Exercises and Solutions
Lecture Design and Correction of Optical Systems – Part 2

Exercise 2-1: Focal Length of a Thick Lens

Use the formulas of the paraxial raytrace procedure
\[ y_j = y_{j-1} - d_{j-1}U_j \]
\[ i_j = c_j y_j - U_{j-1}, \quad i'_j = \frac{n_j}{n'_j} i_j \]
\[ U' = U_j + i_j - i'_j \]
to derive the equation for the focal length of a thick lens in air.

Exercise 2-2: Axial Image Shift in a Medium

Calculate the axial shift of the image position, if a raybundel with aperture \( \sin(u) \) in a medium with refractive index \( n \) is focussed into a medium with index \( n' \) exact and in paraxial approximation.

Exercise 2-3: Lagrange Invariant for Illumination System

An object with 2.5 mm diameter should be illuminated with a numerical aperture of \( NA = 0.3 \). If the aplanatic corrected illumination system can accept a numerical aperture of \( NA = 0.9 \) of the light source, what is the minimum size of the radiating area of the lamp?
Exercise 2-4: Defocussed Telescope

An inverted afocal Galilean telescope is given which reduces the diameter of an incoming beam by a factor of 5. Both used lenses have one plane surface, the positive lens with \( f_1 = 100 \text{ mm} \) is made of a material with refractive index \( n_1 = 1.5 \), the negative lens with \( f_2 = -20 \text{ mm} \) has \( n_2 = 2.0 \). Sketch the system with an orientation of the lenses, which is beneficial for the correction. Determine the bending and the magnification parameter of both lenses. Calculate the paraxial ABCD-matrix of the system for an arbitrary distance \( z \) between the lenses. If the distance \( z \) between both lenses is misadjusted by \( \Delta z \), the system is no longer afocal. Calculate the change of the magnification and the residual outgoing ray angle for the case of a misadjustment of \( \Delta = 0.1 \text{ mm} \) and an incoming beam diameter of 20 mm.

Exercise 2-5: Ball Lens

Derive the focal length of a ball lens. What is especially the formula of the focal length for the refractive index \( n = 1.5 \) ?
If the ball lens is used symmetrical for an object in a finite distance, what is the overall length of the imaging system?
Derive the condition, that must be fulfilled, that an incoming plane wave is focussed onto the back vertex point of a ball lens. What is the focal length of the ball lens for this special setup? Where is the principal plane in this case?