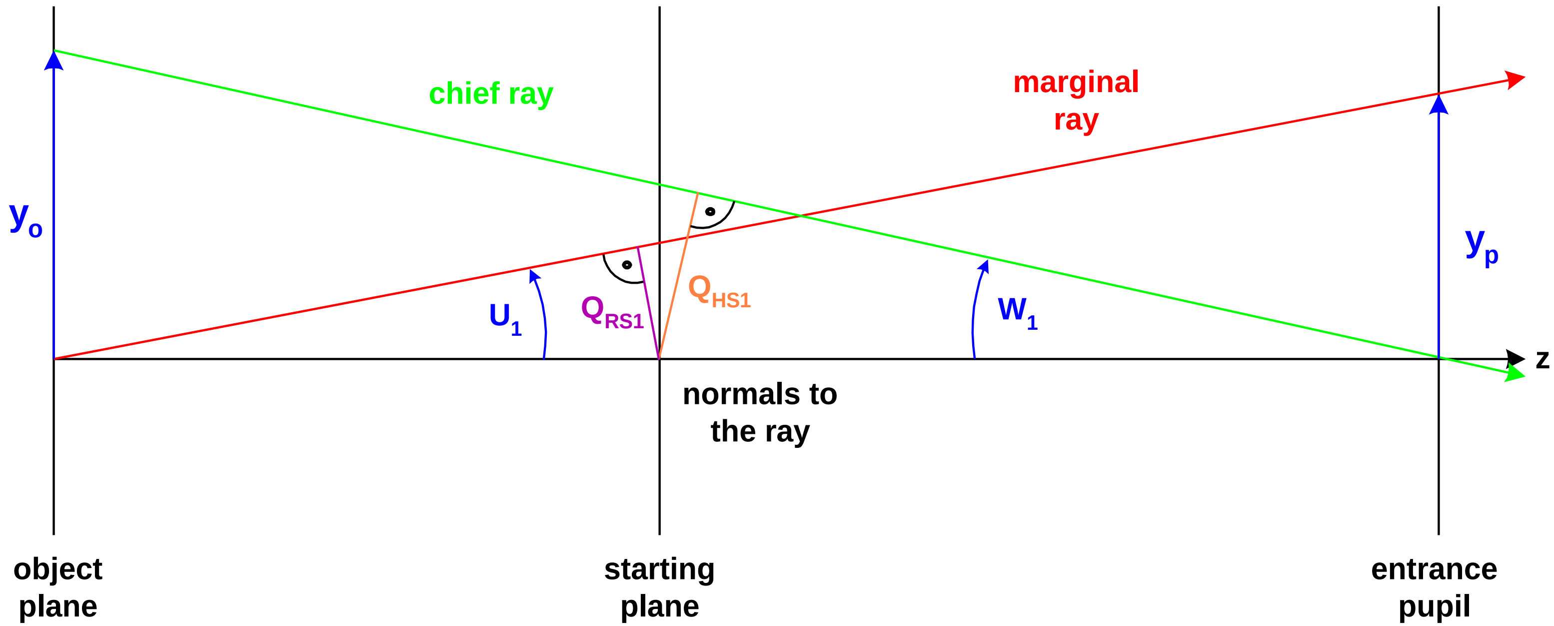
$$U_j' = U_{j-1} + i_j - i_j' = y_j c_j - i_j'$$

Normal Distance of Rays to Surface Vertex

Institute of Applied Physics
Friedrich-Schiller-Universität Jena

- Projection of ray to surface vertex point
  length: Q

- In the paraxial regime identical with ray intersection height y



Exact Meridional Q-U-Raytrace Method

- Exact raytrace scheme in the meridional plane
- Ray description parameters:
  - angle u with optical axis
  - distance Q between ray and surface vertex point
- Set of formulas:

(1) angle of incidence

$$\sin i = Qc - \sin u \qquad \cos i = \sqrt{1-\sin^2 i}$$

(2) refraction

$$\sin i' = \frac{n}{n'} \sin i \qquad \cos i' = \sqrt{1-\sin^2 i'}$$

(3) new angle of ray

$$u' = u - i + i'$$

(4) auxiliary parameter

$$G = \frac{Q}{\cos u + \cos i}$$

(5) distance to vertex

$$Q' = \frac{\sin i' - \sin u'}{c} = Q \frac{\cos u'}{\cos u} = \frac{G}{\cos u' + \cos i'} \qquad Q_{j+1} = Q_j - d \sin u'$$

(6) intersection length

$$L_{j+1} = L_j - d$$

(7) final distance

$$Q_1 = L \sin u$$

Exact Meridional Q-U-Raytrace Method (2)

- Set of formuals (2):

(8) final intersection length

$$L' = \frac{Q'}{\sin u'}$$

(9) intersection point

$$y = \frac{\sin(u+i)}{c} = G \cdot [1 + \cos(u+i)]$$

$$z = \frac{1 - \cos(u+i)}{c} = G \cdot \sin(u+i)$$

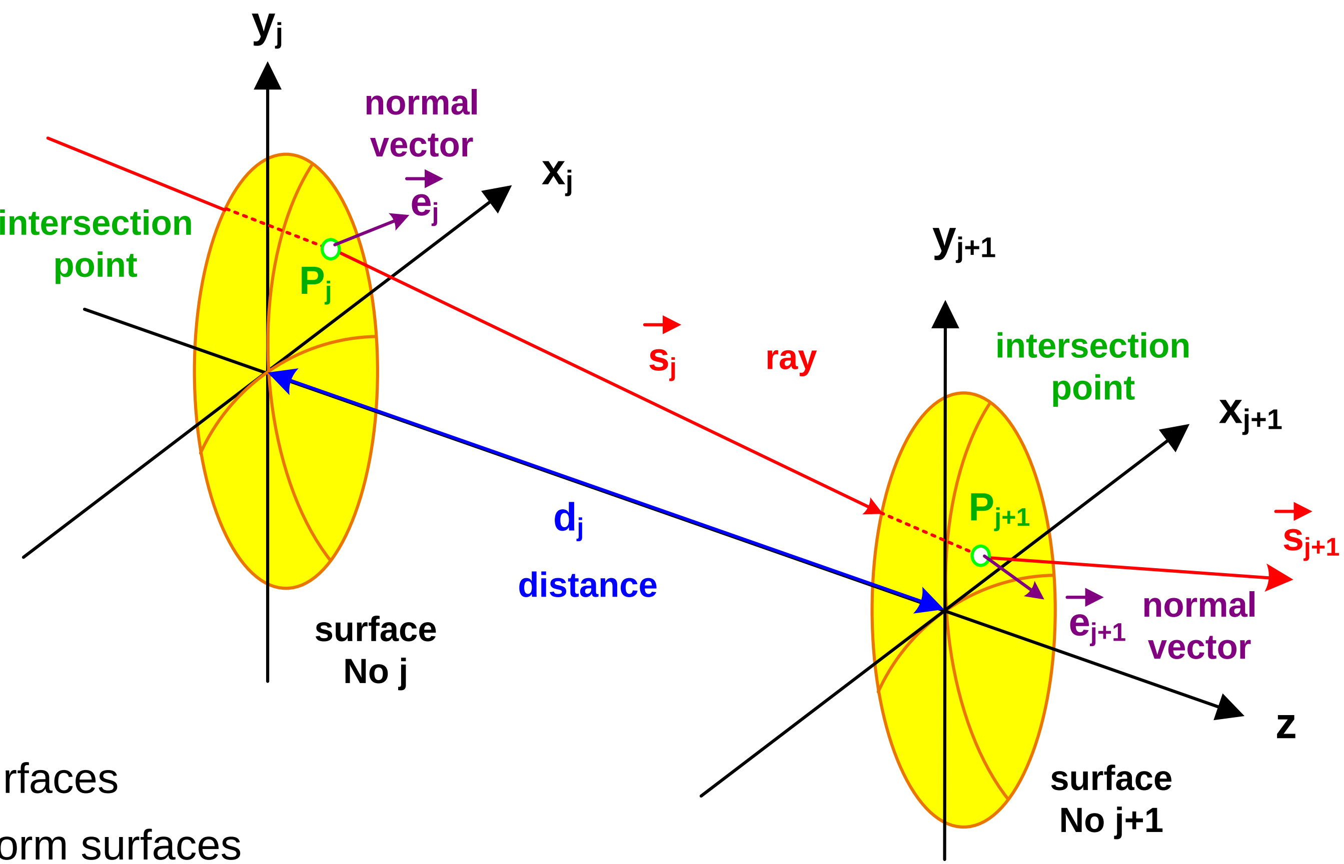
(10) failure condition 1:
ray don‘t hit surface

$$|\sin i| > 1$$

(11) failure condition 2:
total internal reflection

$$|\sin i'| > 1$$

Vectorial Raytrace



- General 3D geometry
- Tilt and decenter of surfaces
- General shaped free form surfaces
- Full description with 3 components
- Global and local coordinate systems

Vectorial Raytrace Formulas

Institute of Applied Physics
Friedrich-Schiller-Universität Jena

- Restrictions:
  - surfaces of second order, fast analytical calculation of intersection point possible
  - homogeneous media

- Direction unit vector of the straight ray

$$\vec{s}_j = \begin{pmatrix} \xi_j \\ \eta_j \\ \varsigma_j \end{pmatrix}$$

- Vector of intersection point on a surface

$$\vec{r}_j = \begin{pmatrix} x_j \\ y_j \\ z_j \end{pmatrix}$$

- Ray equation with skew thickness d_{sj}
  index j of the surface and the space behind

$$\vec{r}_j = \vec{r}_{j-1} + d_{s,j-1} \cdot \vec{s}_{j-1}$$

- Equation of the surface 2.order
  The coefficients H, F, G contains the surface shape parameters

$$H_j d_{s,j-1}^2 + 2F_j d_{s,j-1} - G_j = 0$$

Vectorial Raytrace Formulas (2)

- Special case spherical surface with
  curvature c = 1/R
  Coefficients H, G, F

$$H_j = -c_j$$

$$G_j = c_j\left(x_j^2 + y_j^2 + z_j^2\right) - 2z_j$$

$$F_j = \varsigma_j - c_j\left(x_j \xi_j + y_j \eta_j + z_j \varsigma_j\right)$$

  Unit vector normal to the surface

$$\vec{e}_j = \begin{pmatrix} -c_j x_j \\ -c_j y_j \\ 1 - c_j z_j \end{pmatrix}$$

- Insertion of the ray equation into surface equation:
  skew thickness

$$d_{s,j-1} = \frac{G_j}{F_j + \sqrt{F_j^2 + H_j G_j}}$$

- Angle of incidence

$$\cos i_j = \vec{s}_j \cdot \vec{e}_j$$

- Refraction

$$\cos i'_j = \sqrt{1 - \left(\frac{n_j}{n_{j+1}}\right)^2 \left(1 - \cos^2 i_j\right)}$$

  or
  reflection

$$\cos i'_j = -\cos i_j$$

Vectorial Raytrace Formulas (3)

Institute of Applied Physics
Friedrich-Schiller-Universität Jena

- Auxiliary parameter

$$\Phi_j = n_{j+1} \cos i'_j - n_j \cos i_j$$

- New ray direction vector

$$\vec{s}_{j+1} = \frac{n_j}{n_{j+1}} \vec{s}_j + \frac{\Phi_j}{n_{j+1}} \vec{e}_j$$

Conic Sections

Institute of Applied Physics
Friedrich-Schiller-Universität Jena

- Explicite surface equation, resolved to z

  Parameters: curvature c = 1 / R

  conic parameter κ

$$z = \frac{c\left(x^2+y^2\right)}{1+\sqrt{1-(1+\kappa)c^2\left(x^2+y^2\right)}}$$

- Influence of k on the surface shape

Parameter	Surface shape
κ = - 1	paraboloid
κ < - 1	hyperboloid
κ = 0	sphere
κ > 0	oblate ellipsoid (disc)
0 > κ > - 1	prolate ellipsoid (cigar)

- Relations with axis lengths a,b of conic sections

$$\kappa = \left(\frac{a}{b}\right)^2 - 1 \qquad c = \frac{b}{a^2} \qquad b = \frac{1}{\left|c(1+\kappa)\right|} \qquad a = \frac{1}{\left|c\sqrt{\left|1+\kappa\right|}\right|}$$

Aspherical Surface Types

- Conic section

  Special case spherical

$$z = \frac{c\left(x^2+y^2\right)}{1+\sqrt{1-(1+\kappa)c^2\left(x^2+y^2\right)}}$$

- Cone

$$z = \frac{\sqrt{x^2+y^2}}{\theta}$$

- Toroidal surface with radii R_x and R_y in the two section planes

$$z = R_y - \sqrt{\left(R_y - R_x + \sqrt{R_x^2 - x^2}\right)^2 - y^2}$$

- Generalized onic section without circular symmetry

$$z = \frac{c_x x^2 + c_y y^2}{1+\sqrt{1-(1+\kappa_x)c_x^2 x^2 - (1+\kappa_y)c_y^2 y^2}}$$

- Roof surface

$$z = \left| y \right| \cdot \tan\theta$$

Aspherical Surfaces with Rotational Symmetry

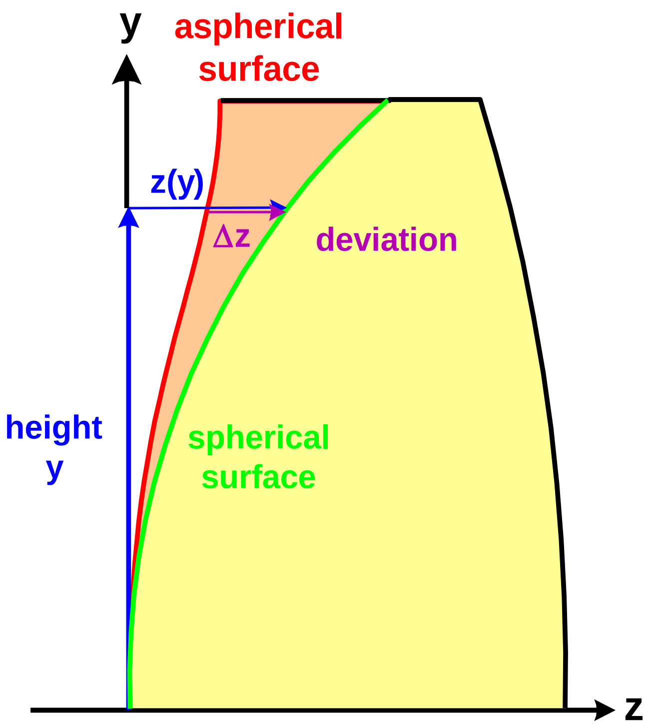
Institute of Applied Physics
Friedrich-Schiller-Universität Jena

- Classical representation of an aspheric surface with rotational symmetry:
  - basic shape of a conic section
  - correction of real sag height with Taylor expansion

$$z(y) = \frac{c \cdot y^2}{1+\sqrt{1-(1+\kappa)c^2 y^2}} + \sum_{k=1} c_k y^{2k+2}$$

- Aspherical constants c_j
  Usually only even orders, no cusp on axis

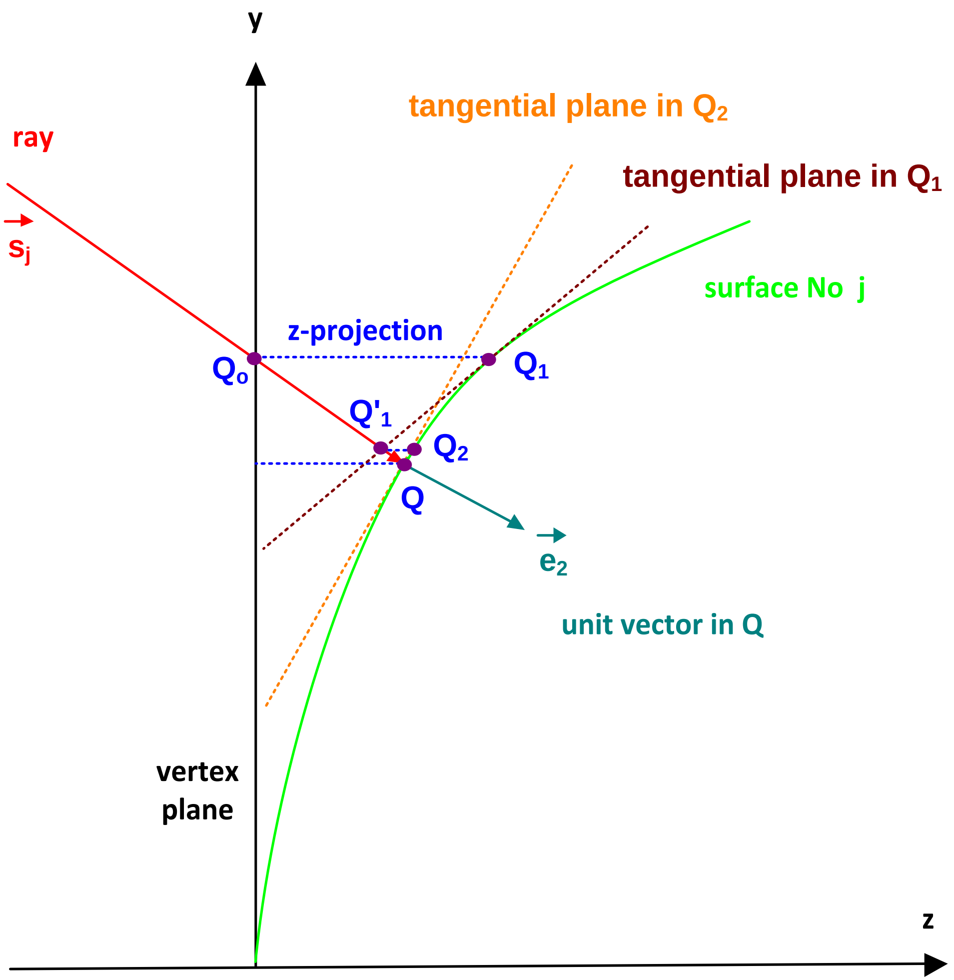
- Problems with this representation:
  - deviation Δz not perpendicular to surface
  - Taylor expansion terms not orthogonal
  - oscillations of higher order terms



Numerical Iterative Raytrace at Aspheres

- Calculation of the intersection point with an aspherical surface:
  - no fast analytical calculation possible
  - iterative numerical computation is used
  - often problems with stability for steep aspheres

- General scheme:
  - intersection point with vertex plane Q_0
  - projection onto surface, point Q_1
  - determine the tangential plane in Q1
  - intersection with tangential plane Q'_1
  - projection onto surface, point Q_2
  - …

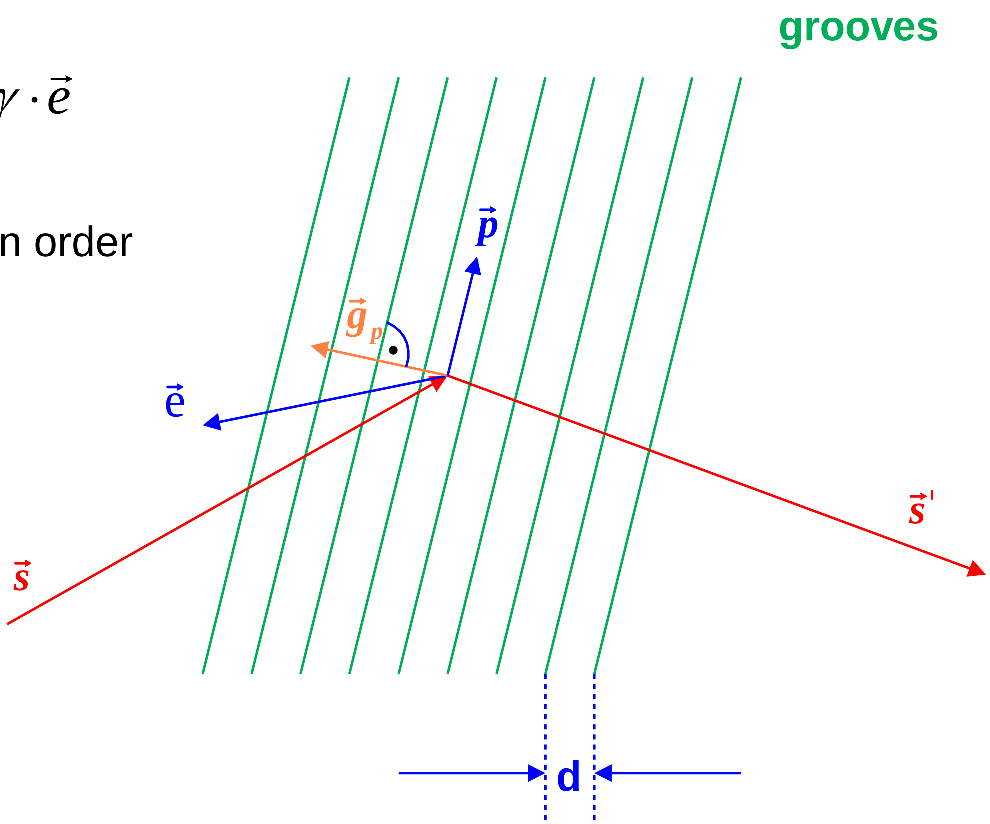


Diffracting Surfaces

- Surface with grating structure:
  new ray direction follows the grating equation
- Local approximation in the case of space-varying grating width

$$\vec{s}' = \frac{n}{n'} \cdot \vec{s} + \frac{m \lambda g}{n' d} \cdot \hat{\vec{g}} + \gamma \cdot \vec{e}$$

- Raytrace only into one desired diffraction order
- Notations:
  g : unit vector perpendicular to grooves
  d : local grating width
  m : diffraction order
  e : unit normal vector of surface
- Applications:
  - diffractive elements
  - line gratings
  - holographic components

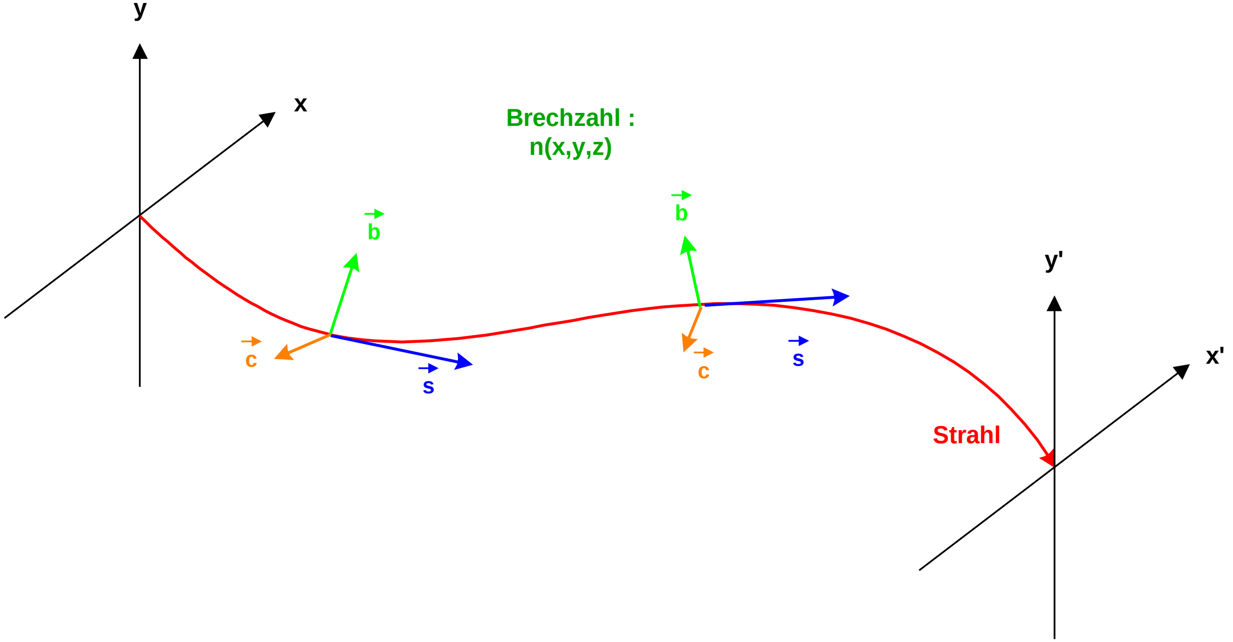


Raytracing in Grin Media

Institute of Applied Physics
Friedrich-Schiller-Universität Jena

- Ray: in general curved line
- Numerical solution of Eikonal equation
- Step-based Runge-Kutta algorithm
  4th order expansion, adaptive step width
- Large computational times necessary for high accuracy

$$\frac{d^2\vec{r}}{dt^2} = n \cdot \nabla n = \vec{D} = \begin{pmatrix} n\frac{\partial n}{\partial x} \\ n\frac{\partial n}{\partial y} \\ n\frac{\partial n}{\partial z} \end{pmatrix}$$



Description of Grin Media

- Analytical description of grin media by Taylor expansions of the function n(x,y,z)
- Separation of coordinates

$$n = n_{o,\lambda} + c_1 h + c_2 h^2 + c_3 h^4 + c_4 h^6 + c_5 h^8 + c_6 z + c_7 z^2 + c_8 z^3 + c_9 z^4$$
$$+ c_{10} x + c_{11} x^2 + c_{12} x^3 + c_{13} y + c_{14} y^2 + c_{15} y^3$$

- Circular symmetry, nested expansion with mixed terms

$$n = n_{o,\lambda} + c_1 h^2 + c_2 h^4 + c_3 h^6 + c_4 h^8 + z\left(c_5 + c_6 h^2 + c_7 h^4 + c_8 h^6 + c_9 h^8\right)$$
$$+ z^2\left(c_{10} + c_{11} h^2 + c_{12} h^4 + c_{13} h^6 + c_{14} h^8\right) + z^3\left(c_{15} + c_{16} h^2 + c_{17} h^4 + c_{18} h^6 + c_{19} h^8\right)$$

- Circular symmetry only radial

$$n = n_{o,\lambda} \sqrt{1 + c_2 (c_1 h)^2 + c_3 (c_1 h)^4 + c_4 (c_1 h)^6 + c_5 (c_1 h)^8 + c_6 (c_1 h)^{10}}$$

- Only axial gradients

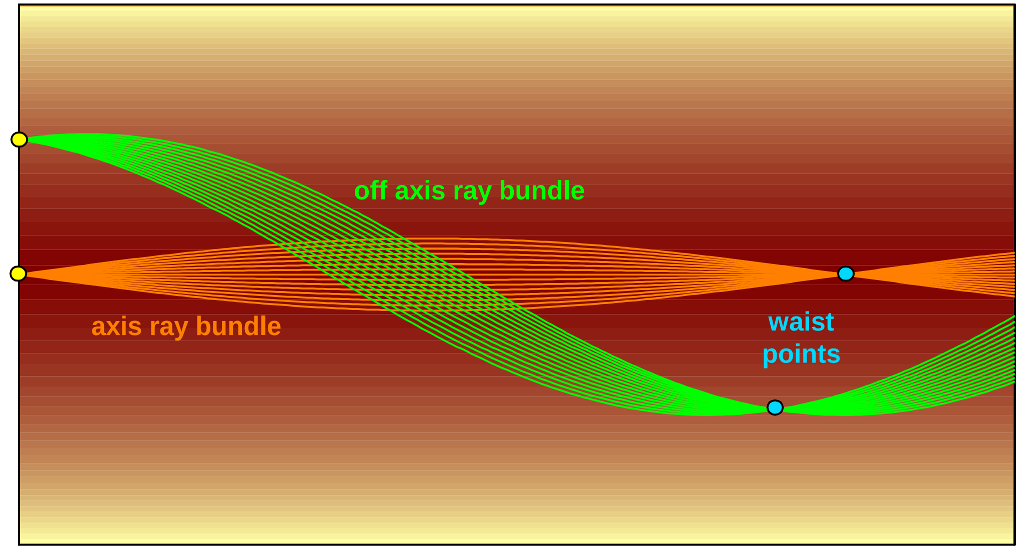
$$n = n_{o,\lambda} \sqrt{1 + c_2 (c_1 z)^2 + c_3 (c_1 z)^4 + c_4 (c_1 z)^6 + c_5 (c_1 z)^8}$$

- Circular symmetry, separated, wavelength dependent

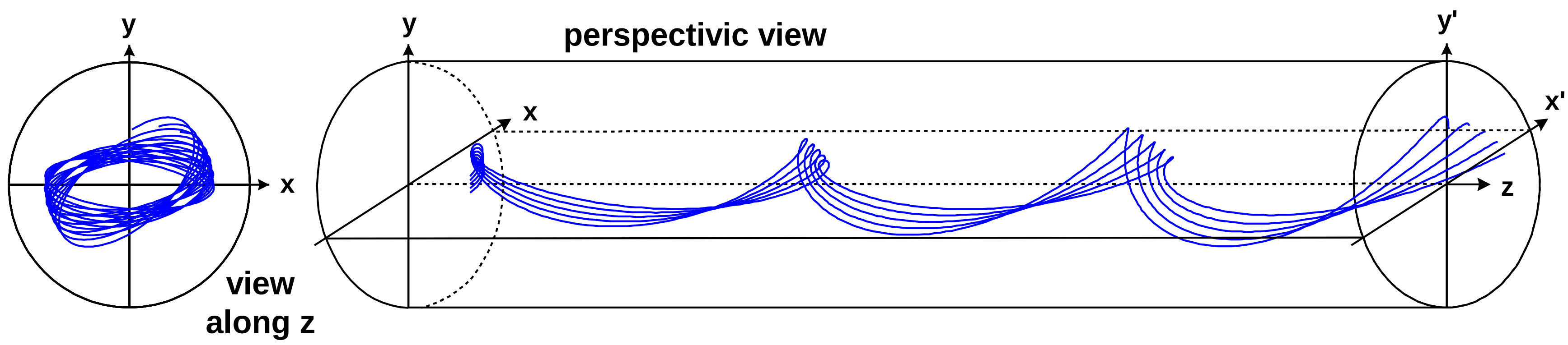
$$n = n_{o,\lambda} + c_{1,\lambda} h^2 + c_{2,\lambda} h^4 + c_{3,\lambda} h^6 + c_{4,\lambda} h^8 + c_{5,\lambda} z + c_{6,\lambda} z^2 + c_{7,\lambda} z^3$$

Gradient Lenses

- Refocusing in parabolic profile



- Helical ray path in 3 dimensions

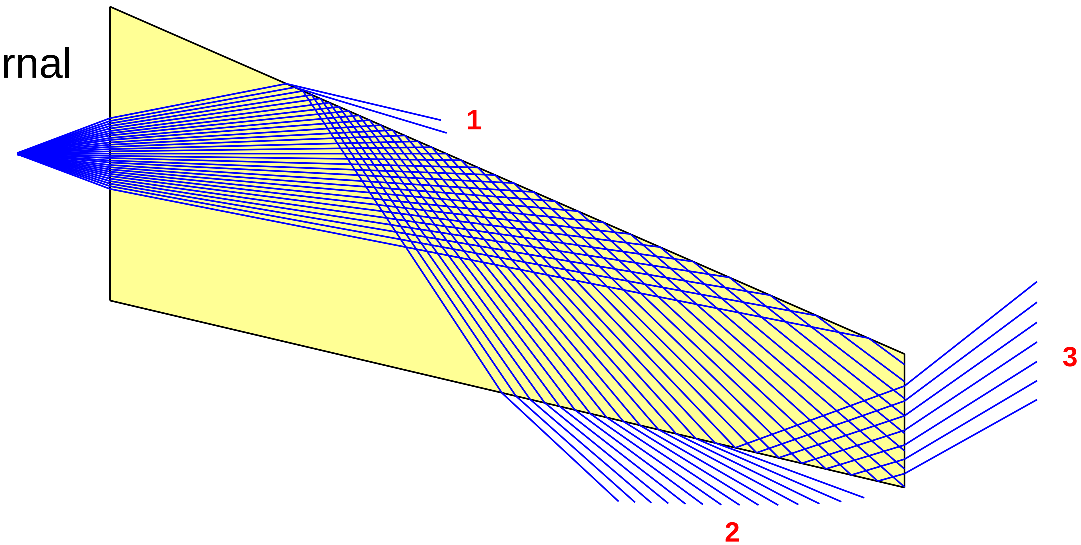


Non-Sequential Raytrace

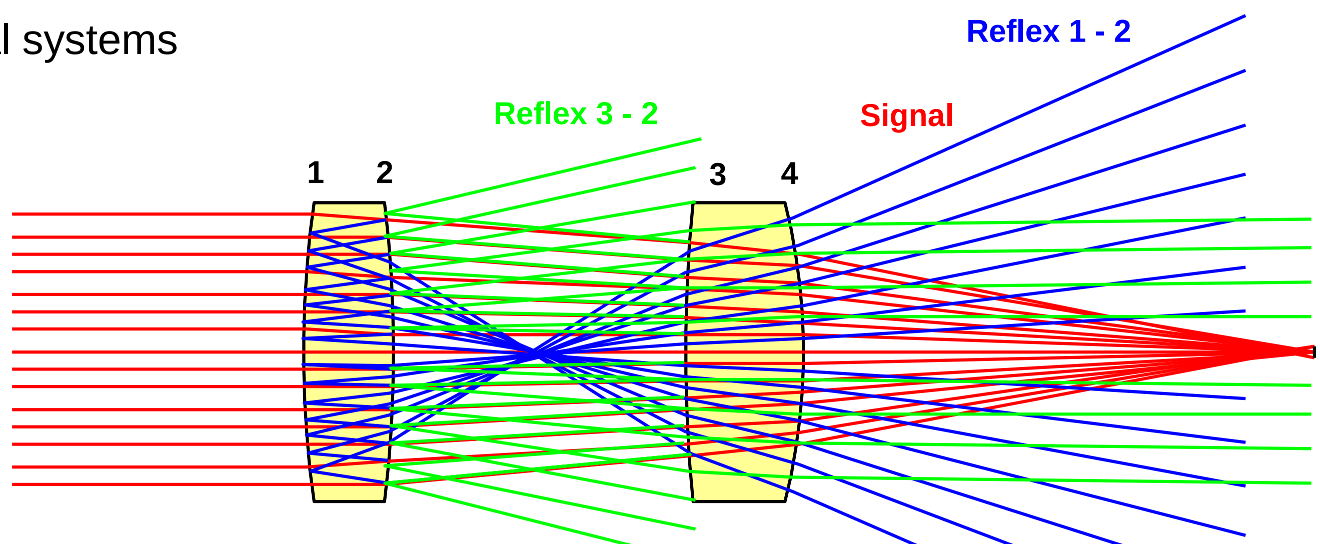
- Conventional raytrace:
  - the sequence of surface hits of a ray is pre-given and is defined by the index vector
  - simple and fast programming of the surface-loop of the raytrace
- Non-sequential raytrace:
  - the sequence of surface hits is not fixed
  - every ray gets ist individual path
  - the logic of the raytrace algorithm determines the next surface hit at run-time
  - surface with several new directions of the ray are allowed:
    1. partial reflection, especially Fresnel-formulas
    2. statistical scattering surfaces
    3. diffraction with several grating orders or ranges of deviation angles
- Many generalizations possible:
  several light sources, segmented surfaces, absorption, …
- Applications:
  1. illumination modelling
  2. statistical components (scatter plates)
  3. straylight calculation

Nonsequential Raytrace: Examples

1. Prism with total internal reflection



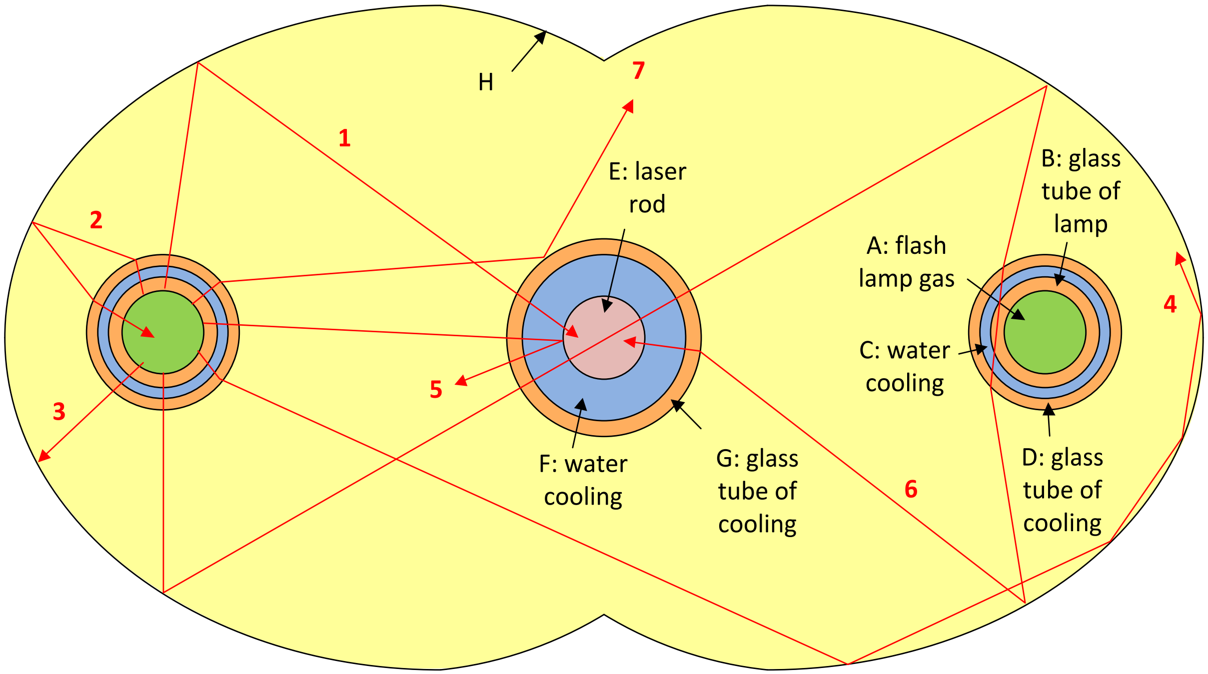
2. Ghost images in optical systems with imperfect coatings



Non-Sequential Raytrace: Examples

3. Illumination systems, here:
   - cylindrical pump-tube of a solid state laser
   - two flash lamps (A, B) with cooling flow tubes (C, D)
   - laser rod (E) with flow tube (F, G)
   - double-elliptical mirror for refocussing (H)

   Different ray paths possible



Institute of Applied Physics
Friedrich-Schiller-Universität Jena

Summary of Important Topics

- Law of refraction, scalar form, nonlinearity of the sine-function causes aberrations
- Law of refraction, vectorial form, basis of general raytrace
- Fresnel formulas, determine the reflection and refraction amplitude, phase ans polarization behavior
- Special care with Fresnel formulas: representations for energy or intensity, intensity changes due to change of beam cross section
- Special case: Brewster angle, reflected light totally polarized
- Total internal reflection: surface wave, finite field penetration into next medium
- Model of optical systems is usually based on raytrace
- Raytracing, several sets of formulas, based on a mathematical system description
- Paraxial formulas, linear, basis for reference on perfect case
- 3D formulas, raytrace of today
- Surfaces of 2nd order: analytical intersection calculation, fast
- General surfaces: iterative numerical calculation, more complicated and slow
- Pitfalls: ray doesn‘t meet surface, total internal relfection, selection of proper intersection point
- General raytrace scheeme: 1 step transition to next surface, 2 refraction and change of direction
- Non-sequential raytrace: sequence of surface ray hits can change for every ray