Computational Photonics
Seminar 05, 08 June 2015

Finite-Difference Time-Domain Method (FDTD)

- Learn how to implement a 1D version of FDTD
- Extend the code to 3D problems
- (voluntary) learn how to save simulation results in movie format
changing of index notation to integer indices

\[ E_{z,i}^{n+1} \approx E_{z,i}^n + \frac{1}{\varepsilon_0\varepsilon_i} \frac{\Delta t}{\Delta x} \left[ H_y^{i+\frac{1}{2}} - H_y^{i-\frac{1}{2}} \right] - \frac{\Delta t}{\varepsilon_0\varepsilon_i} j_z^{i+\frac{1}{2}} \]

\[ H_y^{i+\frac{1}{2}} \approx H_y^{i-\frac{1}{2}} + \frac{1}{\mu_0} \frac{\Delta t}{\Delta x} \left[ E_z^{i+1} - E_z^i \right] \]
3D FDTD: Electric field components
changing of index notation to integer indices

\[
E_x^{n+1}_{i,j,k} = E_x^n_{i,j,k} + \frac{\Delta t}{\varepsilon_0 \varepsilon_{i,j,k}} \left( \frac{H_z^n_{i,j,k+1} - H_z^n_{i,j,k}}{\Delta y} - \frac{H_y^n_{i,j,k+1} - H_y^n_{i,j,k}}{\Delta z} - j_x^n_{i,j,k} \right)
\]

\[
E_y^{n+1}_{i-1,j+1,k} = E_y^n_{i-1,j+1,k} + \frac{\Delta t}{\varepsilon_0 \varepsilon_{i-1,j+1,k}} \left( \frac{H_x^n_{i-1,j+1,k+1} - H_x^n_{i-1,j+1,k}}{\Delta z} - \frac{H_z^n_{i,j,k+1} - H_z^n_{i,j,k}}{\Delta x} - j_y^n_{i-1,j+1,k} \right)
\]

\[
E_z^{n+1}_{i-1,j,k+1} = E_z^n_{i-1,j,k+1} + \frac{\Delta t}{\varepsilon_0 \varepsilon_{i-1,j,k+1}} \left( \frac{H_y^n_{i,j,k+1} - H_y^n_{i-1,j,k+1}}{\Delta x} - \frac{H_x^n_{i-1,j,k+1} - H_x^n_{i,j,k}}{\Delta y} - j_z^n_{i-1,j,k+1} \right)
\]
3D FDTD: Magnetic field components
changing of index notation to integer indices

\[
H_x^{n+1}_{i-1, j+1, k+1} = H_x^n_{i-1, j+1, k+1} + \frac{\Delta t}{\mu_0} \left( \frac{E_y^n_{i-1, j+1, k+1} - E_y^n_{i-1, j+1, k}}{\Delta z} - \frac{E_z^n_{i-1, j+1, k+1} - E_z^n_{i-1, j+1, k+1}}{\Delta y} \right)
\]

\[
H_y^{n+1}_{i, j, k+1} = H_y^n_{i, j, k+1} + \frac{\Delta t}{\mu_0} \left( \frac{E_z^n_{i, j, k+1} - E_z^n_{i-1, j, k+1}}{\Delta x} - \frac{E_x^n_{i, j, k+1} - E_x^n_{i, j, k}}{\Delta z} \right)
\]

\[
H_z^{n+1}_{i, j+1, k} = H_z^n_{i, j+1, k} + \frac{\Delta t}{\mu_0} \left( \frac{E_x^n_{i, j+1, k} - E_x^n_{i, j+1, k}}{\Delta y} - \frac{E_y^n_{i, j+1, k} - E_y^n_{i, j+1, k}}{\Delta x} \right)
\]
3D FDTD: Field discretization and boundary conditions

Number of grid points in Yee grid:

\[
\begin{align*}
E_x^{n+\frac{1}{2}} (N_x, N_y + 1, N_z + 1) & \quad H_x^n (N_x + 1, N_y, N_z) \\
E_y^{n+\frac{1}{2}} (N_x + 1, N_y, N + 1) & \quad H_y^n (N_x, N_y + 1, N_z) \\
E_z^{n+\frac{1}{2}} (N_x + 1, N_y + 1, N) & \quad H_z^n (N_x, N_y, N_z + 1)
\end{align*}
\]

Boundary conditions for \( E \) und \( H \) fields:

\[
\begin{align*}
E_x (\cdot, 1, :) &= 0, & E_x (\cdot, \cdot, 1) &= 0 \\
E_x (\cdot, N_y + 1, :) &= 0, & E_x (\cdot, \cdot, N_z + 1) &= 0 \\
E_y (1, :, :) &= 0, & E_y (\cdot, \cdot, 1) &= 0 \\
E_y (N_x + 1, :, :) &= 0, & E_y (\cdot, \cdot, N_z + 1) &= 0 \\
E_z (1, :, :) &= 0, & E_z (\cdot, 1, :) &= 0 \\
E_z (N_x + 1, :, :) &= 0, & E_z (\cdot, N_y + 1, :) &= 0
\end{align*}
\]
Task I: Implementation of 1D FDTD method

Physical problem:
- Simulate propagation of an ultrashort pulse in a dispersion-free dielectric medium $\varepsilon(x)$
- See what happens when the pulse hits the interface between two different dielectric media

Excitation:
- current source (1A/m$^2$) with frequency $5\times10^{15}$Hz (red light) with delta-shaped spatial profile and Gauss-shaped temporal pulse profile of width $w_0=1$ fs

Simulation grid:
- spatial size of 18 $\mu$m with discretization $\Delta x=30$nm and metallic walls ($E_z=0$ at the boundaries)
- temporal size of 60 fs with discretization $\Delta t=\Delta x/(2c)$

Result output:
- plot $E_z$ and $H_y$ for every 5$^{th}$ calculation step

Useful Matlab functions:
- round, drawnow, subplot

$c=2.99792458\times10^8$, $\mu_0=4\pi\times10^{-7}$, $\varepsilon_0=1/(c^2\mu_0)$
Task I: Implementation of 1D FDTD method

function \([Ez,Hy,X,T] = \text{fdtd}_{1\text{d}}(\text{eps}_\text{rel}, \text{grid}_\text{size}, \text{time}_\text{span},...\]
\quad \text{source}_\text{frequency}, \text{source}_\text{position}, \text{source}_\text{pulse}_\text{length}\)

% FUNCTION CALL:
% \([Ez,Hy,X,T] = \text{fdtd}_{1\text{d}}(\text{eps}_\text{rel}, \text{grid}_\text{size}, \text{time}_\text{span},...\]
\quad \text{source}_\text{frequency}, \text{source}_\text{position}, \text{source}_\text{pulse}_\text{length}\)

% VARIABLES:
% \text{eps}_\text{rel} \quad \text{epsilon distribution in the simulation area}
% \text{grid}_\text{size} \quad \text{spatial size of simulation grid}
% \text{time}_\text{span} \quad \text{time span of simulation}
% \text{source}_\text{frequency} \quad \text{frequency of current source}
% \text{source}_\text{position} \quad \text{spatial position of current source}
% \text{source}_\text{pulse}_\text{length} \quad \text{temporal width of Gauss-shaped source}(w_0)
Physical problem:
- Watch radiation characteristics of pulsed Hertzian dipole

Simulation grid:
- spatial size of 101x121x3 grid points with discretization $\Delta x=\Delta y=\Delta z=30\text{nm}$
- metallic wall boundary conditions
- temporal size of 10 fs with discretization $\Delta t=\Delta x/(2c)$

Excitation:
- current source (1A/m$^2$) with frequency $5\times10^{15}\text{Hz}$ (CW red light)
- delta-shaped spatial profile in center of computation grid

Result output:
- graphical representation of $H_z$ in the x-y plane centered in the middle along the z direction
  (plot every 5$^{th}$ calculation step)

Possibly useful Matlab functions:
- pcolor, mod

Task II: Implementation of 3D FDTD method
function
\texttt{fdtd\_3d}(\texttt{eps\_rel}, \texttt{dr}, \texttt{time\_span}, \texttt{source\_frequency}, \texttt{source\_pulse\_length}, \texttt{jx}, \texttt{jy}, \texttt{hz}, \ldots \texttt{plot\_field\_component}, \texttt{plotlayer})

\% FUNCTION CALL
\% \texttt{fdtd\_3d}(\texttt{eps\_rel}, \texttt{dr}, \texttt{time\_span}, \texttt{source\_frequency}, \texttt{source\_pulse\_length}, \texttt{jx}, \texttt{jy}, \texttt{hz}, \ldots \texttt{plot\_field\_component}, \texttt{plotlayer})
\%

\% VARIABLES:
\% \texttt{eps\_rel}  epsilon distribution in the simulation area (3D array)
\% \texttt{dr}    spatial discretization of simulation grid
\% \texttt{time\_span}  time span of simulation
\% \texttt{source\_frequency}  frequency of current source
\% \texttt{source\_pulse\_length}  temporal width of Gaussian pulse
\% \texttt{jx}, \texttt{jy}, \texttt{hz}  components of source current (3D array)
\% \texttt{plot\_field\_component}  field component to be plotted
\% \texttt{plotlayer}  index of z slice, in which result is plotted in 2D
Task III*: FDTD movie

voluntary task

Output the result of the FDTD as a playable movie file!

Remark:
This functionality will depend on the video codecs installed on the specific system!

Possibly useful MATLAB functions:
• switch, avifile, getframe, addframe, close
Homework 4 (8 June 2015)

• Solve at least tasks I & II.
• Prepare a one page report about your solution with a figure of some calculated example.
• Submit your m-files of your program together with your one page report electronically to teaching-nanooptics@uni-jena.de by June 19, 2015.
• Please put everything together in one single email which contains your name (FAMILY NAME, Given Name) and matriculation number.
• Late submissions will not be accepted!
• On June 20 the solutions of the tasks will be available online at the lectures homepage www.iap.uni-jena.de/teaching >>> Computational Photonics.
• You are expected to solve the task yourself and a declaration of independent work must be signed by every student at the end of the semester.