Advanced Lens Design

Lecture 14: Diffractive elements
2013-01-28
Herbert Gross
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Contents

1. Basic principle
2. Diffraction orders and efficiency
3. Calculation models
4. Miscellaneous
5. Correction
6. Broadband solutions
7. Manufacturing
Deviation of Light

Mechanisms of light deviation and ray bending

- Refraction
- Reflection
- Diffraction according to the grating equation
- Scattering (non-deterministic)

\[
n \cdot \sin \theta = n' \cdot \sin \theta' \\
\theta = - \theta' \\
g \cdot \left( \sin \theta - \sin \theta_o \right) = m \cdot \lambda
\]
Diffractive Optics:

- Local micro-structured surface
- Location of ray bending: macroscopic carrier surface
- Direction of ray bending: local grating micro-structure
- Local aspherical phase effect

\[ \Phi(x, y) = k \cdot \sum_{j,m} c_{jm} \cdot x^j \cdot y^m \]

Local grating constant, \( q \) ist the wrapping factor

\[ g(x, y) = \frac{2\pi \cdot q}{|\nabla \Phi(x, y)|} \]
Diffractive Elements

- Original lens height profile $h(x)$
- Wrapping of the lens profile: $h_{red}(x)$  Reduction on maximal height $h_{2\pi}$
- Digitalization of the reduced profile: $h_{q}(x)$
- Usually all diffraction orders are obtained simultaneously
- Blazed structure: suppression of perturbing orders
- Unwanted orders: false light, contrast and efficiency reduced
- Efficiency optimized for one desired central wavelength
- Strong dispersion for other wavelengths
- Reduction of efficiency for deviating wavelengths

(figure: scalar approximation)
Efficiency of Binary DOE's

- Binarized diffractive elements have a reduced efficiency.
- The efficiency grows theoretically with the number of quantization levels.
- For 8 levels the efficiency is \( \eta = 0.95 \).
- In reality for a larger number of levels the efficiency shrinks due to tolerances.
- Estimation of N levels in scalar approximation.

\[
\eta = \sin^2 \left( \frac{1}{N} \right)
\]
Fresnel Zone Plate

- Circular rings at radii
  \[ r_m = \sqrt{2m \cdot f \cdot \lambda} \]

- Classical Fresnel zone lens:
  only rings with same sign of phase have transmission 1

- Modern zone plate (Wood):
  phase steps of \( \pi \) at the rings,
  improved power transmission
Diffractive Lens

- Diffractive Fresnel lens
- Zone rings with radii
- Blaze in every zone (surface slope)

\[ r_k = \sqrt{2\pi \cdot m \cdot k \cdot f \cdot \lambda} \]

\[ h_k = \frac{m \cdot \lambda}{n - \cos \theta_k} = (r_{k+1} - r_k) \cdot \tan \psi_k \]
Surface with grating structure:
new ray direction follows the grating equation

Local approximation in the case of space-varying grating width

\[ \tilde{s}' = \frac{n}{n'} \cdot \tilde{s} + \frac{m \lambda g}{n' d} \cdot \hat{g} + \gamma \cdot \hat{e} \]

Raytrace only into one desired diffraction order

Notations:
g : unit vector perpendicular to grooves
d : local grating width
m : diffraction order
e : unit normal vector of surface

Applications:
- diffractive elements
- line gratings
- holographic components
Modellierung Diffractive Elements

- Discrete topography on the surface
- Phase unwrapped to get a smooth surface
- Only one desired order can be calculated with raytrace
- Model approximation according to Sweatt: same refraction by small height and high index
Phase function redistributed:
large index / small height
typical: \( n = 10000 \)

Calculation in conventional software with raytrace possible

\[
\varphi(x, y) = 2\pi \cdot n \cdot z(x, y) = 2\pi \cdot n^* \cdot z^*(x, y)
\]
Fresnel zone plate: works as lens
- p-phase jumps at Fresnel zones,
  - only odd zones are considered
  - no destructive interference
- Refractive: no surface structure
- Blazed diffractive: only one diffraction order, for one wavelength and one angle of incidence
- Binary diffractive: several orders simultaneously
Transition Refractive - Diffractive

- Phase of refracting blaze grating: convolution of prism transmission with periodic interference function

\[ T(x) = T_{\text{prism}}^{(r/l)}(x) \otimes T_{\text{periodic}}^{(\text{diff})} \]

- Notation:
  - \( L \) total width
  - \( g \) length of period
  - \( \psi \) blaze angle
  - \( m \) index of periods
  - \( q \) order of phase jump with height \( h \)

- Complete transmission:
  1. one period with linear phase
  2. total width
  3. periodicity of cell

\[
T(x) = \text{rect}\left(\frac{x}{g}\right) \cdot e^{\frac{2\pi i \cdot \psi \cdot n - 1}{\lambda}} \otimes \text{rect}\left(\frac{x}{L}\right) \cdot \sum_{m} \delta(x - mg)
\]
Transition Refractive - Diffractive

- Higher number of periods
- Increasing order $q$
- Widths of orders decrease
- Limiting case: one prism, slit diffraction
- Lens with diffractive structured surface: hybrid lens
- Refractive lens: dispersion with Abbe number $\nu = 25...90$
- Diffractive lens: equivalent Abbe number
  \[ \nu_d = \frac{\lambda_d}{\lambda_F - \lambda_C} = -3.453 \]
- Combination of refractive and diffractive surfaces: achromatic correction for compensated dispersion
- Usually remains a residual high secondary spectrum
- Broadband color correction is possible but complicated
Dispersion by grating diffraction:
Abbe number

Relative partial dispersion

Consequence:
Large secondary spectrum

ν-P-diagram

\[ ν_e = \frac{λ_e}{λ_F' - λ_C'} = -3.330 \]

\[ P_{g,F'} = \frac{λ_g - λ_{F'}}{λ_{F'} - λ_{C'}} = 0.2695 \]
- Combination of DOE and aspherical carrier
- Upper part: conventional lens

- Lower part: lens with diffractive surface
Data:
- $\lambda = 193$ nm
- $\text{NA} = 0.65$
- $\beta = 50$
- $s_{\text{free}} = 7.8$ mm

Properties:
- short total track
- extreme large free working distance
- few lenses
Diffractive Optics: Hybrid Lens

- Straylight suppression
Broad Band DOE: Nearly Index Matching

- Nearly index matched materials

\[ \frac{\lambda_e}{\lambda_{F'} - \lambda_{C'}} \left( \frac{n_1(\lambda_e) - 1}{\nu_1} - \frac{n_2(\lambda_e) - 1}{\nu_2} \right) = n_1(\lambda_e) - n_2(\lambda_e) \]

- Height of second layer

\[ h = \frac{\lambda_e}{n_1(\lambda_e) - n_2(\lambda_e)} \]

- Efficiency

\[ \eta_{nim} = \text{sinc}^2 \left( \pi \cdot \frac{\lambda_o}{\lambda} \cdot \frac{n_1(\lambda) - n_2(\lambda)}{n_1(\lambda_o) - n_2(\lambda_o)} - \pi \cdot m \right) \]
Color Correction of a Hybrid System

- Principle of achromatic correction
- Ratio of Abbe numbers defines refractive power distribution
- Diffractive element: Abbe number $\nu = -3.45$

$$F_{\text{refr}} = F \cdot \frac{V_{\text{glas}}}{V_{\text{glas}} - V_{\text{diff}}}$$

$$F_{\text{refr}} = -F \cdot \frac{V_{\text{diff}}}{V_{\text{glas}} - V_{\text{diff}}}$$

- Diffractive element gets only approx. 5% of the refractive power

Diagram showing the correction of a hybrid system with bending angles and refractive and diffractive solutions.
Dispersion by grating diffraction:
Abbe number: small and negative!

Relative partial dispersion

Consequence:
Large secondary spectrum

ν-P-diagram

\[ \nu_e = \frac{\lambda_e}{\lambda_{F'} - \lambda_C} = -3.330 \]

\[ P_{g,F'} = \frac{\lambda_g - \lambda_{F'}}{\lambda_{F'} - \lambda_C} = 0.2695 \]
- Achromatic doublet
  Ratio of Abbe numbers and refractive powers

- Visible: 5% refractive power of DOE
  Infrared: strong variations depending on substrat material
Spherical Hybrid Achromate

- Classical achromate:
  - two lenses, different glasses
  - strong curved cemented surface

- Hybride achromate:
  - one lens
  - one surface spherical with diffractive structure
  - tolerances relaxed
Aspherical Hybrid Achromate

- Types of achromates
  a) Spherical surfaces un cemented
  b) Hybrid with aspherical and diffractive surface

- Relaxed tolerances for hybrid solution
Primary Aberrations of a Diffractive Lens

- Expansion of the optical pathlength for one field point:
  Primary Seidel aberrations:

\[ W(r) = \frac{2\pi}{\lambda} \left( \frac{r^2}{2f} - \frac{r^4}{8f^3} + \frac{wr^3}{2f^2} - \frac{3w^2r^2}{4f} \right) \]

- No field curvature
- No distortion (stop at lens)
- Ray bending in a plane corresponds to linear collineation
- Equivalent bending of lens

\[ X = 2f \cdot c_{\text{diff}} = \frac{c_{\text{diff}}}{m\lambda \cdot a_2} \]
Optimization of a Hybrid Lens

- Seidel spherical aberration of a hybrid lens ($A_4$: aspherical equivalent asphericity)

$$S_{sph} = S_{ref} + S_{dif} = \frac{y^4 F_{ref}^3}{4} \left[ \frac{n+2}{n(n-1)^2} + \left( \frac{n}{n-1} \right)^2 - \frac{4(n+1)}{n(n-1)} + \frac{3n+2}{n} \right] - 8m\lambda \cdot A_4 y^4$$

- Optimal bending: choice of $A_4$

$$A_4 = \frac{F_{ref}^3}{32m\lambda} \left[ \frac{n+2}{n(n-1)^2} + \left( \frac{n}{n-1} \right)^2 - \frac{4(n+1)}{n(n-1)} + \frac{3n+2}{n} \right]$$

- Large spherochromatism

$$\Delta s_{gauss} = \frac{\Delta s_{ref, sph}}{v_{dif}}$$

- Optimization of refractive power of the DOE (10% instead of 5%)

$$F_{dif} = F \cdot \left( \frac{v_{dif}}{v_{dif} - v_{ref}} - \frac{v_{ref}}{v_{dif} - v_{ref}} \cdot \frac{\Delta s_{ref, sph}}{f} \right)$$

$$F_{ref} = -F \cdot \left( \frac{v_{ref}}{v_{dif} - v_{ref}} - \frac{v_{ref}}{v_{dif} - v_{ref}} \cdot \frac{\Delta s_{ref, sph}}{f} \right)$$
- Large depth of relief: blaze-wavelengths numbers of orders: m, p

\[ \lambda_{\text{blaze}} = m \cdot \frac{\lambda_o}{n(\kappa_o) - 1} \cdot \frac{n(\lambda_{\text{blaze}}) - 1}{p} \approx \frac{m \cdot \lambda_o}{p} \]

- Gives a smooth uniform broad band efficiency (scalar approximation \( g \gg \lambda \))

![Graph showing efficiency against wavelength for different orders](image-url)
Broad Band Achromatization

- Different approaches for broad band achromatic correction with DOE
- Problems: 1. Efficiency
  2. Secondary spectrum

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<th>Bi-Blazing strategy</th>
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<tr>
<td>Broad band efficiency</td>
<td>high</td>
<td>medium</td>
<td>high</td>
<td>medium</td>
</tr>
<tr>
<td>Minimal number of components</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Minimal number of materials</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
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<tr>
<td>Deep structures necessary</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
<td>possible</td>
</tr>
<tr>
<td>Special effort for centering and adjustment</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
<td>no</td>
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- Division of aperture into rings with blazing structures for different wavelengths
- Two wavelength-design: bi-blaze
- Structures complicated, efficiency low
Diffractive Broadband Optic

- Double diffractive element sandwich
  Achromatization by adapted compensation of materials for 2 wavelengths
  Residual dispersion

\[ \Phi = \frac{2\pi}{\lambda} \left( d_1 \cdot \left[ n_1(\lambda) - n_g(\lambda) \right] + d_2 \cdot \left[ n_2(\lambda) - n_g(\lambda) \right] \right) \]

- Optimized depth of structures allows correction for 4 wavelengths

- Realized in Canon photographic lens
  Manufacturing properties critical too expansive for consumer products
Realizations of Diffractive Elements

- **DOE's**
  - **blazed DOE's**
    - Grating of index
      - Example: HOE
  - **quantized DOE's**
    - Surface contour
    - Multi phase level
    - Phase grating
    - Amplitude grating
    - Binary grating
Discretisation and Wrapping

- Phase reduced by \(2\pi m\)
  
  \(m\): wrapping height
  
  perhaps rather small feature sizes at the boundary (large slope)

- Changed wrapping number \(m\) from center to boundary to prevent from small sizes

- Discrete binarization, constant lateral feature size
- Scaling error in mask

- Typical Tolerances

Tolerances of Diffractive Elements

- Ideal profile
- Edge profile rounded
- Profile discrete approximated
Diffractive Optics: Tolerances

- Error of slope angle
- Significant decrease in efficiency
Diffractive Optics: Rounding of Edges

- Round edges in a Fresnel lens:
  - enlarged diffraction fringes
  - Encircled energy broadened

\[ E(r) \]

\[ \text{Log Ipsf(x) E(r)} \]

\[ \text{a b c d} \]
Structured Surfaces in 2D

- Diffractive elements
- Arrays
- Mot’s eye coatings
- Free form beam profiler
Structured Surfaces in 2D

- Statistical micro lens arrays for homogenizing
- Diffractive beam shaper
- Cylinder lens bar

Ref: Axetris Corp.

Ref: R. Steiner
Photonic Crystals

- Sub-wavelength structures

Ref: S. Richter

Ref: J. Joannopoulos
Different profile shapes of linear gratings
- General diffractive elements

Ref: Axetris Corp.

Ref: R. Steiner