Medical Photonics Lecture
Optical Engineering

Lecture 14: Metrology
2017-02-02
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- Collimated incident light
- Calibrated collimator with focal length $f_c$ and test chart with size $y$
- Selection of sharp image plane
- Analysis of image size

$$f' = -f_c \cdot \frac{y'}{y}$$
Setup with distance object-image $L > 4f$
- Known location of the principal plane $P$ of the system
- Distance $d_p$ between principal planes
- Selection of two system locations with sharp image
- Relative axial shift $D$ between the two setups

$$f = \frac{L - d_H}{4} - \frac{D^2}{4(L - d_H)}$$
Telecentric movable measurement microscope with offset $y$

Focusing of two different test charts with sizes $y_1$ and $y_2$

Determination of the focal length by

\[ \tan u = \frac{y}{f} = \frac{y_2 - y_1}{e} \]
- Setup with fiber and plane mirror for autocollimation
- Change of distance between test lens and fiber
- Analysis of the recoupled power into the fiber (confocal) gives the focal point

Measurement of Focal Length by Confocal Setup
- Afocal setup with sharp image plane
- Measurement of long focal lengths
- Insertion of test system in collimated light segment and refocussing
- Applying the lens makers formula

\[ \frac{1}{f} = \frac{1}{f_2} - \frac{x}{f_2^2} \]
- Setup with collimator and two Ronchi rulings
- System under test is inserted
- Grating period $d$ and azimuthal angle $\theta$ between the gratings
- Moire pattern is rotated by angle $\alpha$, if test lens acts as focussing element
- Radius of curvature $R$ or focal length

\[ R = \frac{d}{\theta \cdot \tan \alpha} \]
Setup with Ronchi grating in collimated light gives a series of Talbot images.
The Talbot planes are imaged by the system under test.
Analysis of the image plane by lens formula gives the desired focal length.
By use of several planes, the position of the principal plane can be eliminated.
A second Ronchi grating can be used to find the accurate image planes.

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Criteria for best focus:

1. Paraxial centre of curvature for the paraxial spherical wave of an on-axis object point.
2. Maximum of the Strehl ratio
3. Smallest rms-value of the wave aberration
4. Highest contrast of the modulation of an object feature of given spatial frequency
5. Highest value of the slope of an edge
6. Highest value of the entropy of the detected digital image

Requirements for focus detection procedure

1. Steep curve dependency to get high accuracy
2. Robust definition to deliver a large dynamic range
3. Suppression of side lobe effects to guarantee an unambiguous solution
4. High frequency pre-filtering to be noise insensitive
Determination of Best Focus

- Blur of defocussed plane

- Minimum of image entropy

\[ E = - \sum w_j \cdot \log_2 w_j \]

- Maximum of image contrast
Determination of Best Focus

- Phase analysis by Zernike coefficient $c_4$

$$c_4 = -\frac{1}{4n\lambda} \cdot \Delta z \cdot NA^2$$

Measurement with two Ronchi gratings
Determination of Best Focus

- Measurement by image analysis:

  1. Maximum gradient of edges
     \[ g = |\nabla I(x, y)| = \sqrt{\left(\frac{\partial I}{\partial x}\right)^2 + \left(\frac{\partial I}{\partial y}\right)^2} \]

  2. Power of gradients
     \[ G = \iiint |\nabla I(x, y)|^2 \, dx \, dy \]

  3. Laplacian
     \[ L = \iiint |\nabla^2 I(x, y)|^2 \, dx \, dy \]
- Measurement for systems in air via the nodal planes
- Imaging of a test pattern with a collimator onto a detector
- Invariant lateral image location for rotated system around the nodal point
- Critical: vignetting effects for large angles
Setup of the test lens with different object locations: axial shift $D$

Analysis of the lens imaging formula

$$\frac{1}{a_j + \Delta} + \frac{1}{a_j' - \Delta} = \frac{1}{f}$$

Minimizing the error of several measurements $j$

$$\delta = f \cdot (a_j + a_j') + \Delta \cdot (a_j - a_j') - a_j a_j' + \Delta^2$$
Measurement of Pupil Size

- Setup with collimating auxiliary lens

\[
\tan u = \frac{D}{2f}
\]

- Determination with measuring microscope (dynameter)
- Setup with Ronchi grating
- Measurement of the lateral shift of higher diffraction orders at distance $z$

- High-NA in microscopy: $NA > 1$
  Measurement of total internal reflection of fluorescence light
- Measurement of object sided telecentricity errors by lateral shift of image location during defocussing

- High accuracy measurement by interferometry and measurement of Zernike coefficients $c_{2/3}$. 
Measurement of Lens Position

- Measurement of reflexes at lens vertex points
- Analysis of confocal signal in autocollimation
- Avoiding spherical aberration induced errors by ring illumination

![Diagram of lens measurement setup]

- Laser source
- Beam splitter
- Confocal pinholes
- Movable sensor for focusing on surface vertex
- Confocal difference signal
- Graph showing changes in signal (\(\Delta I\)) with distance (z)
- Distance between surface vertices (e.g., \(S_1, S_2, S_3, S_4\))
Autocollimation

- Measurement of tilt errors (plane or spherical surface) in autocollimation
- Projection of the cross
- Observation of lateral shift in Fourier plane \[ x = f \cdot 2\varphi \]
Test of Prism Angles

- Measurement of 90° angle in air
- Wedge interferogram
- Kink of fringes gives angle error

\[ \alpha = \frac{m \cdot \lambda}{2D} \]
- Projection of test marker
- Autocollimation of sharp image, focal point coincides with center of curvature of surface with radius $r$
- Rotation of test system: tilt of surface induces a lateral shift of the image
- Problems with inner surfaces

$$v = \beta \cdot v_M = 2 \cdot \beta \cdot r \cdot \kappa$$
Measurement of Centering Errors in Transmission

- Thin collimated beam through lens
- Focussing of the beam onto detector
- Measurement of wedge angle by lateral shift \( v \)
- Tilt angle of lens not detectable
- Not feasible for very short focal lengths

\[
\varphi = (n - 1) \cdot \theta = (n - 1) \cdot (\alpha_1 - \alpha_2) = \frac{v}{f}
\]
Reasons for reduced system transmission:
1. Absorption in the bulk material of the components
2. Scattering in the bulk materials by inclusions or finite scattering parameters
3. Absorption in the coatings of the surfaces
4. Partial reflection or transmission at the coatings at transmissive or reflective surfaces
5. Blocking of light via mechanical or diaphragm parts of the system due to vignetting
6. Scattering of light by local surface imperfections or non-perfect polished surfaces
7. Deflection of light by diffraction of the light at edges
8. Deflection of light in unwanted higher orders of diffractive elements

Usually strong dependency on:
1. field position
2. wavelength of light
3. used pupil location
4. polarization

Critical:
1. absolute values for test lens
2. influence of auxiliary components
3. change of vignetting and incidence angles
**Measurement of Transmission**

*Measurement of transmission:*

a) Calibration setup

b) Measurement setup

\[
T = \frac{P_{in}}{P_{out}}
\]

*Reasons for measurement errors:*

1. Absorption in the component materials
2. Absorption in the coatings
3. Finite reflectivity of the coatings
4. Vignetting of the aperture bundle for oblique chief rays
5. Natural vignetting according for oblique chief rays and projection of tilted planes
6. False light from surrounding light sources, which reach the image plane
7. Scattering of light at components of the system mechanical design
8. False light due to ghost images or narcissus in infrared systems
Measurement of unwanted light: 2 different approaches:

1. object area black, surrounded by bright source
detection of irradiance in image region

2. intensive isolated point light source in the object plane at different locations
detection of artificial distributions in the image area: glare spread function
Tactile Measurement

- Scanning method
  - Sapphire sphere probes shape
  - slow
  - only some traces are measured

- Universal coordinate measuring machine (CMM) as basic engine

- Contact can damage the surface

- Accuracy 0.2 μm in best case

Ref: H. Hage / R. Börret
Tactile Measurement

- Influence of the tast sphere: filtering of higher spatial frequencies due to finite size
- Gradient of skew touching geometry can be calibrated and corrected
- Periods of same size as the sphere: error in amplitude
- High frequencies: not detected
Measurement by Fringe Projection

- Projection of a light sheet onto a deformed surface

- Corresponds to one fringe in more complicated pattern projection
Example Fringe projection

- Monochromatic illuminated technical surface
3D Shape Measurement for Biometry

- Colored biometric fringe projection
- Projection of a 2D triangular pattern
Shape Measurement by Fringe Projection

- Shape measurement of a surface
- Projection of a fringe pattern onto the surface
- Observation of the fringe deformation by a camera
  A shift corresponds to a change in depth
- Non-trivial image processing
Fringe Projection

Data evaluation
- Fringe period
  \[ d_x = \frac{d}{\cos \theta_1} \]
- Lateral shift
- Corresponding depth value \( z \)

\[ u = z \cdot (\cos \theta_1 + \cos \theta_2) \]

\[ f(x) = \frac{u}{d_x} = \frac{z \cdot (\cos \theta_1 + \cos \theta_2)}{\frac{d}{\cos \theta_1}} = \frac{z}{d} \cdot \frac{\sin(\theta_1 + \theta_2)}{\cos \theta_2} \]
- Projection of a narrow beam onto a surface
  Backscattering of light
- Observation under an oblique angle:
  Lateral shift of image point corresponds to depth $\Delta z$
- Scattered light amplitude depends strongly on material properties
- Problems with polished surfaces

\[ \Delta z = \frac{C \cdot \lambda}{2\pi \cdot \sin \theta \cdot \sin u} \]
- Measurement of the refractive index of a liquid
- Thin film of test liquid between prisms, adjustment of total internal reflection
- Special setup with direct sight prisms, no color fringes
Ellipsometry for Polarization Measurement

- Measurement of material data by polarization
- Incident linear polarized light
- Reflected light elliptical polarized
- Compensation of ellipticity, quantitative determination of null-test parameter
Two Beam Interference

- Two beam interference of two waves:
  - propagation in the same direction
  - same polarization
  - phase difference smaller than axial length of coherence

- Coherent superposition of waves

- Difference of phase / path difference

- Number of fringes
  location of same phase

- Contrast

\[
I = |E_1 + E_2|^2
= I_1 + I_2 + 2\sqrt{I_1 \cdot I_2} \cdot \cos \Delta \varphi_{12}
\]

\[
\Delta s = \frac{\lambda}{2\pi} \cdot \Delta \varphi_{12}
\]

\[
N = \frac{\Delta \varphi_{12}}{2\pi} = \frac{\Delta s}{\lambda}
\]

\[
K = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} = \frac{2\sqrt{I_1 \cdot I_2}}{I_1 + I_2}
\]
Testing with Fizeau Interferometer

- Long common path, quite insensitive setup
- Autocollimating Fizeau surface quite near to test surface, short cavity length
- Imaging of test surface on detector
- Straylight stop to block unwanted light
- Curved test surface: auxiliary objective lens (aplanatic, double path)
- Highest accuracy
Testing with Twyman-Green Interferometer

- Short common path, sensible setup
- Two different operation modes for reflection or transmission
- Always factor of 2 between detected wave and component under test

1. mode: lens tested in transmission auxiliary mirror for auto-collimation
2. mode: surface tested in reflection auxiliary lens to generate convergent beam
Test by Newton Fringes

- Reference surface and test surface with nearly the same radii
- Interference in the air gap
- Reference flat or curved possible
- Corresponds to Fizeau setup with contact
- Broad application in simple optical shop test
- Radii of fringes

\[ r_m = \sqrt{mR\lambda} \]

Ref: W. Osten
Interferograms of Primary Aberrations

- Spherical aberration $1\lambda$
- Astigmatism $1\lambda$
- Coma $1\lambda$

Defocussing in $\lambda$:
- $-1$
- $-0.5$
- $0$
- $+0.5$
- $+1$
Problems in real world measurement:

- Edge effects
  Definition of boundary

- Perturbation by coherent stray light

- Local surface error are not well described by Zernike expansion

- Convolution with motion blur

Ref: B. Dörband
Critical definition of the interferogram boundary and the Zernike normalization radius in reality
- Color fringes of a broadband interfergram

Ref: B. Dörband
Shearing Interferometer

- Separation of wavefront: self reference
- Interferograms are looking completely different
- Aperture reduced due to shear
- Splitting and shift of wavefront:
  - by thin plate
  - by grating
- Focussing onto a transparent plate with pinhole
- Pinhole creates a reference spherical wave
- Optimization of contrast:
  - size of pinhole
  - numerical aperture
  - transparency of the plate
- Very stable setup

**Point Diffraction Interferometer**

- transparent plate with pinhole
- wavefront under test
- reference wavefront
Temporal Coherence

- Radiation of a single atom:
  
  Finite time $\Delta t$, wave train of finite length,
  
  No periodic function, representation as Fourier integral
  
  with spectral amplitude $A(\nu)$

- Example rectangular spectral distribution

- Finite time of duration: spectral broadening $\Delta \nu$, schematic drawing of spectral width

- Corresponding axial coherence time

  $$\tau_c = \frac{1}{\Delta \nu}$$

- Axial coherence length

  $$l_c = c \cdot \tau_c$$

\[ E(t) = \int A(\nu) \cdot e^{2\pi i \nu t} \, d\nu \]

\[ A(\nu) = \frac{\sin(\pi \cdot \nu \cdot \Delta t)}{\pi \cdot \nu \cdot \Delta t} \]

$\Delta \nu = 1/\Delta t$
Lateral and Axial Resolution

- Intensity distributions
- Aberration-free Airy pattern: lateral resolution
  \[ D_{\text{Airy}} = \frac{1.22 \cdot \lambda}{NA} \]
- Axial resolution
  \[ R_E = \frac{n \cdot \lambda}{NA^2} \]

Ref: U. Kubitschek
OCT Setup

- Basic principle of OCT
- Michelson interferometer
- Time domain signal
Example:
- sample with two reflecting surfaces

1. Spatial domain

2. Complete signal

3. Filtered signal
   - high-frequency content removed

Ref: M. Kaschke
Example of OCT Imaging

Example:
Fundus of the human eye
Optical Coherence Tomography

- Dimensions of OCT imaging:
  a) only depth (A-scan), one-dimensional
  b) depth and one lateral coordinate B-scan), two-dimensional
  c) all three coordinates, volume imaging

Ref: M. Kaschke
White Light Interferometry

- Examples

Ref: R. Kowarschik
Knife Edge Method

- Moving a knife edge perpendicular through the beam cross section

- Relationship between power transmission and intensity: Abel transform for circular symmetry

\[ P(x) = 2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{I(r) r dr}{\sqrt{r^2 - \xi^2}} d\xi \]

- Example: geometrical spot with spherical aberration

before caustic
zone rays below
near paraxial focus
- Measurement of an edge image
- Evaluating the derivative:
  Line spread function
- Fourier transform:
  optical transfer function

\[ I_{LSF}(x') = \frac{d I_{ESF}(x')}{dx'} \]

\[ H_{OTF}(s) = \hat{F}[I_{LSF}(x')] \]
Setup:
Imaging of a grating

Possible realizations:
1. Density type grating, the sine wave is modelled by gray levels
2. Area type gratings, the sine wave is modelled by geometrical sine-shaped structures

Area coded sine grating:
- Lenslet array divides the wavefront into subapertures
- Every lenslet generates a single spot in the focal plane
- The averaged local tilt produces a transverse offset of the spot center
- Integration of the derivative matrix delivers the wave front $W(x,y)$
Hartmann Shack Wavefront Sensor

- Typical setup for component testing

- Lenslet array

![Diagram of typical setup for component testing]

![Diagram of lenslet array]
Spot Pattern of a HS - WFS

- Aberrations produce a distorted spot pattern
- Calibration of the setup for intrinsic residual errors
- Problem: correspondence of the spots to the subapertures

a) spherical aberration  
b) coma  
c) trefoil aberration
Real Measurement of a HS-WFS

- Problem in practice: definition of the boundary
Separated spots in case of diffraction

\[
d_{s}^{(gesamt)} = \frac{d_{2}}{d_{1}} \cdot d_{s} + \left( d_{1} + d_{2} \right) \cdot \frac{D_{\text{obj}}}{f} + \left( d_{1} + d_{2} \right) \cdot \frac{2.44 \cdot \lambda}{d_{s}}
\]
Hartmann Method

- Real pinhole pattern with signal
- Problems with cross talk and threshold