Design and Correction of Optical Systems

Lecture 10: Correction principles I
2018-06-18
Herbert Gross

Summer term 2018
<table>
<thead>
<tr>
<th>Date</th>
<th>Session Title</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>09.04.</td>
<td>Basics</td>
<td>Law of refraction, Fresnel formulas, optical system model, raytrace, calculation approaches</td>
</tr>
<tr>
<td>16.04.</td>
<td>Materials and Components</td>
<td>Dispersion, anormal dispersion, glass map, liquids and plastics, lenses, mirrors, aspheres, diffractive elements</td>
</tr>
<tr>
<td>23.04.</td>
<td>Paraxial Optics</td>
<td>Paraxial approximation, basic notations, imaging equation, multi-component systems, matrix calculation, Lagrange invariant, phase space visualization</td>
</tr>
<tr>
<td>30.04.</td>
<td>Optical Systems (Yi Zhong)</td>
<td>Pupil, ray sets and sampling, aperture and vignetting, telecentricity, symmetry, photometry</td>
</tr>
<tr>
<td>07.05.</td>
<td>Geometrical Aberrations</td>
<td>Longitudinal and transverse aberrations, spot diagram, polynomial expansion, primary aberrations, chromatical aberrations, Seidels surface contributions</td>
</tr>
<tr>
<td>14.05.</td>
<td>Wave Aberrations (Johannes Stock)</td>
<td>Fermat principle and Eikonal, wave aberrations, expansion and higher orders, Zernike polynomials, measurement of system quality</td>
</tr>
<tr>
<td>28.05.</td>
<td>PSF and Transfer function</td>
<td>Diffraction, point spread function, PSF with aberrations, optical transfer function, Fourier imaging model</td>
</tr>
<tr>
<td>04.06.</td>
<td>Further Performance Criteria</td>
<td>Rayleigh and Marechal criteria, Strehl definition, 2-point resolution, MTF-based criteria, further options</td>
</tr>
<tr>
<td>11.06.</td>
<td>Optimization and Correction</td>
<td>Principles of optimization, initial setups, constraints, sensitivity, optimization of optical systems, global approaches</td>
</tr>
<tr>
<td>18.06.</td>
<td>Correction Principles I (Johannes Stock)</td>
<td>Symmetry, lens bending, lens splitting, special options for spherical aberration, astigmatism, coma and distortion, aspheres</td>
</tr>
<tr>
<td>25.06.</td>
<td>Correction Principles II</td>
<td>Field flattening and Petzval theorem, chromatical correction, achromate, apochromate, sensitivity analysis, diffractive elements</td>
</tr>
<tr>
<td>02.07.</td>
<td>Optical System Classification</td>
<td>Overview, photographic lenses, microscopic objectives, lithographic systems, eyepieces, scan systems, telescopes, endoscopes</td>
</tr>
<tr>
<td>09.07.</td>
<td>Special System Examples</td>
<td>Zoom systems, confocal systems</td>
</tr>
</tbody>
</table>
Contents

1. Introduction
2. Sensitivity and improvement process
3. Special options for spherical aberration
4. Astigmatism
5. Symmetry
6. Coma
7. Distortion
8. Aspheres and higher orders
System Design Phases

1. Paraxial layout:
   - specification data, magnification, aperture, pupil position, image location
   - distribution of refractive powers
   - locations of components
   - system size diameter / length
   - mechanical constraints
   - choice of materials for correcting color and field curvature

2. Correction/consideration of Seidel primary aberrations of 3rd order for ideal thin lenses, fixation of number of lenses

3. Insertion of finite thickness of components with remaining ray directions

4. Check of higher order aberrations

5. Final correction, fine tuning of compromise

6. Tolerancing, manufactability, cost, sensitivity, adjustment concepts
# Correction Effectiveness

- Effectiveness of correction features on aberration types

<table>
<thead>
<tr>
<th>Aberration</th>
<th>Primary Aberration</th>
<th>5th</th>
<th>Chromatic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spherical Aberration</td>
<td>(a)</td>
<td>(c)</td>
<td>e (f)</td>
</tr>
<tr>
<td>Coma</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Astigmatism</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Petzval Curvature</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distortion</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>5th Order Spherical</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Axial Color</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Lateral Color</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Secondary Spectrum</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Spherochromatism</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

**Lens Bending**

- Makes a good impact.
- Makes a smaller impact.
- Makes a negligible impact.
- Zero influence.

**Power Splitting**

**Power Combination**

**Distances**

**Stop Position**

**Refractive Index**

**Dispersion**

**Relative Partial Disp.**

**Material**

- GRIN
- Cemented Surface
- Aplanatic Surface
- Aspherical Surface
- Mirror
- Diffractive Surface
- Symmetry Principle
- Field Lens

Ref: H. Zügge
Usefull options for accelerating a stagnant optimization:

- split a lens
- increase refractive index of positive lenses
- lower refractive index of negative lenses
- make surface with large spherical surface contribution aspherical
- break cemented components
- use glasses with anomalous partial dispersion
Operationen with zero changes in first approximation:

1. Bending a lens.
2. Flipping a lens into reverse orientation.
3. Flipping a lens group into reverse order.
4. Adding a field lens near the image plane.
5. Inserting a powerless thin or thick meniscus lens.
6. Introducing a thin aspheric plate.
7. Making a surface aspheric with negligible expansion constants.
8. Moving the stop position.
9. Inserting a buried surface for color correction, which does not affect the main wavelength.
10. Removing a lens without refractive power.
11. Splitting an element into two lenses which are very close together but with the same total refractive power.
12. Replacing a thick lens by two thin lenses, which have the same power as the two refracting surfaces.
13. Cementing two lenses a very small distance apart and with nearly equal radii.
Structural Changes for Correction

- Lens bending

- Lens splitting

- Power combinations

- Distances

Ref: H. Zügge
Lens Removal

- Removal of a lens by vanishing of the optical effect
- For single lens and cemented component
- Problem of vanishing index:
  Generation of higher orders of aberrations

a) Geometrical changes: radius and thickness
   1) adapt second radius of curvature
   2) shrink thickness to zero

b) Physical changes: index
Sensitivity by large Incidence

- Small incidence angle of a ray: small impact of centering error
- Large incidence angle of a ray:  
  - strong non-linearity range of $\sin(i)$  
  - large impact of decenter on ray angle

Ref: H. Sun
Sensitivity of a System

- **Sensitivity/relaxation:**
  Average of weighted surface contributions of all aberrations

- **Correctability:**
  Average of all total aberration values

- **Total refractive power**
  \[ F = F_1 + \sum_{j=2}^{k} \omega_j F_j \]

- **Important weighting factor:**
  ratio of marginal ray heights
  \[ \omega_j = \frac{h_j}{h_1} \]

---

1. **Sph**
2. **Coma**
3. **Ast**
4. **CHL**

**incidence angle**
Sensitivity of a System

- Quantitative measure for relaxation

\[ A_j = \omega_j \cdot \frac{F_j}{F} = \frac{h_j \cdot F_j}{h_1 \cdot F} \]

with normalization

\[ \sum_{j=1}^{k} A_j = 1 \]

- Non-relaxed surfaces:
  1. Large incidence angles
  2. Large ray bending
  3. Large surface contributions of aberrations
  4. Significant occurrence of higher aberration orders
  5. Large sensitivity for centering

- Internal relaxation can not be easily recognized in the total performance

- Large sensitivities can be avoided by incorporating surface contribution of aberrations into merit function during optimization
Sensitivity of a System

Representation of wave

Seidel coefficients $[\lambda]$

Double Gauss 1.4/50

Ref: H. Zügge
Relaxed System

- Example: achromate with cemented/splitted setup
- Equivalent performance
- Inner surfaces of splitted version more sensitive

a) Cemented achromate $f=100\, \text{mm}$, $\text{NA}=0.1$

b) Splitted achromate $f=100\, \text{mm}$, $\text{NA}=0.1$

Ref: H. Zügge
Relaxed Design

- Photographic lens comparison
- Data:
  - F# = 2.0
  - f = 50 mm
  - Field 20°
- Same size and quality
- Considerably tighter tolerances in the first solution

Ref: D. Shafer
Example: Sensitivity of Endoscopic Systems

- Data: $W_j$, $S_j$, $i_{MR}^2$, $i_{CR}^2$, Sph, $\Gamma$, $A$
- Relay systems
  1. ZeBase M 29, Takahashi, Olympus
  
  2. Hopkins modified

  3. Kidger
Reality:
- as-designed performance: not reached in reality
- as-built-performance: more relevant

Possible criteria:
1. Incidence angles of refraction
2. Squared incidence angles
3. Surface powers
4. Seidel surface contributions
5. Permissible tolerances

Special aspects:
- relaxed systems does not contain higher order aberrations
- special issue: thick meniscus lenses
Further Parameter of Sensitivity

- Possible further criteria for modified merit function to obtain relaxed systems
  1. \( \cos-\Gamma \)-factor of ray bending

\[
\Gamma_j = n_j \cdot \vec{s}_j \cdot \vec{e}_j - n_j \cdot \vec{s}_j \cdot \vec{e}_j
\]

2. Squared sum of incidence angles

\[
\theta = \sqrt{\frac{1}{2N} \cdot \sum_{j=1}^{N} (i_j^2 + i_j')^2}
\]

Target: minimum value for \( i, i' \)

3. Optimization of performance and performance change simultaneously

\[
D = \sqrt{\sum_{m=1}^{M} \left( \frac{\partial M}{\partial p_m} \cdot \Delta p_m \right)^2}
\]
- Relaxed system: as built performance improved
- Typically:
  - no or weak correlation to designed performance
  - weak decrease in nominal performance possible
Design Solutions and Sensitivity

- Focussing 3 lens with NA = 0.335
- Spherical correction with/without compensation
- Red surface: main correcting surface
- Counterbeding every lens in one direction

<table>
<thead>
<tr>
<th>counterbending</th>
<th>Dspot</th>
<th>SPH-min</th>
<th>SPH-max</th>
</tr>
</thead>
<tbody>
<tr>
<td>no</td>
<td>10.9</td>
<td>0.63</td>
<td>3.7</td>
</tr>
<tr>
<td>L1 +</td>
<td>0.38</td>
<td>4</td>
<td>151</td>
</tr>
<tr>
<td>L1 -</td>
<td>0.28</td>
<td>12</td>
<td>105</td>
</tr>
<tr>
<td>L2 -</td>
<td>0.19</td>
<td>14</td>
<td>95</td>
</tr>
<tr>
<td>L2 +</td>
<td>0.65</td>
<td>4</td>
<td>292</td>
</tr>
<tr>
<td>L3 -</td>
<td>0.18</td>
<td>5</td>
<td>151</td>
</tr>
<tr>
<td>L3 +</td>
<td>0.50</td>
<td>5</td>
<td>151</td>
</tr>
</tbody>
</table>
Microscopic Objective Lens

- Incidence angles for chief and marginal ray
- Aperture dominant system
- Primary problem is to correct spherical aberration
Photographic lens

- Incidence angles for chief and marginal ray
- Field dominant system
- Primary goal is to control and correct field related aberrations: coma, astigmatism, field curvature, lateral color
Symmetrical Dublet

- Variable focal length
  \( f = 15 \ldots 200 \text{ mm} \)

- Invariant:
  object size \( y = 10 \text{ mm} \)
  numerical aperture \( NA = 0.1 \)

- Type of system changes:
  - dominant spherical for large \( f \)
  - dominant field for small \( f \)

- Data:

<table>
<thead>
<tr>
<th>No</th>
<th>focal length [mm]</th>
<th>Length [mm]</th>
<th>spherical ( c_3 )</th>
<th>field curvature ( c_4 )</th>
<th>astigmatism ( c_5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>808</td>
<td>3.37</td>
<td>-2.01</td>
<td>2.27</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>408</td>
<td>1.65</td>
<td>1.19</td>
<td>-4.50</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td>206</td>
<td>1.74</td>
<td>3.45</td>
<td>-7.34</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>75</td>
<td>0.98</td>
<td>3.93</td>
<td>2.31</td>
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<tr>
<td>5</td>
<td>15</td>
<td>59</td>
<td>0.20</td>
<td>16.7</td>
<td>-5.33</td>
</tr>
</tbody>
</table>

\( f = 200 \text{ mm} \)
\( f = 100 \text{ mm} \)
\( f = 50 \text{ mm} \)
\( f = 20 \text{ mm} \)
\( f = 15 \text{ mm} \)
Field vs Aperture Correction

- Remote pupil system with decreasing field for one wavelength and 4 lenses
- Aperture maximum value for overall diffraction limited correction
- Large field angles: lenses bended towards pupil
- Old achromate move towards usual achromate
- Effect of bending a lens on spherical aberration
- Optimal bending:
  Minimize spherical aberration
- Dashed: thin lens theory
  Solid: think real lenses
- Vanishing SPH for n=1.5
  only for virtual imaging
- Correction of spherical aberration possible for:
  1. Larger values of the magnification parameter |M|
  2. Higher refractive indices

Ref: H. Zügge
Aplanatic Surfaces with Vanishing Spherical Aberration

- Aplanatic surfaces: zero spherical aberration:
  1. Ray through vertex \( s' = s = 0 \)
  2. concentric
  3. Aplanatic \( s' = s\) und \( u = u' \)
  \( ns = n's' \)

- Condition for aplanatic surface:
  \[ r = \frac{ns}{n+n'} = \frac{n's'}{n+n'} = \frac{ss'}{s+s'} \]

- Virtual image location

- Applications:
  1. Microscopic objective lens
  2. Interferometer objective lens

![Diagram showing linear growing aberrations with deviation of object location and no shift invariance in z](image-url)
Aplanatic lenses

- Combination of two spherical corrected surfaces: one concentric and one aplanatic surface: zero contribution of the whole lens to spherical aberration

- Not useful:
  1. aplanatic-aplanatic
  2. concentric-concentric bended plane parallel plate, nearly vanishing effect on rays
- Impact of aplanatic lenses in microscopy on magnification

\[ m = \frac{y'}{y} = \frac{n s'}{n' s} \]

- Three cases of typical combinations

| A-C | \( m_{AC} = n \) | A-V | \( m_{AV} = n^2 \) | A-C \ A-V | \( m = n^3 \) |
- Example lens
- Aperture NA = 0.5
- Spherical correction with one surface
- Examples of large-working-distance objective lenses
- Aplanatic-concentric shell-lenses in the front group
- Large diameter of the lens
- Correction of spherical aberration: Splitting of lenses
- Distribution of ray bending on several surfaces:
  - smaller incidence angles reduces the effect of nonlinearity
  - decreasing of contributions at every surface, but same sign
- Last example (e): one surface with compensating effect
Correcting Spherical Aberration: Power Splitting

Splitting of lenses and appropriate bending:
1. compensating surface contributions
2. Residual zone errors
3. More relaxed setups preferred, although the nominal error is larger

Ref: H. Zügge
- Better correction for higher index
- Shape of lens / best bending changes from
  1. nearly plane convex for n = 1.5
  2. meniscus shape for n > 2

Ref: H. Zügge
Correcting spherical aberration by cemented doublet:
- Strong bended inner surface compensates
- Solid state setups reduces problems of centering sensitivity
- In total 4 possible configurations:
  1. Flint in front / crown in front
  2. bi-convex outer surfaces / meniscus shape
- Residual zone error, spherical aberration corrected for outer marginal ray

Ref: H. Zügge
Better correction for high index also for multiple lens systems

Example: 3-lens setup with one surface for compensation
Residual aberrations is quite better for higher index
IR Objective Lens

- Aperture f/1.5
- Spectral 8 - 12 µm
- 2 lenses aspherical
Compound Systems

- System groups:
  1. Afocal zoom telescope
  2. Scanning group
  3. Reimager
- Suitable for cooled matrix detector
- Bending effects astigmatism
- For a single lens 2 bending with zero astigmatism, but remaining field curvature
Principle of Symmetry

- Perfect symmetrical system: magnification \( m = -1 \)
- Stop in centre of symmetry
- Symmetrical contributions of wave aberrations are doubled (spherical)
- Asymmetrical contributions of wave aberration vanishes \( W(-x) = -W(x) \)
- Easy correction of:
  coma, distortion, chromatical change of magnification
Even Aberrations in Symmetrical Systems

- Aberrations with even symmetry are doubled
- Spherical aberration, Astigmatism, field curvature, axial chromatical aberration

spherical aberration in an symmetrical system

\[ W = c_4 \cdot Z_4 + c_9 \cdot Z_9 \]

\[ W = 2c_4 \cdot Z_4 + 2c_9 \cdot Z_9 \]

Ref: M. Seesselberg
Odd Aberrations in Symmetrical Systems

- Aberrations with odd symmetry are vanishing
- Coma, distortion, transverse chromatical aberration

\[ W = c_8 \cdot Z_8 + c_{15} \cdot Z_{15} \]

\[ W = -c_8 \cdot Z_8 - c_{15} \cdot Z_{15} \]

vanishing values

\[ W = 0 \]
Ideal symmetrical systems:
- Vanishing coma, distortion, lateral color aberration
- Remaining residual aberrations:
  1. spherical aberration
  2. astigmatism
  3. field curvature
  4. axial chromatical aberration
  5. skew spherical aberration
Wide Angle Lenses - Symmetrical

- Radii of curvature of wide angle camera lenses - symmetrical setups
- Mostly radii 'concentric' towards the stop position
- Locations $z_j$ of surfaces normalized for comparison
- Nearly linear trend, some exceptions near to the pupil
- Stop position centered
Symmetry Principle

- Application of symmetry principle: photographic lenses
- Especially field dominant aberrations can be corrected
- Also approximate fulfillment of symmetry condition helps significantly: quasi symmetry
- Realization of quasi-symmetric setups in nearly all photographic systems

Ref: H. Zügge
Offner-System

- Concentric system of Offner:
  relation

\[ d_1 = d_2 = \frac{r_1}{2} = r_2 \]

- Due to symmetry:
  Perfect correction of field aberrations in third order
- Catadioptric system with $m = -1$ according Dyson
  
  Advantage: flat field
  Application: lithography and projection

- Relation:
  \[ r_L = \frac{n - 1}{n} \cdot r_M \]

- Residual aberration: astigmatism
Mono-Centric Systems

Offner

Offner-Wynne

Dyson

do-nothing lens (only Petzval)

Sutton ball-lens

Retrofocus I

Retrofocus II
Monocentric System

- Nearly perfect system with 4 monocentric surfaces
- Data:
  - $f = 25 \text{ mm}$
  - $\text{NA} = 0.98$
- Unfortunately image plane not easily accessible
- Only small field possible
- No coma due to monocentric behavior

Mono-Centric Lenses

Schwarzschild

Newton

Stamenov

Schmidt
Bending of an achromate
- optimal choice: small residual spherical aberration
- remaining coma for finite field size

Splitting achromate:
- additional degree of freedom:
  - better total correction possible
  - high sensitivity of thin air space

Aplanatic glass choice:
vanishing coma

<table>
<thead>
<tr>
<th>Achromat bending</th>
<th>Image height:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>y' = 0 mm</td>
</tr>
<tr>
<td>Pupil section:</td>
<td>meridional</td>
</tr>
<tr>
<td>Transverse Aberration:</td>
<td>Δy' 0.05 mm</td>
</tr>
</tbody>
</table>

Ref: H. Zügge
Inner and Outer Coma

- Effect of lens bending on coma
- Sign of coma: inner/outer coma

From: H. Zügge
Coma Correction: Symmetry Principle

- Perfect coma correction in the case of symmetry
- But magnification $m = -1$ not useful in most practical cases

<table>
<thead>
<tr>
<th>Symmetry principle</th>
<th>Image height:</th>
<th>$y' = 19$ mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pupil section:</td>
<td>meridional</td>
<td>sagittal</td>
</tr>
<tr>
<td>Transverse Aberration:</td>
<td>$\Delta y'$</td>
<td>$\Delta y'$</td>
</tr>
<tr>
<td></td>
<td>0.5 mm</td>
<td>0.5 mm</td>
</tr>
</tbody>
</table>

(a) 

(b) 

From: H. Zügge
Coma Correction: Stop Position and Aspheres

- Combined effect, aspherical case prevent correction

<table>
<thead>
<tr>
<th>Plano-convex element exhibits spherical aberration</th>
<th>Sagittal coma $\Delta y'\ 0.5 \text{ mm}$</th>
<th>Spherical aberration corrected with aspheric surface</th>
<th>Sagittal coma $\Delta y'\ 0.5 \text{ mm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Image of Plano-convex element with spherical aberration" /></td>
<td><img src="image2.png" alt="Image of sagittal coma correction" /></td>
<td><img src="image3.png" alt="Image of spherical aberration corrected" /></td>
<td><img src="image4.png" alt="Image of sagittal coma correction" /></td>
</tr>
</tbody>
</table>

Ref: H. Zügge
Distortion and Stop Position

- Sign of distortion for single lens: depends on stop position and sign of focal power
- Ray bending of chief ray defines distortion
- Stop position changes chief ray height at the lens

<table>
<thead>
<tr>
<th>Lens</th>
<th>Stop location</th>
<th>Distortion</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>positive</td>
<td>rear</td>
<td>$V &gt; 0$</td>
<td>telephoto lens</td>
</tr>
<tr>
<td>negative</td>
<td>in front</td>
<td>$V &gt; 0$</td>
<td>loupe</td>
</tr>
<tr>
<td>positive</td>
<td>in front</td>
<td>$V &lt; 0$</td>
<td>retrofocus lens</td>
</tr>
<tr>
<td>negative</td>
<td>rear</td>
<td>$V &lt; 0$</td>
<td>reversed binocular</td>
</tr>
</tbody>
</table>

Ref: H. Zügge
- Example: Achromate
- Balance:
  1. zonal spherical
  2. Spot
  3. Secondary spectrum

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**Coexistence of Aberrations : Balance**

- Example: Achromate
- Balance:
  1. zonal spherical
  2. Spot
  3. Secondary spectrum

---

Ref : H. Zügge
Coexistence of Aberrations: Balance

- Example: Apochromate
- Balance:
  1. Zonal spherical
  2. Spot
  3. Secondary spectrum

![Diagram showing coexistence of aberrations with balance examples for different wavelengths and fields.](image)