Design and Correction of Optical Systems – Part 7

Exercise 7-1: Depth Magnification

The equation for the axial magnification reads

\[ m_z = \frac{n}{n'} \cdot m^2 \]

Derive this equation by differentiation of the lens makers formula. Show that the so called Herrschel parameter

\[ H = \Delta z \cdot n \cdot u^2 \]

is invariant under paraxial imaging conditions.

Exercise 7-2: Mirror Aberrations

A spherical mirror with radius \( r \) and object intersection length \( s \) has for a ray in the height \( y_p \) the wave aberration.

\[ W(y_p) = \frac{y_p^4}{4r} \cdot \left( \frac{1}{r} - \frac{1}{s} \right)^2 \]

for spherical aberration. Derive the general expression of the transverse aberration \( \Delta y' \) in the image plane. Calculate the Airy spot diameter, if the \( z \)-distance at the mirror surface can be neglected.

If the mirror radius is \( r = -500 \) mm, the wavelength \( \lambda = 633 \) nm and the diameter of the incoming beam \( 2y_p = 40 \) mm what is the range of intersections lengths \( s' \), were the system is diffraction limited with a geometrical spot, that is smaller than the diffraction spot size?

Exercise 7-3: Wave Aberrations

The wave aberration of spherical aberration of 5th order reads in polar coordinates on a normalized pupil radius \( W(r) = a_6 \cdot r^6 \) with coefficient \( a_6 \). Calculate the rms-value of the wave aberration. If the deviations are partly compensated by a defocus term, the wave aberration reads \( W(r) = c_6 \cdot \left( r^6 - r^2 \right) \). Calculate the mean value and the rms value for this aberration function. Determine the radius \( r_{\text{min}} \) of the pupil radius, where the aberration of \( W \) has its largest value. What is the value of the wave aberration at half of the pupil radius?
Exercise 7-4: Spherical Aberration of a Plane Parallel Plate

Derive the axial spherical aberration, which corresponds to the difference between the real and the paraxial image location of a plane parallel plate with index $n$ and thickness $d$. Calculate the lowest order of the aberration as a function of a small value of $\sin u$. If the diameter in the gaussian image plane should not be larger than 10 $\mu$m, calculate the greatest possible thickness of a plate in this approximation with refractive index of $n = 1.48$ for a numerical aperture of $\sin u = 0.8$. 