Physical Optics

Lecture 11: PSF engineering
2017-06-14
Herbert Gross
# Physical Optics: Content

<table>
<thead>
<tr>
<th>No</th>
<th>Date</th>
<th>Subject</th>
<th>Ref</th>
<th>Detailed Content</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>05.04.</td>
<td>Wave optics</td>
<td>G</td>
<td>Complex fields, wave equation, k-vectors, interference, light propagation, interferometry</td>
</tr>
<tr>
<td>2</td>
<td>12.04.</td>
<td>Diffraction</td>
<td>B</td>
<td>Slit, grating, diffraction integral, diffraction in optical systems, point spread function, aberrations</td>
</tr>
<tr>
<td>3</td>
<td>19.04.</td>
<td>Fourier optics</td>
<td>B</td>
<td>Plane wave expansion, resolution, image formation, transfer function, phase imaging</td>
</tr>
<tr>
<td>4</td>
<td>26.04.</td>
<td>Quality criteria and resolution</td>
<td>B</td>
<td>Rayleigh and Marechal criteria, Strehl ratio, coherence effects, two-point resolution, criteria, contrast, axial resolution, CTF</td>
</tr>
<tr>
<td>5</td>
<td>03.05.</td>
<td>Polarization</td>
<td>G</td>
<td>Introduction, Jones formalism, Fresnel formulas, birefringence, components</td>
</tr>
<tr>
<td>6</td>
<td>10.05.</td>
<td>Photon optics</td>
<td>D</td>
<td>Energy, momentum, time-energy uncertainty, photon statistics, fluorescence, Jablonski diagram, lifetime, quantum yield, FRET</td>
</tr>
<tr>
<td>7</td>
<td>17.05.</td>
<td>Coherence</td>
<td>G</td>
<td>Temporal and spatial coherence, Young setup, propagation of coherence, speckle, OCT-principle</td>
</tr>
<tr>
<td>8</td>
<td>24.05.</td>
<td>Laser</td>
<td>B</td>
<td>Atomic transitions, principle, resonators, modes, laser types, Q-switch, pulses, power</td>
</tr>
<tr>
<td>9</td>
<td>31.05.</td>
<td>Gaussian beams</td>
<td>D</td>
<td>Basic description, propagation through optical systems, aberrations</td>
</tr>
<tr>
<td>10</td>
<td>07.06.</td>
<td>Generalized beams</td>
<td>D</td>
<td>Laguerre-Gaussian beams, phase singularities, Bessel beams, Airy beams, applications in superresolution microscopy</td>
</tr>
<tr>
<td>11</td>
<td>14.06.</td>
<td>PSF engineering</td>
<td>G</td>
<td>Apodization, superresolution, extended depth of focus, particle trapping, confocal PSF</td>
</tr>
<tr>
<td>12</td>
<td>21.06.</td>
<td>Nonlinear optics</td>
<td>D</td>
<td>Basics of nonlinear optics, optical susceptibility, 2nd and 3rd order effects, CARS microscopy, 2 photon imaging</td>
</tr>
<tr>
<td>13</td>
<td>28.06.</td>
<td>Scattering</td>
<td>G</td>
<td>Introduction, surface scattering in systems, volume scattering models, calculation schemes, tissue models, Mie Scattering</td>
</tr>
<tr>
<td>14</td>
<td>05.07.</td>
<td>Miscellaneous</td>
<td>G</td>
<td>Coatings, diffractive optics, fibers</td>
</tr>
</tbody>
</table>

*D = Dienerowitz  
*B = Böhme  
*G = Gross*
Contents

PSF engineering

- Low Fresnel numbers
- High numerical aperture
- Apodization
- Special profiles and beams
- Superresolution
- Extended depth of focus
Perfect Point Spread Function

Circular homogeneous illuminated aperture:

- Transverse intensity:
  Airy distribution
  Dimension: $D_{\text{Airy}}$
  Normalized lateral coordinate:
  $\nu = \frac{2 \pi x}{\lambda NA}$

- Axial intensity:
  Sinc-function
  Dimension: Rayleigh unit $R_E$
  Normalized axial coordinate
  $u = \frac{2 \pi z n}{\lambda NA^2}$

$$I(0, \nu) = \left( \frac{2 J_1(\nu)}{\nu} \right)^2 I_0$$

$$D_{\text{Airy}} = \frac{1.22 \cdot \lambda}{NA}$$

$$I(u,0) = \left( \frac{\sin\left(\frac{u}{4}\right)}{u/4} \right)^2 I_0$$

$$R_E = \frac{n \cdot \lambda}{NA^2}$$
Axial and Lateral Ideal Point Spread Function

- Comparison of both cross sections

Ref: R. Hambach
Low Fresnel Number Focussing

- Small Fresnel number $N_F$:
  geometrical focal point (center point of spherical wave) not identical with best focus (peak of intensity)

- Optimal intensity is located towards optical system (pupil)

- Focal caustic asymmetrical around the peak location
Low Fresnel Number Focussing

- Example:
  focussing by a micro lens

- Data:
  - radius of aperture: 0.1 mm
  - wavelength: 1 μm
  - focal length: 10 mm
  - Numerical aperture: NA = 0.01
  - Fresnel number: 1

- Results:
  - highest intensity in front of geometrical focus

Ref: R. Hambach
- System with small Fresnel number: Axial intensity distribution $I(z)$ is asymmetric

$$I(z) = I_0 \cdot \left(1 - \frac{u}{2\pi \cdot N_F}\right)^2 \cdot \left(\frac{\sin \frac{u}{4}}{4}\right)^2$$

- Explanation of this fact: The photometric distance law shows effects inside the depth of focus

- Example microscopic 100x0.9 system: $a = 1\text{mm}, z = 100\text{ mm}$: $N_F = 18$
High-NA Focusing

- Transfer from entrance to exit pupil in high-NA:
  1. Geometrical effect due to projection (photometry): apodization

  \[ A = A_0 \cdot \frac{1}{\sqrt{4(1 - s^2 r^2)}} \]
  
  with
  
  \[ s = \sin u = \frac{NA}{n} \]

- Tilt of field vector components

  \[ A = \frac{A_0}{2} \left[ 1 + \sqrt{1 - s^2 r^2} \right] - \left( 1 - \sqrt{1 - s^2 r^2} \right) \cdot \cos 2\theta \]
Vectorial Diffraction at high NA

Linear Polarization

Pupil

\[\text{I}_{\text{ges}}\]
\[\text{I}_{\text{x}}\]
\[\text{I}_{\text{y}}\]
\[\text{I}_{\text{z}}\]
High-NA Point Spread Function

- Comparison of Psf intensity profiles for different models as a function of defocussing

![Graph showing comparison of PSFs for different models](image)

**NA = 0.98**

- **paraxial**
- **high-NA scalar**
- **high-NA vectorial**
Gaussian Illumination

- Known profile of gaussian beams

Ref: R. Hambach
Spherical Aberration

- Axial asymmetrical distribution off axis
- Peak moves

Ref: R. Hambach
Annular Ring Pupil

- Generation of Bessel beams

Ref:

R. Hambach

$r_p$

$r$

$z$

Pupil intensity

Intensity $I(r)$ at focal point, $z=10$

Ref:
Farfield of a ring pupil:
- outer radius \( a_o \)
- inner radius \( a_i \)
- parameter \( \varepsilon = \frac{a_i}{a_o} < 1 \)
- Ring structure increases with \( \varepsilon \)
- Depth of focus increases

\[
\Delta z = \frac{2\lambda}{n\sin^2 u \cdot (1 - \varepsilon^2)}
\]

Application:
Telescope with central obscuration

Intensity at focus

\[
I(x) = \frac{1}{(1 - \varepsilon^2)^2} \left[ \frac{2J_1(x)}{x} - \varepsilon^2 \frac{2J_1(\varepsilon x)}{\varepsilon x} \right]^2
\]
- Ring pupil illumination
- Enlarged depth of focus
- Lateral resolution constant due to large angle incidence
- Can not be understood geometrically
- Encircled energy curve:
  - steps due to side lobes
  - strong spreading, large energy content in the rings

**Graph: PSF for ring-Shaped Pupil**

**Equations:**

\[
E(r)\]

- \(\varepsilon = 0.01\)
- \(\varepsilon = 0.25\)
- \(\varepsilon = 0.35\)
- \(\varepsilon = 0.50\)
- \(\varepsilon = 0.70\)
Point Spread Function with Apodization

- Apodisation of the pupil:
  1. Homogeneous circle → Airy
  2. Gaussian → Gaussian
  3. Ring illumination → Bessel

- Psf in focus: different convergence to zero for larger radii

- Encircled energy: same behavior

- Complicated: Definition of compactness of the central peak:
  1. FWHM: Airy more compact as Gauss Bessel more compact as Airy
  2. Energy 95%: Gauss more compact as Airy Bessel extremly worse
Coherent Beam Profiling

- Circular symmetric setups
- Conservation of energy
- Setup:  
  - 1st asphere angle adapted distribution
  - intensity redistribution
  - 2nd asphere flattens the phase

\[
\int_{r=0}^{r} I_{in}(r) 2\pi r \, dr = \int_{r'=0}^{r'} I_{out}(r') 2\pi r' \, dr'
\]
Beam Profiling Calculation

- Beamprofiling
- Transition: Tophat into Gauss
Super Gaussian Profile

- Intensity profile as generalization of a gaussian profile
- More realistic synthetic shape, not a laser mode
- Description: additional parameter \( m \)
  - \( m = 2 \): Gaussian profile
  - \( m \to \infty \): tophat profile
  - arbitrary \( m \): flattened gaussian profile
- Case study:

\[
I(r) = I_o e^{-2 \left( \frac{r}{w} \right)^m}
\]
Free Space Propagation of Supergaussian Profile

- Free space propagation of a super Gaussian profile
- The profile shape is not invariant
Doughnut Profiles

- Doughnut ring modes:
  - special phase shaping
  - superposition of higher order modes

- Example:
  doughnut creating by twisted screw phase plate
Bessel Beams

- Diffraction of cone-shaped waves
- Generation by:
  - axicons
  - ring apertures
- Diffraction free beams
- Invariant during propagation
- Ring lens creates ring focus
- Fourier lens generates Bessel beam of finite length
- Apodization allows for an improved uniformity
Line Focus Generation with Ringlens

- Intensity

Bessel profile $I(r)$

intensity $I(r,z)$

line profile $I(z)$
Axicon Lens Combination

- Generation of a ring profile
- Axicon: cone surface with peak on axis
- Ringradius in the focal plane of the lens

\[ R = (n - 1) \cdot f \cdot \alpha \]

- Ring width due to diffraction

\[ \Delta R = \frac{1.22 \cdot f \cdot \lambda}{a} \]
Ring Generation by Twist

- Alternative way to generate a ring: spiral phase plate with intermediate twist

- Setup:
  - pair of spiral phase plates generate ring, twisted phase
  - pair of axicons changes aspect ratio and creates focussing with large free working distance
  - Bessel line focus in final plane

- Advantage:
  reduced alignment sensitivity of axicons
Synthetic Aperture

- Single PSF: jump of phase sign between the sidelobes
- Improved resolution by synthetic aperture: overlay of 4 apertures, clever compensation of PSF amplitudes

Ref: A. Decloux
Stimulated Emission Depletion (STED)

- Special geometry:
  1. Pumpbeam Gaussian
  2. Erase beam Doughnut
- Super resolution in central location

Quelle: P. Török
Stimulated Emission Depletion

- Experimental setup
Stimulated Emission Depletion

- Example 1

- Example 2

Quelle: P. Török
Characterization of a Caustic Shape

- Transverse plane:
  Width $\Delta x$, gain factor $G_T$

- Axial direction:
  Width $\Delta z$, gain factor $G_A$

- Height of the focus intensity, Strehl ratio $D_S$

- Relative height of sidelobes,
  Relative height of peak: $M = I_{\text{side}} / I_{\text{peak}}$

- Absolute transmission of power,
  For transmission filter with absorption

- Modified criteria for special applications,
  Example: intensity squared in case of confocal detection
Depth of Focus

- Schematic drawing of the principal ray path in case of extended depth of focus
- Where is the energy going?
- What are the constraints and limitations?
Depth of Focus

- Depth of focus depends on numerical aperture
  
  1. Large aperture: small depth of focus
  2. Small aperture: large depth of focus

Ref: O. Bimber
1. Psf engineering
   - Phase pupil mask
   - Amplitude pupil mask
   - Complex Pupil mask
2. Phase Pupil mask with digital restoration
3. Lippman imaging with arrays (plenoptical principle, integrated imaging)
4. Time multiplexing of a fast variable system

Considered here: cases 1. and 2.
<table>
<thead>
<tr>
<th>Pupil Mask Types for Extended Depth of Focus</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. <strong>Ring pupil</strong></td>
</tr>
<tr>
<td>Poon 1987</td>
</tr>
<tr>
<td>Ojeda-Castaneda 1988,</td>
</tr>
<tr>
<td>2. <strong>Toraldo rings</strong></td>
</tr>
<tr>
<td>Ben-Eliezer 2008, rings with complex</td>
</tr>
<tr>
<td>transmission Demenikov 2009, several rings,</td>
</tr>
<tr>
<td>fractal</td>
</tr>
<tr>
<td>3. <strong>Axicon</strong></td>
</tr>
<tr>
<td>...</td>
</tr>
<tr>
<td>4. <strong>Cubic Phase (CCP)</strong></td>
</tr>
<tr>
<td>Dowski 1994, asymmetric</td>
</tr>
<tr>
<td>Takahashi 2008, modified</td>
</tr>
<tr>
<td>Bagheri 2008, complex</td>
</tr>
<tr>
<td>5. <strong>Logarithmic</strong></td>
</tr>
<tr>
<td>George 2001</td>
</tr>
<tr>
<td>6. <strong>Polynomial, quartic...</strong></td>
</tr>
<tr>
<td>Sheppard 1988, Prasad 2003</td>
</tr>
<tr>
<td>7. <strong>Spoke segments</strong></td>
</tr>
<tr>
<td>Chang 2010</td>
</tr>
<tr>
<td>8. <strong>Analytic rational</strong></td>
</tr>
<tr>
<td>Caron 2008</td>
</tr>
<tr>
<td>9. <strong>Radial cos-mask</strong></td>
</tr>
<tr>
<td>Ojeda-Castaneda 1990</td>
</tr>
<tr>
<td>10. <strong>Diffractive high frequency</strong></td>
</tr>
<tr>
<td>...</td>
</tr>
<tr>
<td>11. <strong>Zernike mask</strong></td>
</tr>
<tr>
<td>...</td>
</tr>
<tr>
<td>12. <strong>Pixelized radial</strong></td>
</tr>
<tr>
<td>...</td>
</tr>
<tr>
<td>13. <strong>Cubic sinusoidal modulated</strong></td>
</tr>
<tr>
<td>Zhao 2010</td>
</tr>
<tr>
<td>14. <strong>Combined representations</strong></td>
</tr>
<tr>
<td>...</td>
</tr>
</tbody>
</table>
1. Transmission - Phase
   - Amplitude

2. Geometry:
   - asymmetric (cubic,...)
   - circular symmetric
   - azimuthal periodic

3. Spatial frequency:
   - smooth (logarithmic, cubic,...)
   - mid frequency (Zernike)
   - high frequency (diffractive)

4. Surface type: - smooth
   - steps in slope (segmented)
   - steps in height (binary)
Special EDF - Criteria

- Uniformity over $z$
- Axial gain factor of focal depth
- Strehl definition
- Peak height of side lobes
- Transverse resolution
- Power in the bucket, central peak power
- Energy transmission
- Contrast of image
- MTF-requirements
- OTF-requirements
Fundamental Properties and Limits of EDF

- Psf caustic engineering:
  Conservation of energy
  Exact uniformity ('light sausage') is physically not possible
  Sidelobes and contrast reduction ('diffraction noise') physically necessary
  Critical transition: inside EDF-interval (uniformity) and outside (no straylight)

- Performance optimization:
  Threshold of detection (eye) suppresses sidelobes
  Confocal sidelobe suppression
  Improvement by axial destruction of coherence
  Digital reconstruction of a nearly perfect image
  Selective detection by human brain (?)

- Critical balance between EDF-factor, resolution and contrast necessary
Toraldo Ring Masks

- Ring shaped masks according to Toraldo:
  - discrete rings
  - absorbing rings or pure phase shifts
  - original setup: only $0/\pi$ values of phase
  - special case
    Fresnel zone plate

- Pure phase rings:
  amplitude of psf

$$E(r') = \sum_{j=1}^{n} e^{2\pi i \Phi_j} \left[ \varepsilon_j^2 \cdot \frac{2J_1(kr' \sin u' \varepsilon_j)}{kr' \sin u' \varepsilon_j} - \varepsilon_{j-1}^2 \cdot \frac{2J_1(kr' \sin u' \varepsilon_{j-1})}{kr' \sin u' \varepsilon_{j-1}} \right]$$
Toraldo Mask for Pupil Engineering

- Here: pure phase mask

Ref: R. Hambach
EDF with Complex Toraldo Mask

- $I(r,z)$
- $I(x)$
- $I(z)$

$I(z)$ depth for 80%: 13 RE
• Absorption filter with profil

• Corresponds to a linear sequence of single psf-peaks

• Transmission quite small

\[ T = \frac{1}{(2m + 1)^2} \]

• Extrem good edf performance

\[
P(r) = \frac{1}{2m+1} \left[ 1 + 2 \sum_{n=1}^{m} (-1)^n \cdot \cos(2\pi n \cdot r^2) \right]
\]
EDF with Chirped Ring Pupil

- Example with $m = 8$ peaks
Pupil Zernike, phase and apodization, depth of focus $12 \, R_E$

**Phase**

**Apodisation**

$x,z$ \, Dz=12.47849 \, Gz=0.142 \, Dr=1.22931 \, Gr=0.686$
System with EDF can be defined as an enlarged constant axial OTF-distribution.

With clear pupil function

With manipulated pupil function

EDF and MTF-Profile

Ref: S. Förster
Continuous Phase Mask for Microscope

MTF and bar pattern as a function of defocus

ideal

with Mask
Phase Mask with cubic polynomial shape

\[ P(x) = \begin{cases} e^{i\alpha x^3} & \text{für } |x| \leq 1 \\ 0 & \text{sonst} \end{cases} \]

Effect of mask:
- depth of focus enlarged
- Psf broadened, but nearly constant
- Deconvolution possible

Problems:
- variable psf over field size
- noise increased
- finite chief ray angle
- broadband spectrum in VIS
- Imageartefacts
Cubic Phase Mask: PSF and OTF

Conventional imaging

System with cubic phase mask
Cubic Phase Mask : Psf

Defocus

primary

with mask

with mask deconvolved
MTF for Cibic Phase Plate

- System with cubic phase plate
- MTF good only along axes
- Poor resolution in 45° directions
- Orientation dependent resolution with image perturbation
Extended Depth of Focus: Microscopic Imaging

- Conventional microscopic image

- Image with phase mask with / without deconvolution

Ref: E. Dowski
- Cubic phase plate
  - small changes of MTF with defocus
  - oscillation of real- and imaginary part:
  - strong change of PTF
- The phase of the Fourier components is not properly reconstructed:
  artefacts in image deconvolution
Real behavior of MTF:
1. invariant MTF for lower spatial frequencies
   but re-magnification also increases noise
2. decrease for higher spatial frequencies:
   reduction in resolution
EDF-Masks

Different types of masks:
1. Shape
2. Phase only - complex
3. Axisymmetry - without symmetry

Ref: S. Förster
EPI-Fluorescence (Molecular Probes FluoCells)

Ref: L. Höring

Standard
NA=0.66

\[
A = \frac{0.61 \cdot \lambda}{NA} = 0.50 \mu m
\]

EDF
NA=0.66
as measured

EDF
NA=0.66
reconstructed

20\mu m