Tilt operator for electromagnetic fields and its application to propagation through plane interfaces

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What Is Field Tilting?

• Assume an electromagnetic field is given on a plane in a homogeneous medium.
• A tilt operator calculates from that the field on a plane which is arbitrarily rotated to the initial plane.
Importance of Field Tilting

- Propagation operators provide fields on parallel planes.

- Various reasons to overcome that:
  - Detection of light on tilted planes
  - Tilted interfaces in case of mirrors, prisms, gratings, ...
**Plane Interface or Grating**

- Single plane wave

![Diagram showing incident plane wave, reflected plane wave, and transmitted plane wave. The diagram also includes a grating with labels indicating Fresnel equations or grating solver like FMM.](image-url)
Plane Interface or Grating

- General beam
• General beam
Importance of Field Tilting

• Propagation operators provide fields on parallel planes.

• Various reasons to overcome that:
  – Detection of light on tilted planes
  – Tilted interfaces in case of mirrors, prisms, gratings, ...
  – Tolerancing
Tolerancing

Free-space propagation

Component operator, e.g. geometrical optics for lens
Tolerancing

**Light Source**

Free-space propagation

Component operator, e.g. geometrical optics for lens
Tolerancing

Free-space propagation + tilt

Component operator, e.g. geometrical optics for lens
Importance of Field Tilting

- Propagation operators provide fields on parallel planes.

- Various reasons to overcome that:
  - Detection of light on tilted planes
  - Tilted interfaces in case of mirrors, prisms, gratings, ...
  - Tolerancing
Existing Tilt Technology


- Problem:
  - Procedure works with a non-equitidistantly sampled field in $k$-space.
  - Such an interpolation is not well defined.

- With our new method we have solved the problem.
Plane Wave Decomposition

- Plane wave decomposition
  - A general beam distribution on $x$-$y$ plane

\[
V_l(\rho) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} A_l(\kappa) e^{i\kappa \cdot \rho} \, dk_x \, dk_y ,
\]

with

\[
A_l(\kappa) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} V_l(\rho) e^{-i\kappa \cdot \rho} \, dx \, dy .
\]

\[
\rho = \begin{pmatrix} x \\ y \end{pmatrix}, \quad \kappa = \begin{pmatrix} k_x \\ k_y \end{pmatrix}
\]
Concept in *k*-Space

- First task: Find *k*-vector positions in both planes
- Second task: Find corresponding values of angular spectrum in initial plane
Concept in $k$-Space

- First task: Find $k$-vector positions in both planes.
Concept in $k$-Space

- First task: Find $k$-vector positions in both planes
- 1st projection: from the initial $\kappa$-plane to the surface of Edward sphere.
Concept in $k$-Space

- First task: Find $k$-vector positions in both planes
- 1st projection: from the initial $\kappa$-plane to the surface of Edward sphere.
- 2nd projection: from the surface of Edward sphere to tilded plane in $k$-space
- Conclusion: Pixel deformed and sampling non-equidistant
Algorithm Flow Tilt Operator

- **input angular spectrum**
- **equidistant \( \kappa \)-grid**
- **find a new equidistant grid**
- **non-equidistant \( \bar{\kappa} \)-grid**

\( \kappa \)-plane

\( \bar{\kappa} \)-plane
Algorithm Flow Tilt Operator

- Input angular spectrum
- Non-equidistant $\kappa$-grid
- Equidistant $\overline{\kappa}$-grid

$T^{-\frac{1}{2}}$
Concept in $k$-Space

- First task: Find $k$-vector positions in both planes
- Second task: Find corresponding values of angular spectrum in initial plane
Algorithm Flow Tilt Operator

- **input angular spectrum**
- **non-equidistant \( \kappa \) - grid**
- **Scaling factors**
- **equidistant \( \bar{\kappa} \) - grid**

\[
\begin{align*}
\kappa \text{- plane} & \quad k_x \quad k_y \\
\bar{\kappa} \text{- plane} & \quad \bar{k}_x \quad \bar{k}_y
\end{align*}
\]
Scaling

- Amplitude scaling takes ratio of pixel areas in tilted and initial plane into account.
- We derived an analytical expression for that!
Scaling Factor Calculation

- Boundaries on the Tilted $\vec{\kappa}$ - plane
  - On the tilted $\vec{\kappa}$ - plane, the projected boundaries are constructed by two curves from ellipse and two lines.
2. Scaling Factor Calculation

- Boundaries on the Tilted $\kappa$-plane
  - On the tilted $\kappa$-plane, the projected boundaries are constructed by two curves from ellipse and two lines.
  - Three steps to calculate the sampling area $s$:
    1. Area $A_+ = |\sin \beta| \int_{p_x-0.5\Delta k_x}^{p_x+0.5\Delta k_x} \sqrt{a_+^2 - k_x^2} \, dk_x$
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2. Scaling Factor Calculation

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  - On the tilted $\kappa$-plane, the projected boundaries are constructed by two curves from ellipse and two lines.

- Three steps to calculate the sampling area $s$:
  1. Area $A_+ = |\sin \beta| \int_{p_x-0.5\Delta k_x}^{p_x+0.5\Delta k_x} \sqrt{a_+^2 - k_x^2} \, dk_x$;
  2. Area $A_- = |\sin \beta| \int_{p_x-0.5\Delta k_x}^{p_x+0.5\Delta k_x} \sqrt{a_-^2 - k_x^2} \, dk_x$;
  3. Area $\Delta A = \cos \beta \Delta k_y \Delta k_x$.

$$S = A_+ + \Delta A - A_-$$
Scaling

- Amplitude scaling takes ratio of pixel areas in tilted and initial plane into account.
- We derived an analytical expression for that!
- Very fast and accurate!
Algorithm Flow Tilt Operator

- **input angular spectrum**
- **interpolation**
- **resampled input spectrum**
- **Scaling factors**
- **output angular spectrum**

- **non-equidistant \( \kappa \)-grid**
- **equidistant \( \overline{\kappa} \)-grid**
Examples

- **Input**
  - 45° - polarized Gaussian
  - Waist Radius 100×100 μm
  - 95×95 sampling points
Examples

- **Input**
  - 45° - polarized Gaussian
  - Waist Radius 100×100 μm
  - 95×95 sampling points

- **Simulations**
  - Input plane x-o-y
  - Tilting around x-axis
  - Output plane x-o-y
  - angle from 0° to 60°
  - With a step of 20°
Examples

Amplitude ($E_x$)

$\kappa_0 = (0, -4.04 \times 10^6)^T \text{ 1/m}$

$\kappa_0 = (0, -7.59 \times 10^6)^T \text{ 1/m}$

$\kappa_0 = (0, -1.02 \times 10^7)^T \text{ 1/m}$

Phase

$\frac{\pi}{2} = (0, 0)$

$\frac{\pi}{2} = (0, -7.59 \times 10^6)$

$\frac{\pi}{2} = (0, -1.02 \times 10^7)$
Algorithm Flow Tilt Operator

- Input angular spectrum
  - Non-equidistant $\kappa$-grid
  - Interpolation
  - Resampled input spectrum
  - Scaling factors
  - Equidistant $\bar{\kappa}$-grid
  - Output angular spectrum
Interpolation Comparison

• Non-equidistant Interpolation
  – To resample the input angular spectrum on a non-equidistant $\mathbf{\kappa}$-grid, certain interpolation techniques are applied
  – Rigorous interpolation: Sinc-pointwise interpolation
  – Approximations: Cubic 4p, Cubic 6p and Cubic 8p interpolation

• Comparison
  – The same input as previous simulations
  – Result from Sinc-pointwise interpolation as a reference
  – Compare the results from other interpolation techniques with the reference
## Interpolation Comparison

### Deviation from the Reference

<table>
<thead>
<tr>
<th>Interpolation Method</th>
<th>Tilting Angle $\beta$ (°)</th>
<th>20</th>
<th>40</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cubic 4P</td>
<td></td>
<td>0.005602 %</td>
<td>0.006940 %</td>
<td>0.000214 %</td>
</tr>
<tr>
<td>Cubic 6P</td>
<td></td>
<td>0.000696 %</td>
<td>0.000861 %</td>
<td>0.000025 %</td>
</tr>
<tr>
<td>Cubic 8P</td>
<td></td>
<td>0.000066 %</td>
<td>0.000076 %</td>
<td>0.000002 %</td>
</tr>
</tbody>
</table>
# Efficiency Tests

<table>
<thead>
<tr>
<th>Methods based on Rigorous Theory</th>
<th>Total Time¹ (s)</th>
<th>Kernel Loop² (s)</th>
<th>Deviation (%)</th>
<th>Sampling Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>SincP.W.</td>
<td>196.570</td>
<td>194.665</td>
<td>0</td>
<td>$207 \times 183$</td>
</tr>
<tr>
<td>Cubic(8p)</td>
<td>2.183</td>
<td>0.199</td>
<td>0.000151</td>
<td></td>
</tr>
<tr>
<td>Cubic(6p)</td>
<td>2.021</td>
<td>0.100</td>
<td>0.00148</td>
<td></td>
</tr>
<tr>
<td>Cubic(4p)</td>
<td>1.976</td>
<td>0.087</td>
<td>0.0169</td>
<td></td>
</tr>
<tr>
<td>Parabasal Approximation</td>
<td>2.104</td>
<td>0.025</td>
<td>0.00523</td>
<td>$207 \times 207$</td>
</tr>
</tbody>
</table>

¹Total Time: contains a FFT and a inverse FFT process.

²Kernel Loop: contains the kernel process of the tilt operation. For example, in the rigorous method the non-equidistant interpolation is performed within this loop.
Plane Interface Propagation

- An illustrative example
  - $x$ – polarized Gaussian input;
  - Propagated field (SPW);
  - Propagated field on interface (tilt);

\[ \kappa_0 = (0, -5.90 \times 10^6) T 1/m \]
An illustrative example
- Reflection at the interface;
- Rotation of reflected field;
- Propagation of rotated field (SPW);

\[
\kappa_0 = (0, -5.90 \times 10^6)^T \frac{1}{m}
\]
An illustrative example

- Transmission at the interface;
- Rotation of transmitted field;
- Propagation of rotated field (SPW).

$$\kappa_0 = (0, -5.90 \times 10^6)^T \frac{1}{m}$$
Plane Interface Propagation: Brewster Angle

- Brewster angle \((n_1 = 1, n_2 = 2)\)
  - At \(\theta_B = 63.43^\circ\) incidence;
  - Input: 45°-polarized Gaussian;
  - Reflection;
  - Transmission.
• Assume reflection interaction with plane interface or grating (zeroth order).
• Assume input and output planes are orthogonal to propagation direction.
• Conclusion in $k$-space: sampling grid is invariant
• No tilt operator necessary.
• Very efficient rigorous reflection algorithm follows.
**Treatment of Smooth Phase Terms**

<table>
<thead>
<tr>
<th>Operators</th>
<th>Analytical treatment of smooth phase</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Linear Phase</td>
</tr>
<tr>
<td><strong>Tilt</strong></td>
<td></td>
</tr>
<tr>
<td>Rigorous</td>
<td>Yes</td>
</tr>
<tr>
<td>Parabasal Approx.</td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Reflection</strong></td>
<td>Yes</td>
</tr>
<tr>
<td><strong>Planar Interface and Grating</strong></td>
<td>Yes</td>
</tr>
</tbody>
</table>

Numerically efficient treatment of operators on basis of parabasal decomposition technique (PDT).
Prism Modeling

- Prism modeling requires a sequence of free space and plane interface propagation steps.
Example: Five Wavelengths with Same Weight

- Input angle: 60°
Conclusion

- Tilt operator of great concern in physical optics modeling, e.g., for rigorous beam propagation through plane interfaces, prisms, and gratings.
- We presented a well-defined and fast rigorous method.
- Operators allow analytical treatment of linear phase terms. That allows efficient handling of smooth phase terms by parabasal field decomposition.
We implement techniques in new programmable components for VirtualLab™. Will be available for all VirtualLab™ users soon.