Design and Correction of Optical Systems

Lecture 2: Materials and Components
2013-04-17
Herbert Gross
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<tr>
<th>Week</th>
<th>Date</th>
<th>Topic</th>
<th>Details</th>
</tr>
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<tr>
<td>1</td>
<td>10.04</td>
<td>Basics</td>
<td>Law of refraction, Fresnel formulas, optical system model, raytrace, calculation approaches</td>
</tr>
<tr>
<td>2</td>
<td>17.04</td>
<td>Materials and Components</td>
<td>Dispersion, anormal dispersion, glass map, liquids and plastics, lenses, mirrors, aspheres, diffractive elements</td>
</tr>
<tr>
<td>3</td>
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<td>Paraxial approximation, basic notations, imaging equation, multi-component systems, matrix calculation, Lagrange invariant, phase space visualization</td>
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<td>Longitudinal and transverse aberrations, spot diagram, polynomial expansion, primary aberrations, chromatical aberrations, Seidels surface contributions</td>
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<td>6</td>
<td>22.05</td>
<td>Wave Aberrations</td>
<td>Fermat principle and Eikonal, wave aberrations, expansion and higher orders, Zernike polynomials, measurement of system quality</td>
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<tr>
<td>7</td>
<td>29.05</td>
<td>PSF and Transfer function</td>
<td>Diffraction, point spread function, PSF with aberrations, optical transfer function, Fourier imaging model</td>
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<tr>
<td>8</td>
<td>05.06</td>
<td>Further Performance Criteria</td>
<td>Rayleigh and Marechal criteria, Strehl definition, 2-point resolution, MTF-based criteria, further options</td>
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<tr>
<td>9</td>
<td>12.06</td>
<td>Optimization and Correction</td>
<td>Principles of optimization, initial setups, constraints, sensitivity, optimization of optical systems, global approaches</td>
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<tr>
<td>10</td>
<td>19.06</td>
<td>Correction Principles I</td>
<td>Symmetry, lens bending, lens splitting, special options for spherical aberration, astigmatism, coma and distortion, aspheres</td>
</tr>
<tr>
<td>11</td>
<td>26.06</td>
<td>Correction Principles II</td>
<td>Field flattening and Petzval theorem, chromatical correction, achromate, apochromate, sensitivity analysis, diffractive elements</td>
</tr>
<tr>
<td>12</td>
<td>03.07</td>
<td>Optical System Classification</td>
<td>Overview, photographic lenses, microscopic objectives, lithographic systems, eyepieces, scan systems, telescopes, endoscopes</td>
</tr>
<tr>
<td>13</td>
<td>10.07</td>
<td>Special System Examples</td>
<td>Zoom systems, confocal systems</td>
</tr>
</tbody>
</table>
Contents

1. Refraction
2. Fresnel formulas
3. Optical systems
4. Raytrace
5. Calculation approaches
## Important Test Wavelengths

<table>
<thead>
<tr>
<th>$\lambda$ in [nm]</th>
<th>Name</th>
<th>Color</th>
<th>Element</th>
</tr>
</thead>
<tbody>
<tr>
<td>248.3</td>
<td>UV</td>
<td>Hg</td>
<td></td>
</tr>
<tr>
<td>280.4</td>
<td>UV</td>
<td>Hg</td>
<td></td>
</tr>
<tr>
<td>296.7278</td>
<td>UV</td>
<td>Hg</td>
<td></td>
</tr>
<tr>
<td>312.5663</td>
<td>UV</td>
<td>Hg</td>
<td></td>
</tr>
<tr>
<td>334.1478</td>
<td>UV</td>
<td>Hg</td>
<td></td>
</tr>
<tr>
<td>365.0146</td>
<td>i</td>
<td>UV</td>
<td>Hg</td>
</tr>
<tr>
<td>404.6561</td>
<td>h</td>
<td>violett</td>
<td>Hg</td>
</tr>
<tr>
<td>435.8343</td>
<td>g</td>
<td>blau</td>
<td>Hg</td>
</tr>
<tr>
<td>479.9914</td>
<td>F'</td>
<td>blau</td>
<td>Cd</td>
</tr>
<tr>
<td>486.1327</td>
<td>F</td>
<td>blau</td>
<td>H</td>
</tr>
<tr>
<td>546.0740</td>
<td>e</td>
<td>grün</td>
<td>Hg</td>
</tr>
<tr>
<td>587.5618</td>
<td>d</td>
<td>gelb</td>
<td>He</td>
</tr>
<tr>
<td>589.2938</td>
<td>D</td>
<td>gelb</td>
<td>Na</td>
</tr>
<tr>
<td>632.8</td>
<td></td>
<td></td>
<td>HeNe-Laser</td>
</tr>
<tr>
<td>643.8469</td>
<td>C'</td>
<td>rot</td>
<td>Cd</td>
</tr>
<tr>
<td>656.2725</td>
<td>C</td>
<td>rot</td>
<td>H</td>
</tr>
<tr>
<td>706.5188</td>
<td>r</td>
<td>rot</td>
<td>He</td>
</tr>
<tr>
<td>852.11</td>
<td>s</td>
<td>IR</td>
<td>Cä</td>
</tr>
<tr>
<td>1013.98</td>
<td>t</td>
<td>IR</td>
<td>Hg</td>
</tr>
<tr>
<td>1060.0</td>
<td></td>
<td></td>
<td>Nd:YAG-Laser</td>
</tr>
</tbody>
</table>
Chromatical performance evaluation of optical systems:
Usage of one main (central) wavelength and two secondary wavelengths

<table>
<thead>
<tr>
<th>Main wavelength</th>
<th>1st secondary wavelength</th>
<th>2nd secondary wavelength</th>
</tr>
</thead>
<tbody>
<tr>
<td>e</td>
<td>546.07</td>
<td>F'</td>
</tr>
<tr>
<td>green</td>
<td>480.0</td>
<td>bue</td>
</tr>
<tr>
<td>green</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>587.56</td>
<td>F</td>
</tr>
<tr>
<td>yellow</td>
<td>486.1</td>
<td>blue</td>
</tr>
<tr>
<td>yellow</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Additional definition of wavelengths at the boundaries of the used spectral range, e.g.
- one further wavelength near to the UV edge (g, i)
- one further wavelength near to the IR-edge (s,t)
atomic model for the refractive index: oscillator approach of atomic field interaction

sellmeier dispersion formula: corresponding function

special case of coupled resonances: example quartz, degenerated oscillators
Dispersion

- Dispersion: Refractive index changes with wavelength

- Normal dispersion: larger index $n$ for shorter wavelengths, Ray bending of blue rays stronger than red

\[
\frac{dn}{d\lambda} < 0
\]

- Notice: Diffraction dispersion is anomalous with $dn/d\lambda > 0$
  The different sign allows for chromatic correction in diffractive elements.

\[
\Delta n = n(\lambda_1) - n(\lambda_2)
\]
Dispersion formulas

- Schott formula
  empirical

- Sellmeier
  Based on oscillator model

- Bausch-Lomb
  empirical

- Herzberger
  Based on oscillator model

- Hartmann
  Based on oscillator model

\[ n = \sqrt{a_o + a_1 \lambda^2 + a_2 \lambda^{-2} + a_3 \lambda^{-4} + a_4 \lambda^{-6} + a_5 \lambda^{-8}} \]

\[ n(\lambda) = \sqrt{A + B \frac{\lambda^2}{\lambda^2 - \lambda_1^2} + C \frac{\lambda^2}{\lambda^2 - \lambda_2^2}} \]

\[ n(\lambda) = \sqrt{A + B \lambda^2 + C \lambda^4 + \frac{D}{\lambda^2} + \frac{E \lambda^2}{(\lambda^2 - \lambda_o^2) + \frac{F \lambda^2}{\lambda^2 - \lambda_o^2}}} \]

\[ n(\lambda) = a_o + a_1 \lambda^2 + \frac{a_2}{\lambda^2 - \lambda_o^2} + \frac{a_3}{\left(\lambda^2 - \lambda_o^2\right)^2} \]

*mit* \( \lambda_o = 0.168 \, \mu m \)

\[ n(\lambda) = a_o + \frac{a_1}{a_3 - \lambda} + \frac{a_4}{a_5 - \lambda} \]
Dispersion and Abbe number

- **Description of dispersion:**
  
  Abbe number 
  \[ \nu(\lambda) = \frac{n(\lambda) - 1}{n_F - n_C} \]

- **Visual range of wavelengths:** typically d,F,C or e,F’,C’ used
  
  \[ \nu_e = \frac{n_e - 1}{n_F - n_C} \]

- **Typical range of glasses**
  \[ \nu_e = 20 ... 100 \]

- **Two fundamental types of glass:**
  Crown glasses:
  - small, \( n \), large, dispersion low
  Flint glasses:
  - large, \( n \), small, dispersion high

---

![Graph showing refractive index for different glasses](image-url)
Material with different dispersion values:
- Different slope and curvature of the dispersion curve
- Stronger change of index over wavelength for large dispersion
- Inversion of index sequence at the boundaries of the spectrum possible
Abbe Number and Achromatization

- Curvatures $c_j$ of the radii of a lens

- Focal power at the center wavelength $e$ for a thin lens

- Difference in focal powers for outer wavelengths $F'$, $C'$

  with the Abbe number

- Focal length at the center wavelength

- Difference of the focal lengths for outer wavelengths

- Achromatization condition for two thin lenses close together

\[
c_1 = \frac{1}{r_1}, \quad c_2 = \frac{1}{r_2}
\]

\[
F_e = (n_e - 1)(c_1 - c_2) = (n_e - 1) \cdot \Delta c
\]

\[
\Delta F = F_F' - F_C' = (n_{F'} - n_{C'}) \cdot \Delta c = \frac{n_{F'} - n_{C'}}{n_e - 1} \cdot (n_e - 1) \Delta c = \frac{F_e}{v_e}
\]

\[
v_e = \frac{n_e - 1}{n_{F'} - n_{C'}}
\]

\[
f_e = \frac{1}{F_e} = \frac{1}{(n_e - 1) \Delta c}
\]

\[
\Delta f = f_{F'} - f_{C'} = \frac{n_{C'} - n_{F'}}{(n_{F'} - 1)(n_{C'} - 1) \Delta c} \approx \frac{n_{C'} - n_{F'}}{(n_e - 1)^2 \Delta c} = -\frac{f_e}{v_e}
\]

\[
\Delta F = \frac{F_1}{v_1} + \frac{F_2}{v_2} = \frac{1}{f_1 v_1} + \frac{1}{f_2 v_2} = 0
\]
Glass Diagram

- Usual representation of glasses: diagram of refractive index vs dispersion $n(\nu)$
- Left to right: Increasing dispersion decreasing Abbe number
Relative Partial Dispersion

- Relative partial dispersion: Change of dispersion slope with $\lambda$. Different curvature of dispersion curve.

- Definition of local slope for selected wavelengths relative to secondary colors

$$P_{\lambda_1\lambda_2} = \frac{n(\lambda_1) - n(\lambda_2)}{n_F - n_C}$$

- Special $\lambda$-selections for characteristic ranges of the visible spectrum

$\lambda = 656 / 1014$ nm far IR
$\lambda = 656 / 852$ nm near IR
$\lambda = 486 / 546$ nm blue edge of VIS
$\lambda = 435 / 486$ nm near UV
$\lambda = 365 / 435$ nm far UV
Partial Dispersion and Normal Line

- The relative partial dispersion changes approximately linear with the dispersion for glasses

\[ P_{\lambda_1,\lambda_2} = a_{\lambda_1,\lambda_2} \cdot \nu_d + b_{\lambda_1,\lambda_2} \]

- Nearly all glasses are located on the normal line in a P-\(\nu\)-diagram

- The slope of the normal line depends on the selection of wavelengths

- Glasses apart from the normal line shows anomalous partial dispersion \(\Delta P\)

\[ P_{\lambda_1,\lambda_2} = a_{\lambda_1,\lambda_2} \cdot \nu_d + b_{\lambda_1,\lambda_2} + \Delta P_{\lambda_1,\lambda_2} \]

these material are important for chromatical correction of higher order
- Long crown and short flint as special realizations of large $P$
Anomalous Partial Dispersion

- There are some special glasses with a large deviation from the normal line.
- Of special interest: long crowns and short flints.

Diagram showing the dispersion with points representing different glasses, and labels for long crowns, short flints, and heavy flints with character of long crowns.
<table>
<thead>
<tr>
<th>Material</th>
<th>Index at 546 nm</th>
<th>Abbe number</th>
<th>Max Temp</th>
<th>Therm expan $10^{-6}$ K$^{-1}$</th>
<th>Scatt er in %</th>
<th>Trans. 3mm,</th>
<th>Density g/cm$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>PMMA - Polymethyl-Methacrylat</td>
<td>1.49280</td>
<td>57</td>
<td>90</td>
<td>65</td>
<td>2</td>
<td>92</td>
<td>1.19</td>
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<tr>
<td>PC - Polycarbonat - Makrolon, Lexan</td>
<td>1.59037</td>
<td>30</td>
<td>120</td>
<td>69</td>
<td>4</td>
<td>87</td>
<td>1.20</td>
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<tr>
<td>CR39 - DEGBAC - Gießharz</td>
<td>1.5011</td>
<td>57.8</td>
<td>100</td>
<td>120</td>
<td>1</td>
<td></td>
<td>1.32</td>
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<tr>
<td>PS - Polystyrol</td>
<td>1.590</td>
<td>30.8</td>
<td>80</td>
<td>70</td>
<td>3</td>
<td>89</td>
<td>1.06</td>
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<tr>
<td>DPSC - Diphenyl-sulfidcarbonat</td>
<td>1.612</td>
<td>26.0</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>CMMA</td>
<td>1.50</td>
<td>56.0</td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Styrol, SAN</td>
<td>1.566</td>
<td>34.7</td>
<td>95</td>
<td>65</td>
<td>4</td>
<td>90</td>
<td>1.09</td>
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<td>SMA</td>
<td>1.585</td>
<td>31.3</td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<td>SMMA</td>
<td>1.568</td>
<td>33.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>FMS</td>
<td>1.508</td>
<td>34.0</td>
<td></td>
<td></td>
<td></td>
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<td></td>
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<td>SCMA</td>
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<td>42.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
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<td>COC</td>
<td>1.533</td>
<td>56.0</td>
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<td>MR8</td>
<td>1.60</td>
<td>42.0</td>
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<td></td>
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</tr>
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</table>
1. Stress induced birefringence during processing
2. Generation of local inhomogenieties of the refractive index in die casting
3. Water intake (swelling) : change of shape (up to 4%) and decrease in the refractive index
4. Electro-static charge
5. Aging due to cold forming, polymerization, opalescence, yellowing
6. Strong thermal variation of the refractive index
7. Limiting temperature (above the transition temperature the material is destroyed)
   100 … 120 °C
8. For an increased abrasive hardness and for the prevention from charging and swelling ,special coatings may have to be applied.
9. During the cooling process significant changes occur in the volume caused by shrinking.
   There are two different types of plastics
   a. thermosets, shrinking 0.4%...0.7%
   b. thermoplasts, shrinking 4%...14%
Plastic Materials

Plastics in the $n - \nu$ diagram

- **PMMA**
- **CR39**
- **oPC**
- **CMMA**
- **COC**
- **SCMA**
- **FMS**
- **SMMA**
- **SMA**
- **DPSC**
- **MR8**

**Anorganic glasses**
Usage of Plastics in Optical Systems

- Most attractive use of plastics: Consumer optics
  - benefit of light weight
  - critical cost
  - high number of pieces

- Advantages for special components due to manufacturing technique:
  - complex surface shapes, arrays, aspheres
  - for injection moulding cost of complex shape only for master piece

- Typical products with plastics components:
  - Eye glasses
  - binoculars
  - photographic lenses
  - pic-up objective lenses
  - illumination systems
Plastics vs Glass Materials

Comparison plastics with glasses

<table>
<thead>
<tr>
<th>property</th>
<th>unit</th>
<th>range plastics</th>
<th>range glass</th>
</tr>
</thead>
<tbody>
<tr>
<td>refractive index n</td>
<td></td>
<td>1.49...1.61</td>
<td>1.44...1.95</td>
</tr>
<tr>
<td>dispersion ν</td>
<td></td>
<td>25...57</td>
<td>20...90</td>
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<tr>
<td>uniformity of the refractive index</td>
<td></td>
<td>10^{-3}...10^{-4}</td>
<td>10^{-4}...10^{-6}</td>
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<tr>
<td>temperature dependence of the refractive index</td>
<td>10^6*K^{-1}</td>
<td>-100...-160</td>
<td>-10...+10</td>
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<tr>
<td>Vickers hardness</td>
<td>N/mm^2</td>
<td>120...190</td>
<td>3000...7000</td>
</tr>
<tr>
<td>thermal expansion</td>
<td>10^6*grd^{-1}</td>
<td>70...100</td>
<td>5...10</td>
</tr>
<tr>
<td>thermal conductivity</td>
<td>Wm^{-1}grd^{-1}</td>
<td>0.15...0.23</td>
<td>0.5...1.4</td>
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<td>internal transmission in the green range</td>
<td></td>
<td>0.97...0.993</td>
<td>0.999</td>
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<td>stress - optical coefficient</td>
<td>10^{-12}Pa^{-1}</td>
<td>40</td>
<td>3</td>
</tr>
<tr>
<td>stress- birefringence</td>
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<td>5*10^{-5}...10^{-3}</td>
<td>0</td>
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<tr>
<td>density</td>
<td>g/cm^3</td>
<td>1.05...1.32</td>
<td>2.3...6.2</td>
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<tr>
<td>water intake</td>
<td>%</td>
<td>0.1...0.8</td>
<td>0</td>
</tr>
<tr>
<td>material</td>
<td>refractive index</td>
<td>(\lambda)-range ((\mu)m)</td>
<td>UV</td>
</tr>
<tr>
<td>-------------------</td>
<td>-----------------</td>
<td>-------------------------------</td>
<td>----</td>
</tr>
<tr>
<td>MgF(_2)</td>
<td>1.389</td>
<td>0.12 - 9.0</td>
<td>●</td>
</tr>
<tr>
<td>ZnS</td>
<td>2.25</td>
<td>0.4 - 14.5</td>
<td>●</td>
</tr>
<tr>
<td>CaF(_2), calcium fluoride</td>
<td>1.42</td>
<td>0.12 - 11.5</td>
<td>●</td>
</tr>
<tr>
<td>ZnSe</td>
<td>2.44</td>
<td>0.5 - 22.0</td>
<td>●</td>
</tr>
<tr>
<td>MgO</td>
<td>1.69...1.737</td>
<td>0.28 - 9.5</td>
<td>●</td>
</tr>
<tr>
<td>CdTe</td>
<td>2.70</td>
<td>0.9 - 31.0</td>
<td>●</td>
</tr>
<tr>
<td>diamond</td>
<td>2.3757</td>
<td>0.25...3.7, 6.0...</td>
<td>●</td>
</tr>
<tr>
<td>germanium</td>
<td>4.003</td>
<td>2.0...15</td>
<td>●</td>
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<td>silicon</td>
<td>3.433</td>
<td>1.2...15</td>
<td>●</td>
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<tr>
<td>BaF(_2)</td>
<td>1.474</td>
<td>0.18...12</td>
<td>●</td>
</tr>
<tr>
<td>SiO(_2), quartz</td>
<td>1.544</td>
<td>0.15...4.0</td>
<td>●</td>
</tr>
<tr>
<td>Al(_2)O(_3), sapphire</td>
<td>1.769</td>
<td>0.17...5.5</td>
<td>●</td>
</tr>
</tbody>
</table>
Different shapes of singlet lenses:
1. bi-, symmetric
2. plane convex / concave, one surface plane
3. Meniscus, both surface radii with the same sign

Convex: bending outside
Concave: hollow surface

Principal planes $P, P'$: outside for mesicus shaped lenses
Cardinal Elements of a Lens

- **Focal points:**
  1. incoming ray parallel to the axis intersects the axis in F’
  2. ray through F is leaves the lens parallel to the axis

The focal lengths are referenced on the principal planes.

- **Nodal points:**
  Ray through N goes through N’ and preserves the direction.
Cardinal Elements of a Lens

- Principal plane P:
  incoming ray hits intersection point with P is transferred with the same height h to P'

- Special case of incident ray parallel to the axis:
  principal plane P':
  location of apparent ray bending
Main properties of a lens

- Main notations and properties of a lens:
  - radii of curvature $r_1, r_2$
  - curvatures $c$
  - sign: $r > 0$ : center of curvature is located on the right side
  - thickness $d$ along the axis
  - diameter $D$
  - index of refraction of lens material $n$

- Focal length (paraxial)

\[ f = \frac{y_F}{\tan u}, \quad f' = \frac{y}{\tan u'} \]

- Optical power

\[ F = -\frac{n}{f} = \frac{n'}{f'} \]

- Back focal length
  intersection length, measured from the vertex point

\[ s_{F'} = f' + s_p \]
Notations of a lens

P  principal point
S  vertex of the surface
F  focal point
s  intersection point of a ray with axis
f  focal length PF
r  radius of surface curvature
d  thickness SS'
n  refractive index
Bending of a Lens

- **Bending**: change of shape for invariant focal length

- **Parameter of bending**

\[
X = \frac{R_1 + R_2}{R_2 - R_1}
\]
Ray path at a lens of constant focal length and different bending

Quantitative parameter of description $X$:

- The ray angle inside the lens changes
- The ray incidence angles at the surfaces changes strongly
- The principal planes move
  For invariant location of $P$, $P'$ the position of the lens moves

$$X = \frac{R_1 + R_2}{R_2 - R_1}$$
- Changes of the incidence angles at the front and the rear surface of a bended lens

- Figure without sign of incidence angle

- Angle at the second surface depends on the refractive index
Magnification parameter $M$: defines ray path through the lens

$$M = \frac{U' + U}{U' - U} = \frac{1 + m}{1 - m} = \frac{2f}{s} + 1 = \frac{2f}{s'} - 1$$

Special cases:
1. $M = 0$: symmetrical 4f-imaging setup
2. $M = -1$: object in front focal plane
3. $M = +1$: object in infinity

The parameter $M$ strongly influences the aberrations.
Aspheres - Geometry

- Reference: deviation from sphere
- Deviation $\Delta z$ along axis
- Better conditions: normal deviation $\Delta r_s$

![Diagram of aspherical geometry](image)
Conic Sections

- Explicite surface equation, resolved to $z$
  Parameters: curvature $c = 1 / R$
  conic parameter $\kappa$
- Influence of $\kappa$ on the surface shape

\[ z = \frac{c(x^2 + y^2)}{1 + \sqrt{1 - (1 + \kappa)c^2(x^2 + y^2)}} \]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Surface shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa = -1$</td>
<td>paraboloid</td>
</tr>
<tr>
<td>$\kappa &lt; -1$</td>
<td>hyperboloid</td>
</tr>
<tr>
<td>$\kappa = 0$</td>
<td>sphere</td>
</tr>
<tr>
<td>$\kappa &gt; 0$</td>
<td>oblate ellipsoid (disc)</td>
</tr>
<tr>
<td>$0 &gt; \kappa &gt; -1$</td>
<td>prolate ellipsoid (cigar)</td>
</tr>
</tbody>
</table>

- Relations with axis lengths $a, b$ of conic sections

\[ \kappa = \left(\frac{a}{b}\right)^2 - 1 \quad c = \frac{b}{a^2} \quad b = \frac{1}{c(1 + \kappa)} \quad a = \frac{1}{c \sqrt{1 + \kappa}} \]
Simple Asphere – Parabolic Mirror

- Equation
  \[ z = \frac{y^2}{2R_s} \]

- Radius of curvature in vertex: \( R_s \)
- Perfect imaging on axis for object at infinity
- Strong coma aberration for finite field angles

Applications:
1. Astronomical telescopes
2. Collector in illumination systems
Simple Asphere – Elliptical Mirror

- Equation

\[ z = \frac{cy^2}{1 + \sqrt{1 - (1 + \kappa)y^2c^2}} \]

- Radius of curvature \( r \) in vertex, curvature \( c \)
- Eccentricity \( \kappa \)
- Two different shapes: oblate / prolate
- Perfect imaging on axis for finite object and image location
- Different magnifications depending on used part of the mirror
- Applications:
  Illumination systems
- Conic surface as basic shape

- Additional correction of the sag by a Taylor expansion
  Only even powers: no kink at r=0

- Mostly rotational symmetric shape considered

- Problems with this representation:
  1. added contributions not orthogonal, 
     bad performance during optimization
  2. non-normalized representation, coefficients depend on absolute size of 
     the diameter (very small high order coefficients for large diameters)
  3. Oscillatory bahavior, large residual slope error can occur
  4. in optics slope and not sag is relevant
  5. the coefficients can not be measured/are hard to control, 
     tolerancing is critical and complicated
  6. the added sag is along z, more important is a correction perpendicular 
     to the surface for strong aspheres

\[
z(x, y) = \frac{c(x^2 + y^2)}{1 + \sqrt{1 - (1 + \kappa) \cdot c^2 (x^2 + y^2)}} + \sum_{k=2}^{k_{\text{max}}} a_k \cdot (x^2 + y^2)^k
\]

\[
z(r) = \frac{c \cdot r^2}{1 + \sqrt{1 - (1 + \kappa) \cdot c^2 r^2}} + \sum_{k=2}^{k_{\text{max}}} a_k \cdot r^{2k}
\]
Aspheres: Correction of Higher Order

- Correction at discrete sampling
- Large deviations between sampling points
- Larger oscillations for higher orders
- Better description: slope, defines ray bending

\[ y' = c \frac{dz_A}{dy} \]

Corrected points with \( y' = 0 \)

Real asphere with oscillations

Corrected points residual angle deviation

Perfect correcting surface

Points with maximal angle error

Paraxial range

Residual spherical transverse aberrations
- Improvement by higher orders
- Generation of high gradients

Aspherical Expansion Order

![Graph showing Δy(r) and D_{rms} [μm]](image)

- 6. order
- 8. order
- 10. order
- 14. order

- k_{max}
- D_{rms} [mm]
Deviation of Light

Mechanisms of light deviation and ray bending

- Refraction
- Reflection
- Diffraction according to the grating equation
- Scattering (non-deterministic)

\[ n \cdot \sin \theta = n' \cdot \sin \theta' \]
\[ \theta = -\theta' \]
\[ g \cdot \left( \sin \theta - \sin \theta_o \right) = m \cdot \lambda \]
Diffracting Surfaces

- Surface with grating structure:
  new ray direction follows the grating equation
- Local approximation in the case of space-varying grating width
  \[ \vec{s}' = \frac{n'}{n} \cdot \vec{s} + \frac{m \lambda g}{n'd} \cdot \hat{g} + \gamma \cdot \hat{e} \]
- Raytrace only into one desired diffraction order
- Notations:
  g : unit vector perpendicular to grooves
d : local grating width
m : diffraction order
e : unit normal vector of surface
- Applications:
  - diffractive elements
  - line gratings
  - holographic components
Diffractive Optics:

- Local micro-structured surface
- Location of ray bending: macroscopic carrier surface
- Direction of ray bending: local grating micro-structure
Summary of Important Topics

- Mathematical description of dispersion: Sellmeier formulas, corresponds to modelling adjacent absorption lines, typically 2-3 terms considered
- Characterization of optical materials: refractive index $n$, Abbe number $\nu$ (dispersion)
- Abbe number: mean slope of dispersion curve, small $\nu$ means large dispersion
- Choice of special wavelengths to describe the dispersion behavior (e.g.: e, F, C)
- Partial dispersion $P$: more refined description of dispersion: curvature
- Normal glass line: most glasses fulfill $P$ proportional to $\nu$
- Glass diagram: $n$-$\nu$-chart, distinction of
  1. crown glasses, $n$ low, $\nu$ high, dispersion low
  2. flint glasses; $n$ high, $\nu$ low, dispersion high
- Plastics materials: for high volume systems, bad performance in comparison to glass (thermal, index low, straylight, transmission)
- Lenses: simple elements, focal length as main parameter
- Lenses: importance of principal plane, plane of artificial ray bending
- Lens bending: constant focal length but varying shape: incidence changed
- Simple aspheres: conic sections, mainly important for mirrors
- Aspherical lenses: one more degree of freedom
- Expansion aspheres: problem of oscillating sag
- Diffractive elements: local ray bending governed by grating equation