Exercise 3-1: Microscope Aplanatic Lens

Consider a microscopic lens with a numerical aperture of NA = 0.6 in air on the object side. The first lens should be an aplanatic-concentric lens to reduce the numerical aperture. The free working distance between the object and the vertex of the first surface is 2 mm.

a) Calculate the radii of curvature of this first lens, if the center thickness is \( d = 1 \text{ mm} \) and a glass with index \( n = 1.6 \) is used.

b) Calculate the location of the image just behind the lens

c) What is the size of the numerical aperture behind the lens?

d) Sketch this system

Solution:

a) the lens must be concentric-aplanatic to reduce the aperture angle: Therefore:
- front surface concentric with \( r_1 = -2 \text{ mm} \)
- rear surface aplanatic with \( s = -2 \text{ mm} \), \( n = 1.6 \), \( n' = 1.0 \), \( d = -1 \text{ mm} \)

\[
\begin{aligned}
r_2 &= \frac{n \cdot (s + d)}{n' + n} = -1.846 \text{ mm} \\

s' &= \frac{n'}{n} (s + d) = -4.8 \text{ mm}
\end{aligned}
\]

c) The concentric surface does not change the aperture angle. The aplanatic surface reduces the aperture angle by the ratio of the refractive indices to

\[
in' u' = \frac{n'}{n} \sin u = \frac{1}{1.6} \cdot 0.6 = 0.375
\]

\[
NA' = 0.375
\]
**Exercise 3-2: Spherical Aberration of a Single Lens**

A thin plane-convex lens with focal length \( f' = 100 \) mm is used to focus a collimated beam with diameter \( D = 10 \) mm and wavelength \( \lambda = 1.06 \) \( \mu \)m.

Draw a sketch of the lens in the optimal setup and explain why this orientation is advantageous. Calculate the magnification parameter \( M \) and the bending parameter \( X \) of the lens.

The surface contribution of the primary spherical wave aberration of a single lens can be expressed by the formula

\[
A_n = \frac{1}{32n(n-1)f'^3} \left[ \frac{n^3}{n-1} + \frac{n+2}{n-1} \left\{ X - \frac{2(n^2-1)}{n+2} \frac{M}{n} \right\}^2 - \frac{n^2(n-1)}{n+2} M^2 \right]
\]

Calculate this coefficient for the refractive indices \( n = 1.4 \) and \( n = 2.0 \). Which choice of refractive index is more advantageous?

Calculate the transverse aberration \( \Delta y' \) in the image plane in the one-dimensional case with the pupil coordinate \( y_p \) by using this formula in the form \( W_{SPH}(y_p) = A_n \cdot y_p^4 \) for both indices. Is the system diffraction limited? Compare the geometrical spot size and the diffraction Airy diameter.

**Solution:**

Sketch of the setup:

In this orientation, the bending of the rays is distributed on two surfaces. This gives smaller incidence angles and a smaller impact of the non-linearity of the law of refraction, which generates aberrations.

The bending parameter is: \( X = +1 \)

the magnification parameter is: \( M = +1 \)

By inserting the data for \( f, n, X \) and \( M \) we get the coefficients:

1. index \( n = 1.4 \):
   \( A_6 = -4.598 \times 10^{-7} \) mm\(^3\)
2. index \( n = 2.0 \):
   \( A_6 = -1.25 \times 10^{-7} \) mm\(^3\)

The transverse aberration is related by with the wave aberration by

\[
\Delta y' = -R \cdot \frac{\partial W}{\partial y_p} = -f' \cdot 4 y_p^3 \cdot A_n
\]

with reference sphere radius \( R = f' \). This gives for the two cases the geometrical spot sizes

\( n = 1.4 \) \( D_{Geo} = 2\Delta y' = 46 \mu \)m

\( n = 2.0 \) \( D_{Geo} = 2\Delta y' = 12.5 \mu \)m

The Airy diameter is given by
\[ D_{\text{Airy}} = \frac{1.22 \cdot \lambda}{\sin u'} = \frac{2.44 \cdot \lambda \cdot f'}{D_m} = 25.9 \, \mu m \]

Therefore, in the case of the higher index \( n=2 \), the system is diffraction limited, the smaller index delivers a geometrical dominated broadening of the spot size.

**Exercise 3-3: Stop and Aberrations**

Explain, why the z-location of a stop in an optical system influences the aberrations. Divide the 5 primary monochromatic and the two main chromatic aberrations into those, which are constant and those, which are changed, if the system stop is moved along z. It is assumed, that the size of stop is adapted during the movement to maintain the numerical aperture constant.

**Solution:**

The stop location forces the ray path of the chief ray. Therefore all the field dependent aberrations are influenced. The field curvature as a special case is not influenced due to the equation of Petzval:

- Not influenced: spherical aberration, petzval curvature, axial chromatic
- Influenced: astigmatism, coma, distortion, lateral chromatic