### 6.1 Galilei Telescope

A Galilean telescope consists of an afocal combination of a positive and a negative lens. The system transforms the diameter of a collimated incoming beam by a magnification factor $\Gamma$. The magnification can be expressed by the focal lengths of the lenses as

$$\Gamma = -\frac{f_+}{f_-}$$

(see exercise 10). What is the Petzval condition for a flattened field of view for this system? Derive the formula for the magnification for a Galilei telescope with flat field. If the available glass materials have refractive indices in the range $n = 1.45...2.0$, what is the achievable range of magnifications?

**Solution:**

Magnification factor of the telescope

$$\Gamma = -\frac{f_+}{f_-}$$

Petzval condition for a flat field of view

$$\frac{1}{n_+ f_+} + \frac{1}{n_- f_-} = 0$$

Insertion gives the simple equation

$$\Gamma = \frac{n_-}{n_+}$$

In the two extreme cases indices $n = 1.45 / n = 2.0$ we get

1. Low index negative lens
   $$\Gamma_{\text{min}} = \frac{1.45}{2.0} = 0.725$$
2. High index negative lens
   $$\Gamma_{\text{max}} = \frac{2.0}{1.45} = 1.379$$

### 6.2 Achromate I

Consider an achromate made by a crown glass with index $n_1 = 1.573$ and an Abbe number $v_1 = 57.4$ and a flint glass with the corresponding data $n_2 = 1.689$ and $v_2 = 31.2$. The system should have the focal length $f = 100$ mm and the crown lens should be shaped symmetrical biconvex corresponding to the following figure.
a) Calculate the focal lengths of both thin lenses and the necessary radii of curvature.
b) Calculate the radii of curvature.
c) Why is it not possible to correct this system for spherical aberration at the boundary?
d) Consider a cemented 2-lens-component without achomatization which is used to focus collimated light from an object point in infinity on axis. If the component is turned around, which of the primary aberrations inclusive the two primary chromatical aberrations are changed, which are invariant and which have the constant value of zero?

Solution:

a) Focal length condition
\[ F = F_1 + F_2 = \frac{1}{f'} \]

Achromatic condition
\[ \frac{F_1}{v_1} + \frac{F_2}{v_2} = 0 \]
gives with the material data for crown
\[ n_1 = 1.573 \quad v_1 = 57.4 \]
and flint glass
\[ n_2 = 1689 \quad v_2 = 31.2 \]
the focal powers and focal lengths
\[ F_1 = \frac{F}{1 - v_2 / v_1} = +0.02191 \quad f'_1 = 45.64 \text{ mm} \]
\[ F_2 = \frac{F}{1 - v_1 / v_2} = -0.01191 \quad f'_2 = -83.97 \text{ mm} \]

b) crown lens symmetrical
\[ \frac{1}{f'_1} = (n_1 - 1) \cdot \frac{2}{r_1} \quad r_1 = 2(n_1 - 1)f'_1 = 52.30 \text{ mm} \]
\[ r_2 = -52.30 \text{ mm} \]

flint lens :
\[ \frac{1}{f'_2} = (n_2 - 1) \left( \frac{1}{r_2} - \frac{1}{r_3} \right) \]
resolved for \( r_3 \) with \( r_2 = r_1 \)
\[ r_3 = -\frac{1}{\frac{1}{r_2} - \frac{1}{(n_2 - 1)f'_2}} = -545.0 \text{ mm} \]

c) Since the crown lens should be symmetric, the degree of freedom of bending the component is not available. This prevents the correction of spherical aberration.

d)

<table>
<thead>
<tr>
<th>aberration type</th>
<th>changed</th>
<th>invariant</th>
<th>constant = 0</th>
</tr>
</thead>
</table>


spherical | ✓ |
| coma | ✓ |
| astigmatism | ✓ |
| field curvature | ✓ |
| distortion | ✓ |
| axial chromatic | ✓ |
| lateral chromatic | ✓ |

6.3 Flattening Mirror

It is an advantage to use a collecting mirror for the field flatness correction of an optical system.

a) Use the modelling approach \( n' = -n \) for a mirror and compare a single positive lens in air with a single focussing mirror to explain this approach.

b) Is it an advantage to put the mirror in a medium with \( n > 1 \) ?

c) What happens for a diverging mirror ?

Solution:

a) For the lens, the Petzval radius of curvature reads
\[
\frac{1}{R_{petz}} = -\frac{1}{n \cdot f} < 0
\]
and is always negative. For a mirror, the Petzval formula of a single surface reads
\[
\frac{1}{R_{petz}'} = \frac{n'-n}{nn'r}
\]
With \( n' = -n \) and \( r < 0 \) for a focussing concave mirror we get
\[
\frac{1}{R_{petz}'} = -\frac{2}{n \cdot r} > 0
\]
which means a correction of typical collecting lens contributions.

b) If the mirror is put into a medium with \( n > 1 \), the effect is reduced (due to its occurrence in the denominator), therefore it is better to use the mirror in air.

c) A diverging mirror with \( r > 0 \) also has a negative Petzval contribution and does not help to correct the lens contributions.

6.4 Retro focus lens

An optical system of the retro focus type has the following principle arrangement of a negative and a positive lens. With this setup a working distance can be obtained, which is larger than the focal length of the system.
a) Sketch the location of the back principal plane of the system and the focal length of the system in the figure above.
b) If only one object point on axis at infinity is considered, what are the relevant aberrations of the system performance?
c) Explain, why the second lens is more critical concerning the aberration contributions.
d) What can be done to get a better image quality in comparison to the single biconcave/biconvex lenses shown in the figure?
e) If two lenses are considered, which are not close together, the contribution of a lens to the axial chromatic aberration can be formulated as \( \Delta_{\text{AXCR}} = \frac{y^2}{v \cdot f} \). This means, the square of the marginal ray height \( y \) at the lens works as a weighting factor. Discuss the possibility of an axial chromatic correction of the given system using this property. Consider the special case, that the lenses are thin and the working distance is twice the focal length. Derive the necessary relation of the Abbe numbers for this constellation.
f) Which of the two lenses in part e) must be the crown lens?

Solution:

a) Creative are only axial chromatical aberration and spherical aberration.

b) At the second lens, the marginal ray height is larger and the ray bending is larger. Therefore the nonlinearity of the incidence angles is more critical at this lens.

d) A better performance is obtained, if the lens is splitted in several smoothly bending lenses or to use an aspherical lens. Alternatively, the first lens can be bended properly to correct the spherical aberration of the second lens.

e) The correction of axial chromatical aberrations needs the fulfillment of the condition

\[
\Delta_{\text{AXCR}} = \frac{y_1^2}{v_1 \cdot f_1} + \frac{y_2^2}{v_2 \cdot f_2} = 0
\]

Since \( y_2 \) and \( f_2 \) are positive and larger in value in comparison to \( y_1 \) and \( f_1 < 0 \), it is possible to fulfill the condition by a suitable choice of the Abbe numbers, if the ratio between \( f_2 \) and \( f_1 \) is not too large.
For a factor of 2 between working distance and focal length, we have 
\[ f_2 = -2f_1 \quad \text{and} \quad y_2 = 2y_1 \]
for these numbers, the achromatic condition gives
\[ \Delta_{\text{ach}} = \frac{y_1^2}{f_1} \left( \frac{1}{v_1} - \frac{4}{v_2} \right) = 0 \]
\[ v_1 = \frac{v_2}{4} \]
f) Corresponding to the last equation, the crown lens with the larger Abbe number must be the second (positive lens).

6.5 Field flattening lens

Calculate a thick meniscus shaped lens with refractive index \( n = 1.5 \) with vanishing Petzval curvature contribution, a focal length of \( f = 100 \) mm and a front radius of \( r_1 = +10 \) mm. The back radius and the thickness is to be determined.

Solution:

The Petzval sum reads
\[ \frac{1}{r_p} = -\sum_j \frac{n_j' - n_j}{n_j n_j' r_j} \]
The condition of a vanishing Petzval curvature is given by
\[ \frac{1.5 \cdot r_1}{1.5 - 1} + \frac{1.5 \cdot r_2}{1 - 1.5} = 0 \]
From this equation we get with \( r_1 = 10 \) mm the radius \( r_2 = 10 \) mm.
In the case of a thick lens we have the focal length
\[ f' = \frac{nr_1r_2}{(n-1)[n(r_2 - r_1) + (n-1)d]} \]
If this equation is resolved for \( d \) we get the thickness
\[ d = -\frac{n(r_2 - r_1)}{n-1} + \frac{rr_2n}{(n-1)^2 f'} = \frac{rr_2n}{(n-1)^2 f'} = 6 \text{ mm} \]