Overview

- Time: Wednesday, 8.15 – 9.45
- Location: HS 3, Haus 3
- Web page on IAP homepage under 'learning/materials' provides slides, exercises, solutions, informations
- Seminar: Exercises and solutions of given problems
  - time: Thursday, 16.00 -17.30
  - SR 5, Helmholtzweg 4
  - starting date: 2014-04-17
- Shift of some dates could be possible
- Written examination, 90'
<table>
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<tr>
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<th>Author(s)</th>
<th>Title</th>
<th>Publisher</th>
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<td>1.</td>
<td>Saleh / Teich</td>
<td>Fundamental of Photonics</td>
<td>Wiley</td>
<td>2007</td>
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<td>3.</td>
<td>Mahajan</td>
<td>Optical Imaging and Aberrations</td>
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<td>Part II : Wave Diffraction Optics</td>
<td>SPIE</td>
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<td>5.</td>
<td>Welford</td>
<td>Aberrations of Optical Systems</td>
<td>Hilger</td>
<td>1986</td>
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<td>7.</td>
<td>Malacara/Malacara</td>
<td>Handbook of Lens Design</td>
<td>Dekker</td>
<td>2013</td>
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<td>Kingslake/Johnson</td>
<td>Fundamentals of optical design</td>
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<td>Geary</td>
<td>Lens Design with practical Examples</td>
<td>Willmann-Bell</td>
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<td>12.</td>
<td>Goodman</td>
<td>Introduction to Fourier Optics</td>
<td>Wiley</td>
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<td>Law of refraction, Fresnel formulas, optical system model, raytrace, calculation approaches</td>
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<td>Materials and Components</td>
<td>Dispersion, anormal dispersion, glass map, liquids and plastics, lenses, mirrors, aspheres, diffractive elements</td>
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<td>Geometrical Aberrations</td>
<td>Longitudinal and transverse aberrations, spot diagram, polynomial expansion, primary aberrations, chromatical aberrations, Seidels surface contributions</td>
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<td>Fermat principle and Eikonal, wave aberrations, expansion and higher orders, Zernike polynomials, measurement of system quality</td>
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<td>PSF and Transfer function</td>
<td>Diffraction, point spread function, PSF with aberrations, optical transfer function, Fourier imaging model</td>
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<td>Further Performance Criteria</td>
<td>Rayleigh and Marechal criteria, Strehl definition, 2-point resolution, MTF-based criteria, further options</td>
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<td>Optimization and Correction</td>
<td>Principles of optimization, initial setups, constraints, sensitivity, optimization of optical systems, global approaches</td>
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<td>Correction Principles I</td>
<td>Symmetry, lens bending, lens splitting, special options for spherical aberration, astigmatism, coma and distortion, aspheres</td>
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<td>Field flattening and Petzval theorem, chromatical correction, achromate, apochromate, sensitivity analysis, diffractive elements</td>
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<td>Optical System Classification</td>
<td>Overview, photographic lenses, microscopic objectives, lithographic systems, eyepieces, scan systems, telescopes, endoscopes</td>
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<td>Special System Examples</td>
<td>Zoom systems, confocal systems</td>
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<td>14</td>
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<td>Further Topics</td>
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Contents

1. Refraction
2. Fresnel formulas
3. Optical systems
4. Raytrace
5. Calculation approaches
Law of Refraction

- Angle deviation at skew incidence
- Change of magnification at curved surfaces, lensing effect
**Law of Refraction**

- **Scalar law of refraction** (Snell's law)
  \[ n \cdot \sin i = n' \cdot \sin i' \]

- **Vectorial form**
  \[ \vec{s}' = \frac{n}{n'} \cdot \vec{s} + \left( -\frac{n}{n'} \cdot \vec{e} \cdot \vec{s} + \sqrt{1 - \left( \frac{n}{n'} \right)^2 \cdot \left[ 1 - (\vec{e} \cdot \vec{s})^2 \right]} \right) \cdot \vec{e} \]

- **Special case reflection**
  \[ \vec{s}' = \vec{s} - 2 \cdot \vec{e} \cdot (\vec{e} \cdot \vec{s}) \]

- **All vectors in the plane of incidence**

- **Fundamental basis:**
  - Principle of Fermat
  - Invariance of field components
Law of Refraction

- Simple derivation of the law of refraction:
  - constant optical path length for two rays,
  - optical path length:
    - product of index of refraction times geometrical path length
  \[ n \cdot a = n' \cdot a' \]

- Geometrical condition of triangles
  \[ a = d \cdot \sin i \]
  \[ a' = d \cdot \sin i' \]

- Insertion delivers the law of refraction
  \[ n \cdot \sin i = n' \cdot \sin i' \]
Law of Refraction

- Scalar law of refraction

- Sine is limited by -1…+1:
  1. grazing incidence at \( i = 90^\circ \)
  2. total internal reflection with \( i' = 90^\circ \)

for \( \sin i = \frac{n'}{n} \)

\[
n \cdot \sin i = n' \cdot \sin i'
\]
Fresnel Formulas

- Schematical illustration of the ray refraction (reflection at an interface)
- The cases of s- and p-polarization must be distinguished
Fresnel Formulas

- Electrical transverse polarization
  TE, s- or σ-polarization, E perpendicular to incidence plane
  \[
  E_{1t} = E_{2t}
  \]

- Magnetical transverse polarization
  TM, p- or p-polarization, E in incidence plane
  \[
  \varepsilon_1 \cdot E_{1n} = \varepsilon_2 \cdot E_{2n}
  \]

- Boundary condition of Maxwell equations at a dielectric interface:
  continuous tangential component of E-field
  \[
  r_{TE} = \left. \frac{E_r}{E_e} \right|_{TE} \\
  r_{TM} = \left. \frac{E_r}{E_e} \right|_{TM}
  \]
  \[
  t_{TE} = \left. \frac{E_t}{E_e} \right|_{TE} = r_{TE} + 1 \\
  t_{TM} = \frac{n}{n'} (r_{TM} + 1)
  \]

- Amplitude coefficients for reflected field
  transmitted field

- Reflectivity and transmission of light power
  \[
  R = \frac{P_r}{P_e} = \left| r^2 \right| \\
  T = \frac{P_t}{P_e} = \frac{n' \cdot \cos i'}{n \cdot \cos i} \cdot \left| t^2 \right|
  \]
Fresnel Formulas: Stokes Relations

- Relation between the amplitude coefficients for reflection/transmission:

1. s-components:
   - field components additive
   - minus sign due to phase jump
   
   \[ t_\perp - r_\perp = 1 \]

2. p-components:
   - energy preservation but change of area size due to projection, correction factor, no additivity of intensities
   
   \[ t_\parallel \cdot \frac{\cos \theta'}{\cos \theta} + r_\parallel = 1 \]
Fresnel Formulas

- Coefficients of amplitude for reflected rays, s and p

\[
\begin{align*}
    r_{E\perp} &= -\frac{\sin(i - i')}{{\sin}(i + i')} = \frac{n \cdot \cos i - \sqrt{n^2 - n'^2 \cdot \sin^2 i}}{n \cdot \cos i + \sqrt{n^2 - n'^2 \cdot \sin^2 i}} = \frac{n \cdot \cos i - n' \cdot \cos i'}{n \cdot \cos i + n' \cdot \cos i'} = \frac{k_{ez} - k_{tz}}{k_{ez} + k_{tz}} \\
    r_{E\parallel} &= \frac{\tan(i - i')}{{\tan}(i + i')} = \frac{n'^2 \cdot \cos i - n \cdot \sqrt{n'^2 - n^2 \cdot \sin^2 i}}{n'^2 \cdot \cos i + n \cdot \sqrt{n'^2 - n^2 \cdot \sin^2 i}} = \frac{n' \cdot \cos i - n \cdot \cos i'}{n' \cdot \cos i + n \cdot \cos i'} = \frac{n'^2 \cdot k_{ez} - n^2 \cdot k_{tz}}{n'^2 \cdot k_{ez} + n^2 \cdot k_{tz}}
\end{align*}
\]

- Coefficients of amplitude for transmitted rays, s and p

\[
\begin{align*}
    t_{E\perp} &= \frac{2n \cdot \cos i}{n \cdot \cos i + n' \cdot \cos i'} = \frac{2n \cdot \cos i}{n \cdot \cos i + \sqrt{n'^2 - n^2 \cdot \sin^2 i}} = \frac{2n \cdot \cos i}{n \cdot \cos i + n' \cdot \cos i'} = \frac{2k_{ez}}{k_{ez} + k_{tz}} \\
    t_{E\parallel} &= \frac{2n \cdot \cos i}{n' \cdot \cos i + n \cdot \cos i'} = \frac{2n \cdot \cos i}{n' \cdot \cos i + n \cdot \sqrt{n'^2 - n^2 \cdot \sin^2 i}} = \frac{2n \cdot \cos i}{n' \cdot \cos i + n \cdot \cos i'} = \frac{2n'^2 \cdot k_{ez}}{n'^2 \cdot k_{ez} + n^2 \cdot k_{tz}}
\end{align*}
\]
Fresnel Formulas

- Typical behavior of the Fresnel amplitude coefficients as a function of the incidence angle for a fixed combination of refractive indices

- $i = 0$
  Transmission independent on polarization
  Reflected $p$-rays without phase jump
  Reflected $s$-rays with phase jump of $\pi$
  (corresponds to $r<0$)

- $i = 90^\circ$
  No transmission possible
  Reflected light independent on polarization

- Brewster angle: completely $s$-polarized reflected light
Fresnel Formulas: Energy vs. Intensity

Fresnel formulas, different representations:

1. Amplitude coefficients, with sign

2. Intensity coefficients: no additivity due to area projection

3. Power coefficients: additivity due to energy preservation
Fresnel Formulas

- Reflectivity and transmittivity of power
  \[
  R_\perp = \frac{\sin^2(i-i')}{\sin^2(i+i')}, \quad R_\| = \frac{\tan^2(i-i')}{\tan^2(i+i')} \quad T_\perp = \frac{\sin 2i' \cdot \cos 2i}{\sin^2(i+i')}, \quad T_\| = \frac{\cos 2i \cdot \sin 2i'}{\sin^2(i+i') \cdot \cos^2(i-i')}
  \]

- Arbitrary azimuthal angle $\tau$ of polarization: decomposition of components
  \[
  R = R_\| \cdot \cos^2 \tau_e + R_\perp \cdot \sin^2 \tau_e \quad T = T_\| \cdot \cos^2 \tau_e + T_\perp \cdot \sin^2 \tau_e
  \]

- In case of vanishing absorption: Energy preservation
  \[
  R + T = 1
  \]

- Special case of normal incidence
  \[
  R_\perp = \left(\frac{n-n'}{n+n'}\right)^2 \quad T_\perp = \frac{4n \cdot n'}{(n+n')^2}
  \]

- Typical values for some glasses and optical materials in air

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<th>$n$</th>
<th>$R$</th>
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<tr>
<td>1.4</td>
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<tr>
<td>1.5</td>
<td>4.0 %</td>
</tr>
<tr>
<td>1.8</td>
<td>8.16 %</td>
</tr>
<tr>
<td>2.4</td>
<td>16.96 %</td>
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Transmission in Optical Systems

- Residual reflectivity of the (identical) surfaces in an optical system with $n$ surfaces:
  
  Overall transmission of energy:
  
  \[ T_{ges} = (1 - R)^n \]

- Transmission decreases nonlinear

- Practical consequences:
  1. loss of signal energy
  2. contrast reduction in case of imaging
  3. occurrence of ghost images
Brewster Angle

- Brewster case of reflection: reflected and transmitted ray are perpendicular

\[ i + i' = 90^\circ \]

- Condition of Brewster angle

\[ \tan i_B = \frac{n'}{n} \]

- The reflected light is completely s polarized

- Application: stack of plates under Brewster angle as polarizer
Total Internal Reflection in Fibers

- Total internal reflection between core and cladding in a step index fiber

Ref: M. Kaschke
- Limiting angle $i_c$ of total internal reflection: no light leaves the medium with the higher index

$$R_\perp = R_\parallel = 1$$

- Condition:

$$\sin i_c = \frac{n'}{n}$$
In case of total internal reflection, the Fresnel formulae reads:

\[
r_{\perp} = \frac{n \cos i - \sqrt{-1} \cdot \sqrt{n^2 \sin^2 i - n'^2}}{n \cos i + \sqrt{-1} \cdot \sqrt{n^2 \sin^2 i - n'^2}}
\]

\[
r_{\parallel} = \frac{n^3 \cos i - n'^2 \cdot \sqrt{-1} \cdot \sqrt{n^2 \sin^2 i - n'^2}}{n^3 \cos i + n'^2 \cdot \sqrt{-1} \cdot \sqrt{n^2 \sin^2 i - n'^2}}
\]

The wave penetrates the boundary and generates an evanescent wave which propagates along the boundary.
Evanescent Field

- Visualization of the evanescent wave
- Evanescent field: finite penetration depth
  propagation parallel to the interface plane
- Data: $n = 1.5$, $n' = 1.0$; $\theta = 45^\circ$

Ref: Peatros
• Interface surfaces
  - mathematical modelled surfaces
  - planes, spheres, aspheres, conics, free shaped surfaces,…

• Size of components
  - thickness and distances along the axis
  - transversal size, circular diameter, complicated contours

• Geometry of the setup
  - special case: rotational symmetry
  - general case: 3D, tilt angles, offsets and decentrations, needs vectorial approach

• Materials
  - refractive indices for all used wavelengths
  - other properties: absorption, birefringence, nonlinear coefficients, index gradients,…

• Special surfaces
  - gratings, diffractive elements
  - arrays, scattering surfaces
Modelling of Optical Systems

- Principal purpose of calculations:

  - System, data of the structure (radii, distances, indices, ...)
  - Function, data of properties, quality performance (spot diameter, MTF, Strehl ratio, ...)

- Imaging model with levels of refinement

  - Paraxial model (focal length, magnification, aperture, ...)
  - Analytical approximation and classification (aberrations, ...)
  - Geometrical optics (transverse aberrations, wave aberration, distortion, ...)
  - Wave optics (point spread function, OTF, ...)

Ref: W. Richter
Five levels of modelling:

1. Geometrical raytrace with analysis
2. Equivalent geometrical quantities, classification
3. Physical model: complex pupil function
4. Primary physical quantities
5. Secondary physical quantities

Blue arrows: conversion of quantities
- Ray: straight line between two intersection points
- System: sequence of spherical surfaces
- Data: - radii, curvature $c=1/r$
  - vertex distances
  - refractive indices
  - transverse diameter
- Surfaces of 2nd order:
  Calculation of intersection points
  analytically possible: fast computation
Two step process:
1. Transition to next surface
   intersection point, test of plausibility
2. Dielectric interface
   refraction and new direction
   test of plausibility
Raytrace Formulas

- Different sets of formulas:
  1. Paraxial formulas, for reference of ideal imaging
  2. Meridional formulas, for circular symmetry
  3. Vectorial formulas, 3D systems, state of the art today
  4. Differential formulas, transfer of neighbourhood, ray density, astigmatism

- Special aspects:
  1. Aspherical surfaces, numerical iterative calculation of intersection points
  2. Gradient media, eikonal differential equation, Runge-Kutta numerical stepwise
  3. Diffractive elements, local grating equation
  4. Non-sequential raytrace, illumination and straylight
  5. Scattering surfaces, Monte-Carlo decision for new direction
  6. Photometric correct raytracing, transfer of relative weighting factor
  7. Polarization raytrace, transfer of Jones vector on a ray
  8. Geometrical approximated edge diffraction, ray deviation depends on edge distance
Paraxial y-U-Method

- Paraxial ray trace formulas

- Parameters of ray description:
  1. ray height $y$
  2. ray angle $U$

- Transfer to next surface index $j \rightarrow j+1$

  height

  angle of incidence

  refraction

  new direction angle

$$y_j = y_{j-1} - d_{j-1} \cdot U_{j-1}$$

$$i_j = c_j \cdot y_j - U_{j-1}$$

$$i_j' = \frac{n_j}{n_j'} i_j$$

$$U_{j'} = U_{j-1} + i_j - i_j' = y_j c_j - i_j'$$
- Projection of ray to surface vertex point length: Q

- In the paraxial regime identical with ray intersection height $y$
Exact Meridional Q-U-Raytrace Method

- Exact raytrace scheme in the meridional plane
- Ray description parameters:
  - angle $u$ with optical axis
  - distance $Q$ between ray and surface vertex point

- Set of formulas:
  1. angle of incidence
     \[
     \sin i = Qc - \sin u \quad \cos i = \sqrt{1 - \sin^2 i}
     \]
  2. refraction
     \[
     \sin i' = \frac{n}{n'} \sin i \quad \cos i' = \sqrt{1 - \sin^2 i'}
     \]
  3. new angle of ray
     \[
     u' = u - i + i'
     \]
  4. auxiliary parameter
     \[
     G = \frac{Q}{\cos u + \cos i}
     \]
  5. distance to vertex
     \[
     Q' = \frac{\sin i' - \sin u'}{c} = Q \frac{\cos u'}{\cos u} = \frac{G}{\cos u' + \cos i'}
     \]
  6. intersection length
     \[
     L_{j+1} = L_j - d
     \]
  7. final distance
     \[
     Q_1 = L \sin u
     \]
Exact Meridional Q-U-Raytrace Method (2)

- Set of formulas (2):
  
  (8) final intersection length
  
  \[ L' = \frac{Q'}{\sin u'} \]

  (9) intersection point
  
  \[ y = \frac{\sin(u + i)}{c} = G \cdot [1 + \cos(u + i)] \]

  \[ z = \frac{1 - \cos(u + i)}{c} = G \cdot \sin(u + i) \]

(10) failure condition 1: ray don't hit surface

| \[|\sin i| > 1 \] |

(11) failure condition 2: total internal reflection

| \[|\sin i'| > 1 \] |
- General 3D geometry
- Tilt and decenter of surfaces
- General shaped free form surfaces
- Full description with 3 components
- Global and local coordinate systems
Vectorial Raytrace Formulas

- Restrictions:
  - surfaces of second order, fast analytical calculation of intersection point possible
  - homogeneous media

- Direction unit vector of the straight ray

- Vector of intersection point on a surface

- Ray equation with skew thickness \( d_{sj} \)
  index j of the surface and the space behind

- Equation of the surface 2.order

The coefficients H, F, G contains the surface shape parameters
Vectorial Raytrace Formulas (2)

- Special case spherical surface with curvature $c = \frac{1}{R}$
  Coefficients $H$, $G$, $F$

- Unit vector normal to the surface

- Insertion of the ray equation into surface equation:
  skew thickness

- Angle of incidence

- Refraction
  or
  reflection

$$H_j = -c_j$$
$$G_j = c_j \left(x_j^2 + y_j^2 + z_j^2 \right) - 2z_j$$
$$F_j = \zeta_j - c_j \left(x_j \xi_j + y_j \eta_j + z_j \zeta_j \right)$$

$$\vec{e}_j = \begin{pmatrix} -c_j x_j \\ -c_j y_j \\ 1 - c_j z_j \end{pmatrix}$$

$$d_{s,j-1} = \frac{G_j}{F_j + \sqrt{F_j^2 + H_j G_j}}$$

$$\cos i_j = \vec{s}_j \cdot \vec{e}_j$$

$$\cos i'_j = \sqrt{1 - \left(\frac{n_j}{n_{j+1}}\right)^2 \left(1 - \cos^2 i_j \right)}$$

$$\cos i'_j = -\cos i_j$$
Vectorial Raytrace Formulas (3)

- Auxiliary parameter
  \[ \Phi_j = n_{j+1} \cos i'_j - n_j \cos i_j \]

- New ray direction vector
  \[ \vec{s}_{j+1} = \frac{n_j}{n_{j+1}} \vec{s}_j + \frac{\Phi_j}{n_{j+1}} \vec{e}_j \]
Conic Sections

- Explicite surface equation, resolved to $z$
  
  Parameters: curvature $c = 1 / R$
  
  conic parameter $\kappa$

- Influence of $k$ on the surface shape

- Relations with axis lengths $a, b$ of conic sections

$$\kappa = \left( \frac{a}{b} \right)^2 - 1 \quad c = \frac{b}{a^2} \quad b = \frac{1}{c(1 + \kappa)} \quad a = \frac{1}{c \sqrt{1 + \kappa}}$$

$$z = \frac{c(x^2 + y^2)}{1 + \sqrt{1 - (1 + \kappa)c^2(x^2 + y^2)}}$$

<table>
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<th>Parameter</th>
<th>Surface shape</th>
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<tbody>
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<td>$\kappa = -1$</td>
<td>paraboloid</td>
</tr>
<tr>
<td>$\kappa &lt; -1$</td>
<td>hyperboloid</td>
</tr>
<tr>
<td>$\kappa = 0$</td>
<td>sphere</td>
</tr>
<tr>
<td>$\kappa &gt; 0$</td>
<td>oblate ellipsoid (disc)</td>
</tr>
<tr>
<td>$0 &gt; \kappa &gt; -1$</td>
<td>prolate ellipsoid (cigar)</td>
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</tbody>
</table>
Aspherical Surface Types

- Conic section
  - Special case spherical
  \[ z = \frac{c(x^2 + y^2)}{1 + \sqrt{1 - (1 + \kappa) c^2 (x^2 + y^2)}} \]

- Cone
  \[ z = \frac{\sqrt{x^2 + y^2}}{\theta} \]
  \[ z = R_y - \sqrt{(R_y - R_x + \sqrt{R_x^2 - x^2})^2 - y^2} \]

- Toroidal surface with radii \( R_x \) and \( R_y \) in the two section planes
  \[ z = \frac{c_x x^2 + c_y y^2}{1 + \sqrt{1 - (1 + \kappa_x) c_x^2 x^2 - (1 + \kappa_y) c_y^2 y^2}} \]

- Generalized onic section without circular symmetry

- Roof surface
  \[ z = |y| \cdot \tan \theta \]
Aspherical Surfaces with Rotational Symmetry

- Classical representation of an aspheric surface with rotational symmetry:
  - basic shape of a conic section
  - correction of real sag height with Taylor expansion

\[ z(y) = \frac{c \cdot y^2}{1 + \sqrt{1 - (1 + \kappa)c^2 y^2}} + \sum_{k=1}^{\infty} c_k y^{2k+2} \]

- Aspherical constants \( c_j \)
  Usually only even orders, no cusp on axis

- Problems with this representation:
  - deviation \( \Delta z \) not perpendicular to surface
  - Taylor expansion terms not orthogonal
  - oscillations of higher order terms
Numerical Iterative Raytrace at Aspheres

- Calculation of the intersection point with an aspherical surface:
  - no fast analytical calculation possible
  - iterative numerical computation is used
  - often problems with stability for steep aspheres

- General scheme:
  - intersection point with vertex plane $Q_0$
  - projection onto surface, point $Q_1$
  - determine the tangential plane in $Q_1$
  - intersection with tangential plane $Q'_1$
  - projection onto surface, point $Q_2$
  - ...

![Diagram showing the iterative raytrace process with aspherical surfaces and points of intersection.](image)
Diffracting Surfaces

- Surface with grating structure:
  new ray direction follows the grating equation

- Local approximation in the case of space-varying
  grating width

\[ \bar{s}' = \frac{n}{n'} \cdot \bar{s} + \frac{m \lambda g}{n' d} \cdot \hat{g} + \gamma \cdot \hat{e} \]

- Raytrace only into one desired diffraction order

- Notations:
  - \( g \): unit vector perpendicular to grooves
  - \( d \): local grating width
  - \( m \): diffraction order
  - \( e \): unit normal vector of surface

- Applications:
  - diffractive elements
  - line gratings
  - holographic components
Raytracing in Grin Media

- Ray: in general curved line
- Numerical solution of Eikonal equation
- Step-based Runge-Kutta algorithm
  
  4th order expansion, adaptive step width
- Large computational times necessary for high accuracy

\[
\frac{d^2 \vec{r}}{dt^2} = n \cdot \nabla n = \vec{D} = \begin{pmatrix}
    \frac{\partial n}{\partial x} \\
    \frac{\partial n}{\partial y} \\
    \frac{\partial n}{\partial z}
\end{pmatrix}
\]
Description of Grin Media

- Analytical description of grin media by Taylor expansions of the function \( n(x,y,z) \)
- Separation of coordinates
  \[
  n = n_{o,\lambda} + c_1 h + c_2 h^2 + c_3 h^4 + c_4 h^6 + c_5 h^8 + c_6 z + c_7 z^2 + c_8 z^3 + c_9 z^4
  + c_{10} x + c_{11} x^2 + c_{12} x^3 + c_{13} y + c_{14} y^2 + c_{15} y^3
  \]
- Circular symmetry, nested expansion with mixed terms
  \[
  n = n_{o,\lambda} + c_1 h^2 + c_2 h^4 + c_3 h^6 + c_4 h^8 + z \left( c_5 + c_6 h^2 + c_7 h^4 + c_8 h^6 + c_9 h^8 \right) + z^2 \left( c_{10} + c_{11} h^2 + c_{12} h^4 + c_{13} h^6 + c_{14} h^8 \right) + z^3 \left( c_{15} + c_{16} h^2 + c_{17} h^4 + c_{18} h^6 + c_{19} h^8 \right)
  \]
- Circular symmetry only radial
  \[
  n = n_{o,\lambda} \sqrt{1 + c_2 (c_1 h)^2 + c_3 (c_1 h)^4 + c_4 (c_1 h)^6 + c_5 (c_1 h)^8 + c_6 (c_1 h)^{10}}
  \]
- Only axial gradients
  \[
  n = n_{o,\lambda} \sqrt{1 + c_2 (c_1 z)^2 + c_3 (c_1 z)^4 + c_4 (c_1 z)^6 + c_5 (c_1 z)^8}
  \]
- Circular symmetry, separated, wavelength dependent
  \[
  n = n_{o,\lambda} + c_{1,\lambda} h^2 + c_{2,\lambda} h^4 + c_{3,\lambda} h^6 + c_{4,\lambda} h^8 + c_{5,\lambda} z + c_{6,\lambda} z^2 + c_{7,\lambda} z^3
  \]
Gradient Lenses

- Refocusing in parabolic profile

- Helical ray path in 3 dimensions
Non-Sequential Raytrace

- Conventional raytrace:
  - the sequence of surface hits of a ray is pre-given and is defined by the index vector
  - simple and fast programming of the surface-loop of the raytrace

- Non-sequential raytrace:
  - the sequence of surface hits is not fixed
  - every ray gets its individual path
  - the logic of the raytrace algorithm determines the next surface hit at run-time
  - surface with several new directions of the ray are allowed:
    1. partial reflection, especially Fresnel-formulas
    2. statistical scattering surfaces
    3. diffraction with several grating orders or ranges of deviation angles

- Many generalizations possible:
  - several light sources, segmented surfaces, absorption, …

- Applications:
  1. illumination modelling
  2. statistical components (scatter plates)
  3. straylight calculation
Nonsequential Raytrace: Examples

1. Prism with total internal reflection

2. Ghost images in optical systems with imperfect coatings
3. Illumination systems, here:
   - cylindrical pump-tube of a solid state laser
   - two flash lamps (A, B) with cooling flow tubes (C, D)
   - laser rod (E) with flow tube (F, G)
   - double-elliptical mirror for refocussing (H)

Different ray paths possible
Summary of Important Topics

- Law of refraction, scalar form, nonlinearity of the sine-function causes aberrations
- Law of refraction, vectorial form, basis of general raytrace
- Fresnel formulas, determine the reflection and refraction amplitude, phase and polarization behavior
- Special care with Fresnel formulas: representations for energy or intensity, intensity changes due to change of beam cross section
- Special case: Brewster angle, reflected light totally polarized
- Total internal reflection: surface wave, finite field penetration into next medium
- Model of optical systems is usually based on raytrace
- Raytracing, several sets of formulas, based on a mathematical system description
- Paraxial formulas, linear, basis for reference on perfect case
- 3D formulas, raytrace of today
- Surfaces of 2nd order: analytical intersection calculation, fast
- General surfaces: iterative numerical calculation, more complicated and slow
- Pitfalls: ray doesn’t meet surface, total internal reflection, selection of proper intersection point
- General raytrace scheme: 1 step transition to next surface, 2 refraction and change of direction
- Non-sequential raytrace: sequence of surface ray hits can change for every ray