Institute of
Applied Physics
Friedrich-Schiller-Universität Jena

Optical Design with Zemax

Lecture 9: Imaging

2013-01-08

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9 Imaging
Time schedule

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1	16.10.	Introduction	Introduction, Zemax interface, menues, file handling, preferences, Editors, updates, windows, Coordinate systems and notations, System description, Component reversal, system insertion, scaling, 3D geometry, aperture, field, wavelength
2	23.10.	Properties of optical systems I	Diameters, stop and pupil, vignetting, Layouts, Materials, Glass catalogs, Raytrace, Ray fans and sampling, Footprints
3	30.10.	Properties of optical systems II	Types of surfaces, Aspheres, Gratings and diffractive surfaces, Gradient media, Cardinal elements, Lens properties, Imaging, magnification, paraxial approximation and modelling
4	06.11.	Aberrations I	Representation of geometrical aberrations, Spot diagram, Transverse aberration diagrams, Aberration expansions, Primary aberrations,
5	13.+27.11.	Aberrations II	Wave aberrations, Zernike polynomials, Point spread function, Optical transfer function
6	04.12.	Advanced handling	Telecentricity, infinity object distance and afocal image, Local/global coordinates, Add fold mirror, Vignetting, Diameter types, Ray aiming, Material index fit, Universal plot, Slider,IO of data, Multiconfiguration, Macro language, Lens catalogs
7	11.12.	Optimization I	Principles of nonlinear optimization, Optimization in optical design, Global optimization methods, Solves and pickups, variables, Sensitivity of variables in optical systems
8	18.12.	Optimization II	Systematic methods and optimization process, Starting points, Optimization in Zemax
9	08.01	Imaging	Fundamentals of Fourier optics, Physical optical image formation, Imaging in Zemax
10	15.01.	Illumination	Introduction in illumination, Simple photometry of optical systems, Non-sequential raytrace, Illumination in Zemax
11	22.01.	Correction I	Symmetry principle, Lens bending, Correcting spherical aberration, Coma, stop position, Astigmatism, Field flattening, Chromatical correction, Retrofocus and telephoto setup, Design method
12	29.01.	Correction II	Field lenses, Stop position influence, Aspheres and higher orders, Principles of glass selection, Sensitivity of a system correction, Microscopic objective lens, Zoom system
13	05.02.	Physical optical modelling	Gaussian beams, POP propagation, polarization raytrace, coatings

Contents

9 Imaging
Definitions of Fourier Optics

- Phase space with spatial coordinate x and
  1. angle θ
  2. spatial frequency ν in mm^{-1}
  3. transverse wavenumber k_x

$$\theta_x = \lambda \cdot \nu_x = \frac{k_x}{k_0}$$

$$k = 2\pi\nu$$

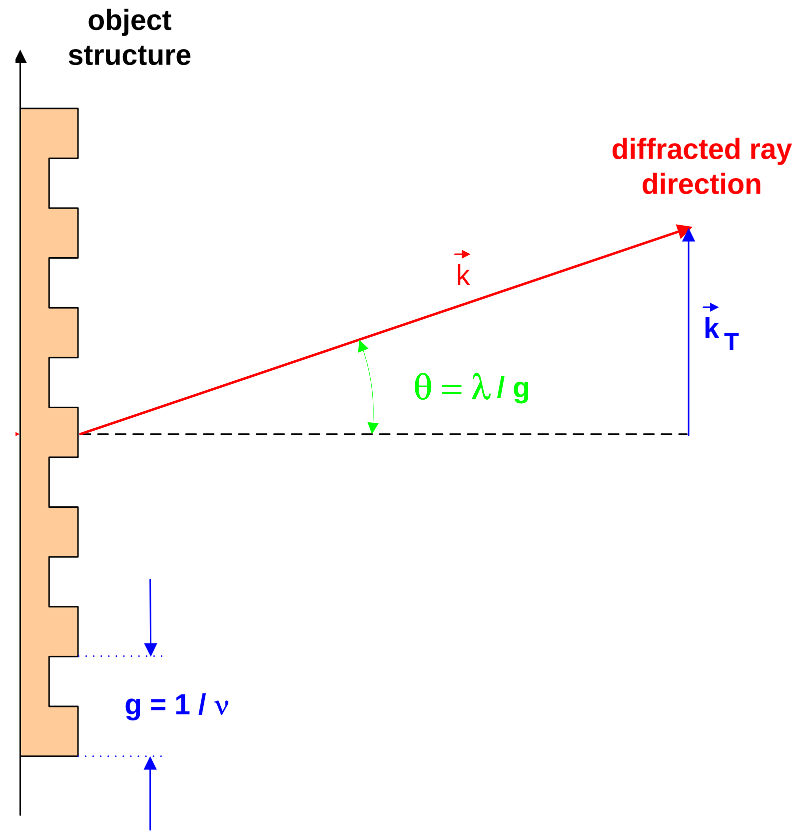
- Fourier spectrum $A(\nu_x, \nu_y) = \hat{F}\left[E(x,y)\right]$

  corresponds to a plane wave expansion

$$A(k_x, k_y, z) = \iint E(x,y,z)\, e^{-i\left(xk_x + yk_y\right)}\, dx\, dy$$

- Diffraction at a grating with period g:
  deviation angle of first diffraction order varies linear with ν = 1/g

$$\sin\theta = \lambda \cdot \frac{1}{g} = \lambda \cdot \nu$$



9 Imaging
Grating Diffraction and Resolution

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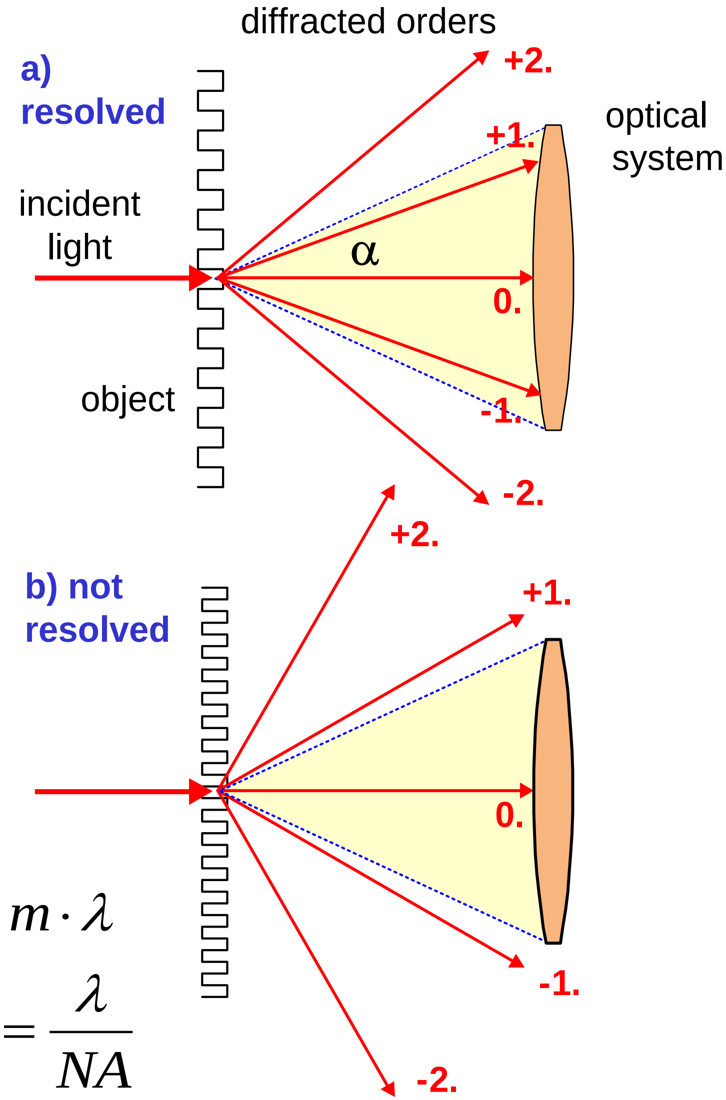
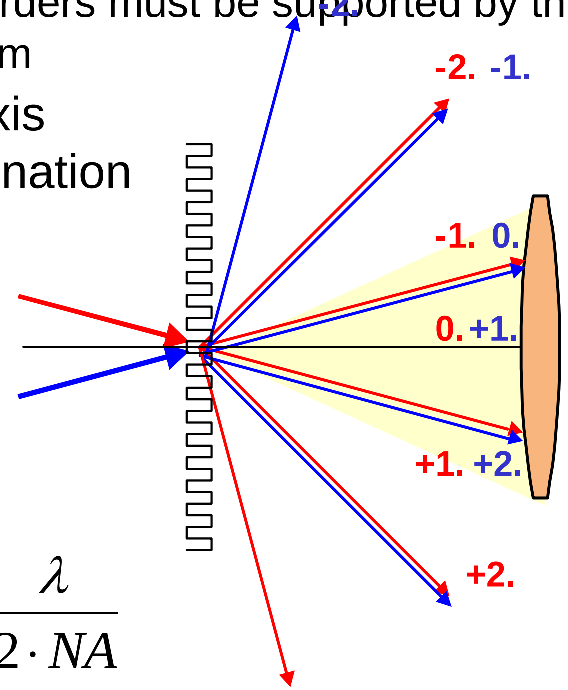


- Arbitrary object expaneded into a spatial frequency spectrum by Fourier transform
- Every frequency component is considered separately
- To resolve a spatial detail, at least two orders must be supported by the system

off-axis illumination



$$g \cdot \sin\alpha = m \cdot \lambda$$

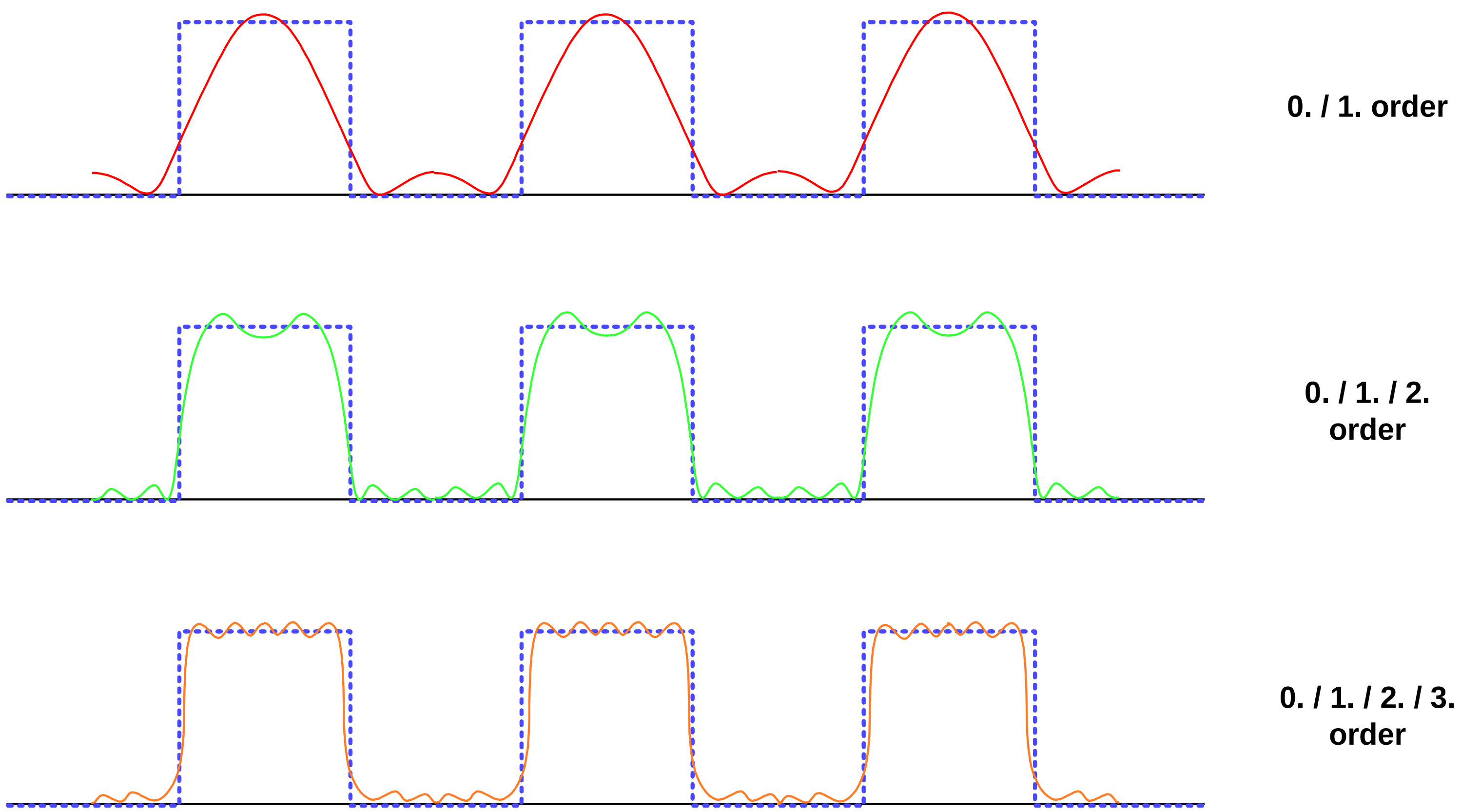
$$g = \frac{\lambda}{\sin\alpha} = \frac{\lambda}{NA}$$

$$g = \frac{\lambda}{2 \cdot NA}$$

Ref: M. Kempe

Resolution: Number of Supported Orders

- A structure of the object is resolved, if the first diffraction order is propagated through the optical imaging system
- The fidelity of the image increases with the number of propagated diffracted orders



9 Imaging
Oblique Illumination in Microscopy

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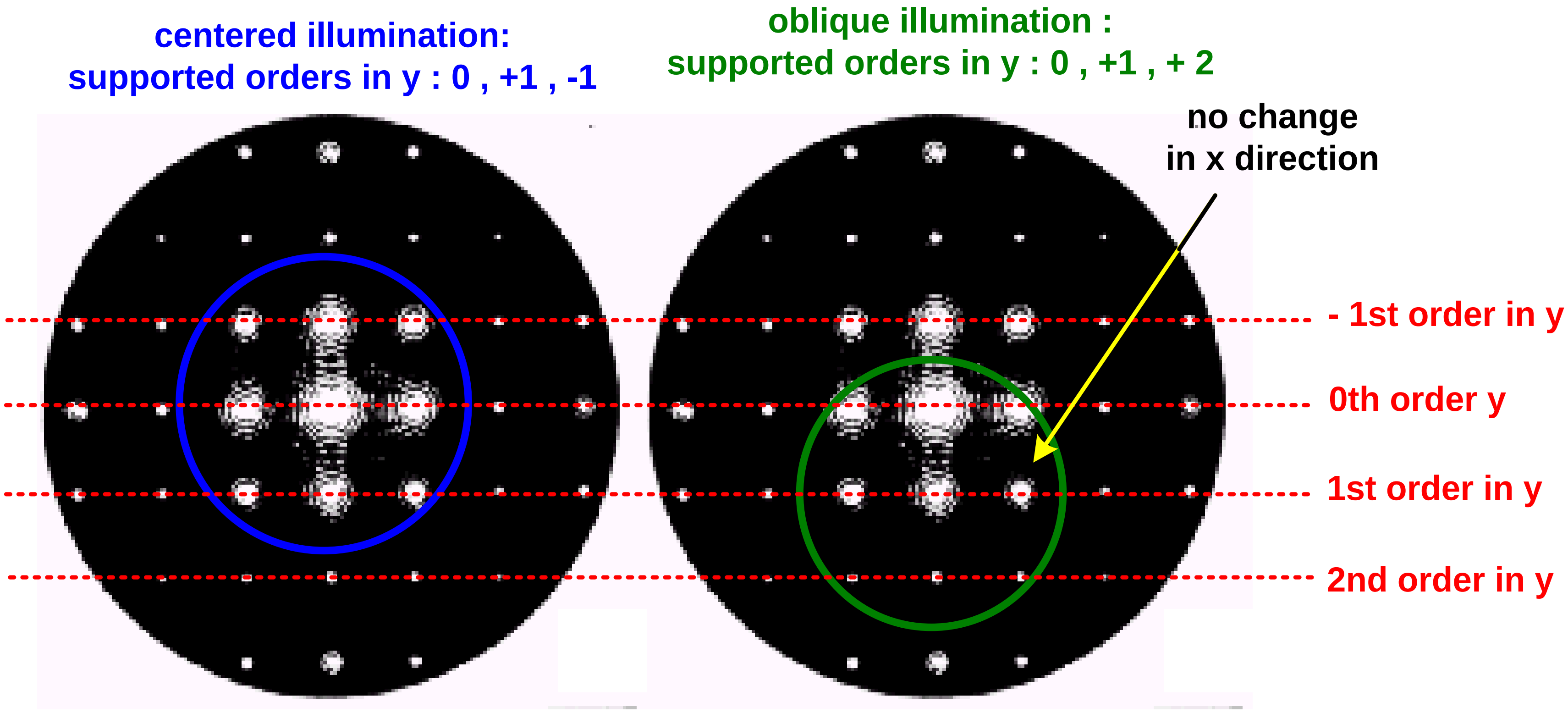
- Improved resolution by oblique illumination in microscopy
- Enhancement only in one direction



9 Imaging
Fourier Optical Fundamentals

- Helmholtz wave equation:
  Propagation with Green's function g,
  Amplitude transfer function, impulse response

$$E(x',y',z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x',y',x,y)\cdot E(x,y)\,dxdy$$

- For shift-invariance:
  convolution

$$E(x',y',z) = g(x-x',y-y',z) * E(x,y)$$

- Green's function of a spherical wave

$$g(\vec{r},\vec{r}\,') = \frac{1}{4\pi\left|\vec{r}-\vec{r}\,'\right|} e^{-ik\left|\vec{r}-\vec{r}\,'\right|}$$

  Fresnel approximation

$$g(x,y,z) = \frac{i}{\lambda z} e^{-ikz} e^{-\frac{i\pi}{\lambda z}(x^2+y^2)}$$

- Calculation in frequency space: product

$$E(\nu_x,\nu_y,z) = G(\nu_x,\nu_y,z)\cdot E(\nu_x,\nu_y)$$

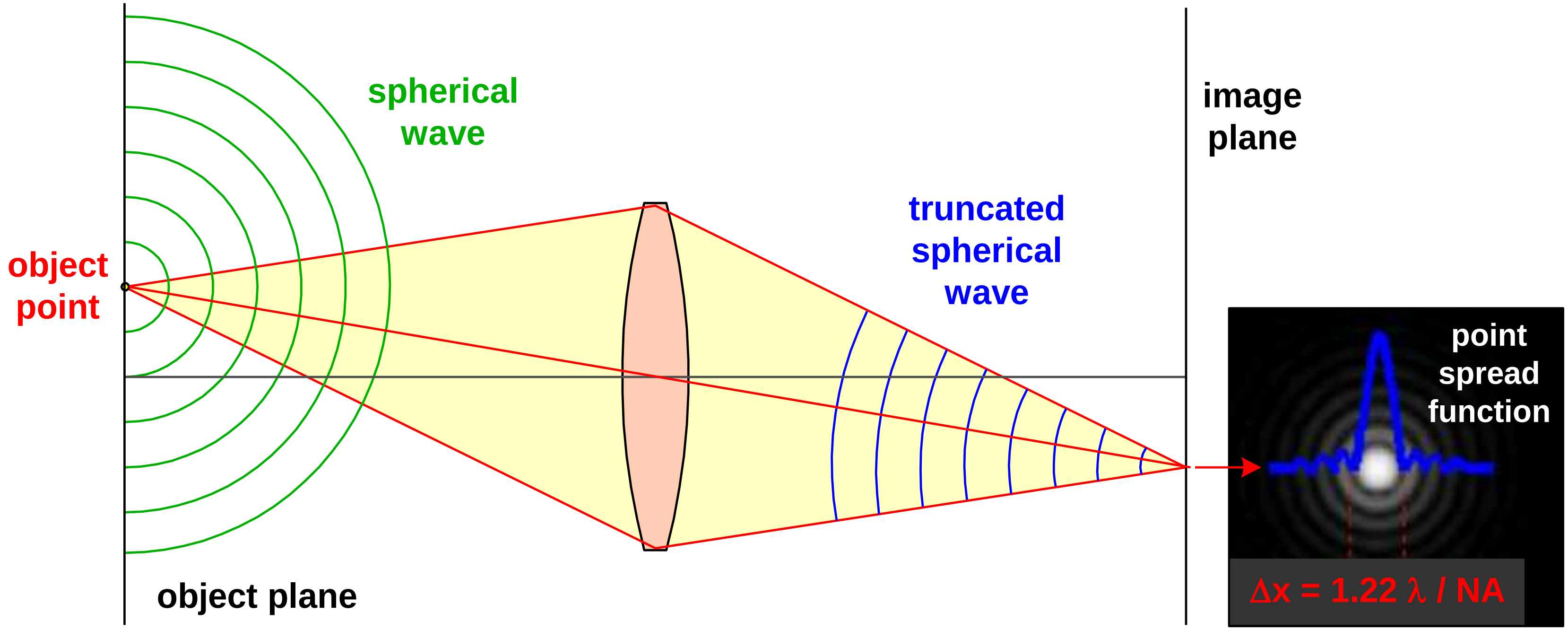
$$G(\nu_x,\nu_y,z) = e^{-ikz+i\pi\lambda z\cdot(\nu_x^2+\nu_y^2)}$$

- Optical systems:
  Impulse response g(x,y) is coherent transfer function, point spread function (PSF).
  $G(\nu_x,\nu_y)$ corresponds to the complex pupil function

- Fourier transform: corresponds to a plane wave expansion

Diffraction at the System Aperture

- Self luminous points: emission of spherical waves
- Optical system: only a limited solid angle is propagated, the truncaton of the spherical wave results in a finite angle light cone
- In the image space: uncomplete constructive interference of partial waves, the image point is spreaded
- The optical systems works as a low pass filter



Optical Transfer Function: Definition

- Normalized optical transfer function (OTF) in frequency space

$$H_{OTF}(v_x,v_y) = \frac{\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} \left|g(x_p,y_p)\right|^2 \cdot e^{-2\pi i\left(x_p v_x + y_p v_y\right)} dx_p dy_p}{\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} \left|g(x_p,y_p)\right|^2 dx_p dy_p}$$

- Fourier transform of the Psf-intensity

$$H_{OTF}(v_x,v_y) = \hat{F}\left[I_{PSF}(x,y)\right]$$

- OTF: Autocorrelation of shifted pupil function, Duffieux-integral

$$H_{OTF}(v_x,v_y) = \frac{\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} P\left(x_p + \frac{\lambda f v_x}{2}, y_p + \frac{\lambda f v_y}{2}\right) \cdot P^*\left(x_p - \frac{\lambda f v_x}{2}, y_p - \frac{\lambda f v_y}{2}\right) dx_p dy_p}{\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} \left|P(x_p,y_p)\right|^2 dx_p dy_p}$$

- Absolute value of OTF: modulation transfer function (MTF)

- MTF is numerically identical to contrast of the image of a sine grating at the corresponding spatial frequency

9 Imaging
Interpretation of the Duffieux Iintegral

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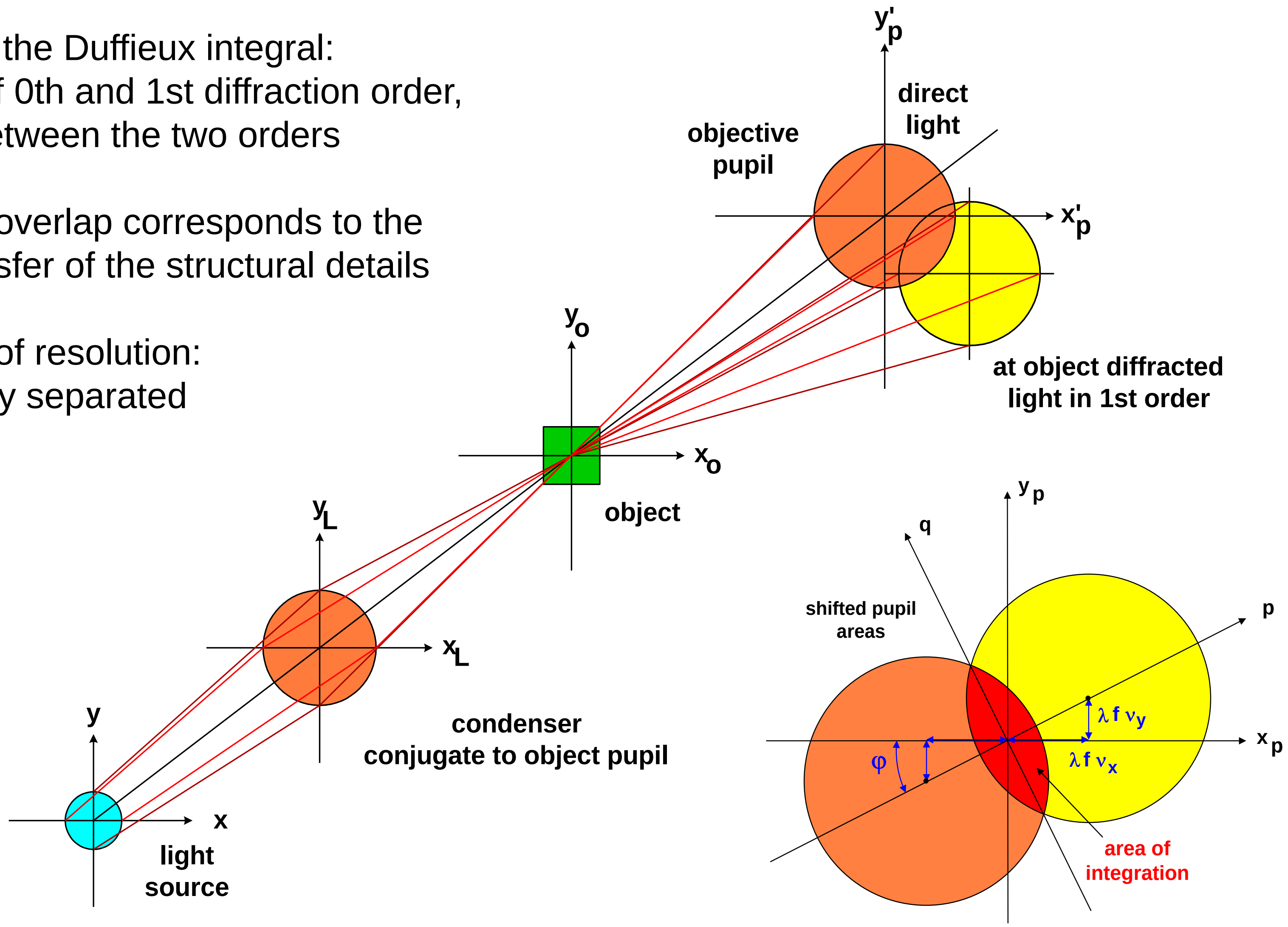
- Interpretation of the Duffieux integral:
  overlap area of 0th and 1st diffraction order,
  interference between the two orders
- The area of the overlap corresponds to the information transfer of the structural details
- Frequency limit of resolution:
  areas completely separated



Optical Transfer Function of a Perfect System

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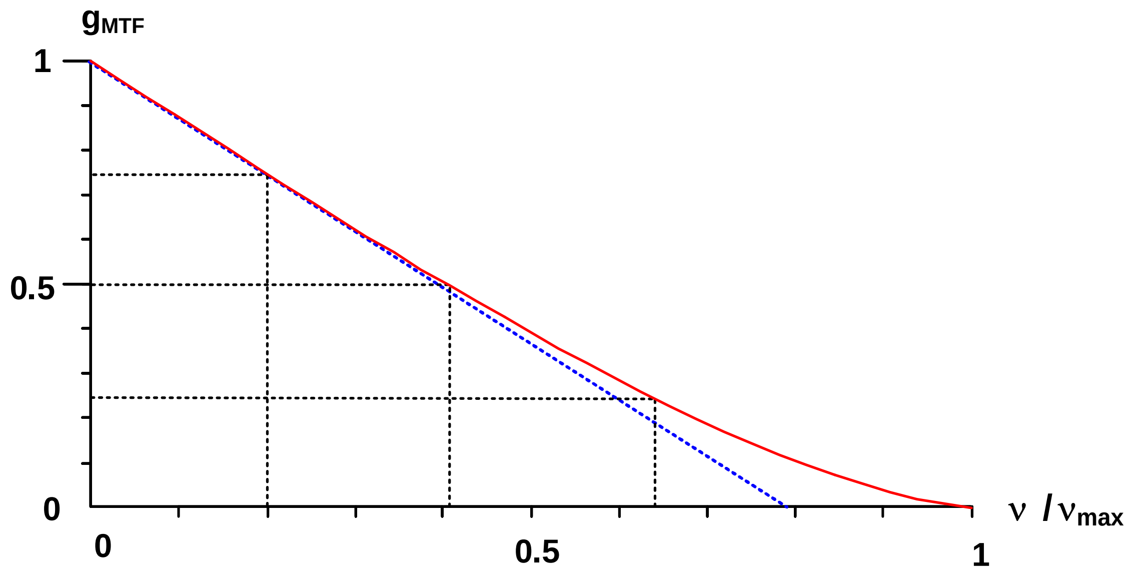
- Aberration free circular pupil:
  Reference frequency

$$\nu_o = \frac{a}{\lambda f} = \frac{\sin u'}{\lambda}$$

- Cut-off frequency:

$$\nu_G = 2\nu_0 = \frac{2na}{\lambda f} = \frac{2n\sin u'}{\lambda}$$

- Analytical representation



$$H_{MTF}(\nu) = \frac{2}{\pi}\left[\arccos\left(\frac{\nu}{2\nu_0}\right) - \left(\frac{\nu}{2\nu_0}\right)\sqrt{1-\left(\frac{\nu}{2\nu_0}\right)^2}\right]$$

- Separation of the complex OTF function into:
  - absolute value: modulation transfer MTF
  - phase value: phase transfer function PTF

$$H_{OTF}(\nu_x,\nu_y) = H_{MTF}(\nu_x,\nu_y) \cdot e^{iH_{PTF}(\nu_x,\nu_y)}$$

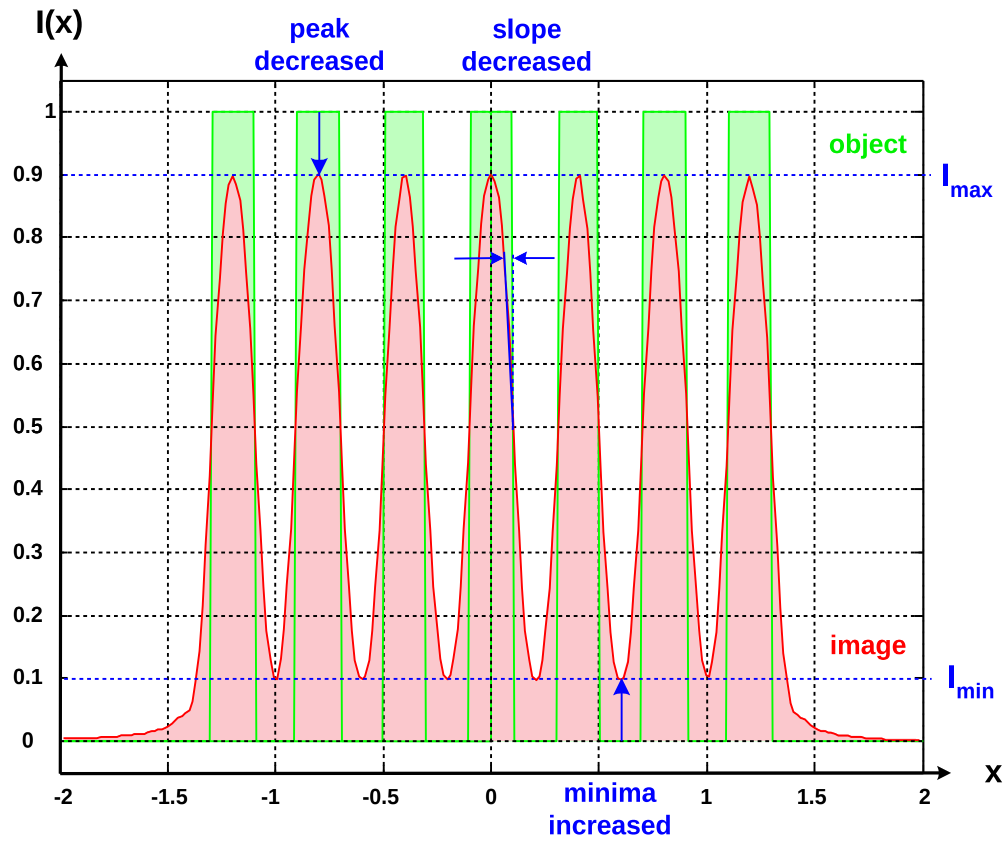
- The MTF-value corresponds to the intensity contrast of an imaged sin grating
- Visibility

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

- The maximum value of the intensity is not identical to the contrast value since the minimal value is finite too

- Concrete values:

ΔI	I_{max}	V
0.010	0.990	0.980
0.020	0.980	0.961
0.050	0.950	0.905
0.100	0.900	0.818
0.111	0.889	0.800
0.150	0.850	0.739
0.200	0.800	0.667
0.300	0.700	0.538



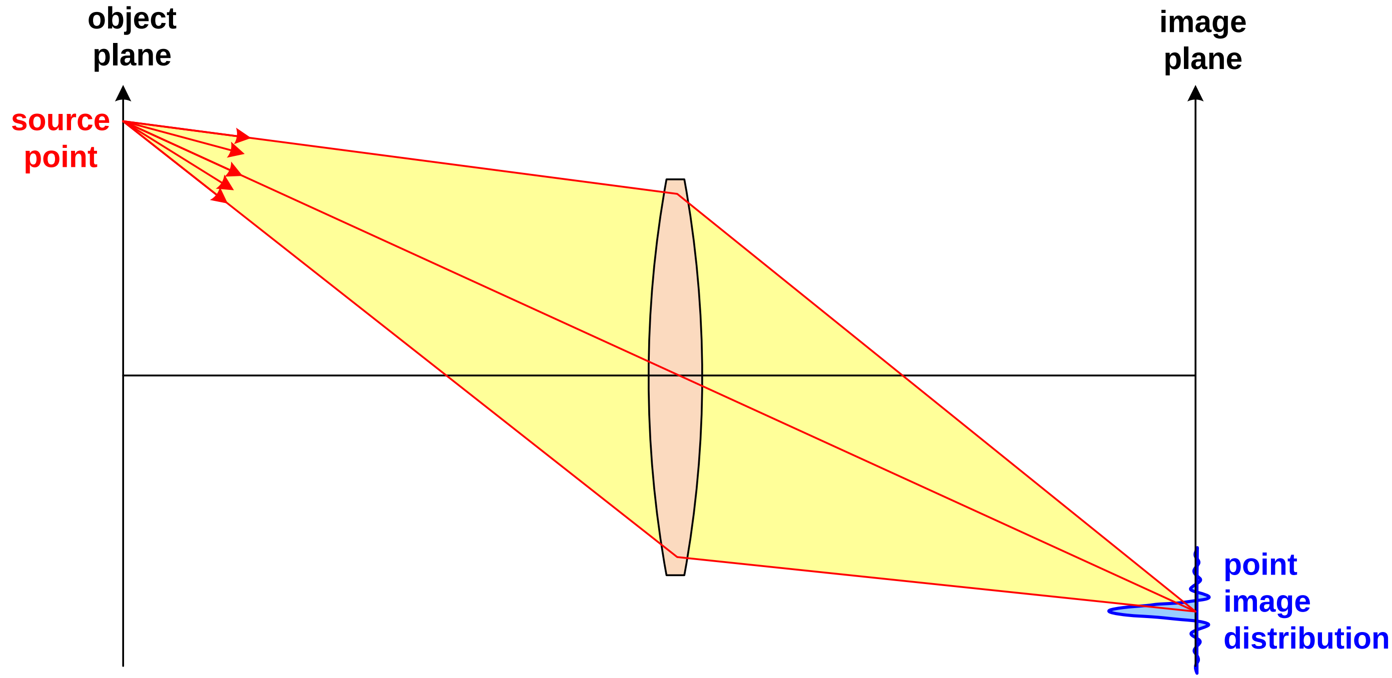
Fourier Optics – Point Spread Function

- Optical system with magnification m
  Pupil function P,
  Pupil coordinates x_p, y_p

$$g_{psf}(x,y,x',y') = N \cdot \iint P(x_p, y_p) \cdot e^{-\frac{ik}{z}\cdot\left[x_p\cdot(x'-mx)+y_p\cdot(y'-my)\right]} dx_p dy_p$$

- PSF is Fourier transform
  of the pupil function
  (scaled coordinates)

$$g_{psf}(x,y) = N \cdot \hat{F}\left[P(x_p, y_p)\right]$$



Fourier Theory of Coherent Image Formation

- Transfer of an extended object distribution I(x,y)

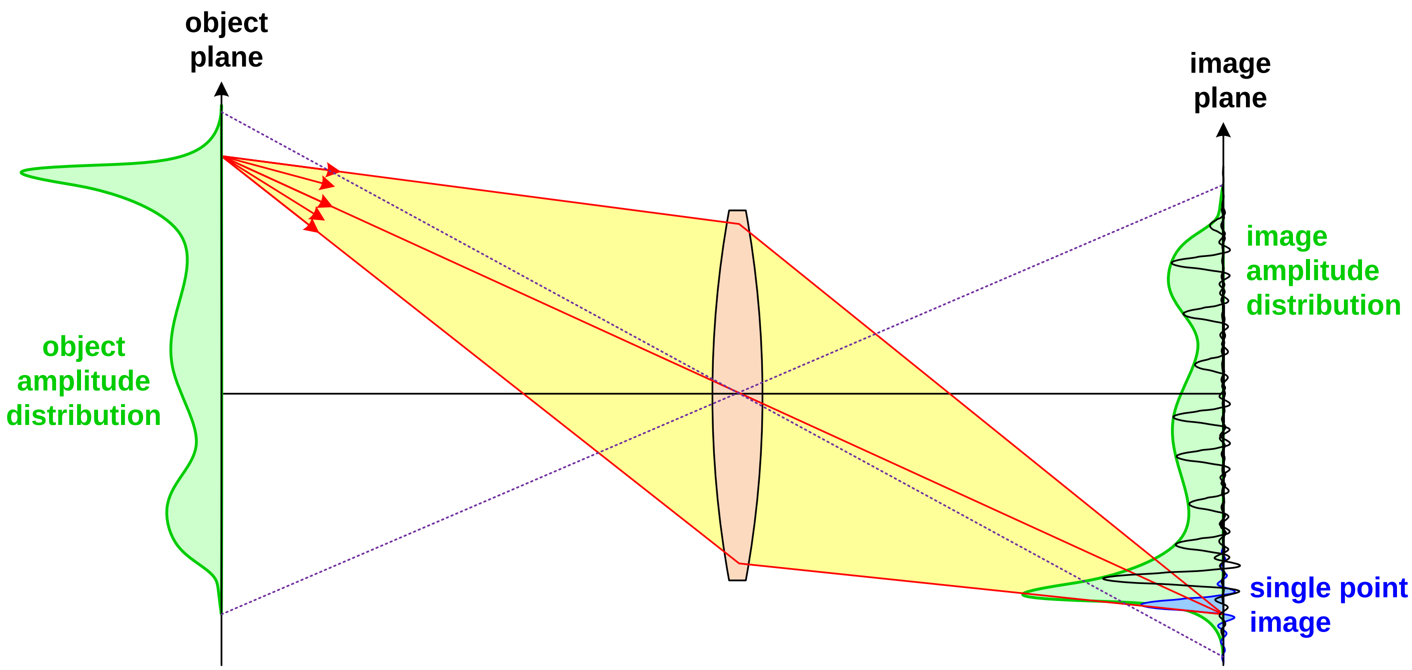
$$E(x',y') = \iint g_{psf}\left(x,y,x',y'\right)\cdot E(x,y)\,dx\,dy$$

- In the case of shift invariance (isoplanasie): coherent convolution of fields

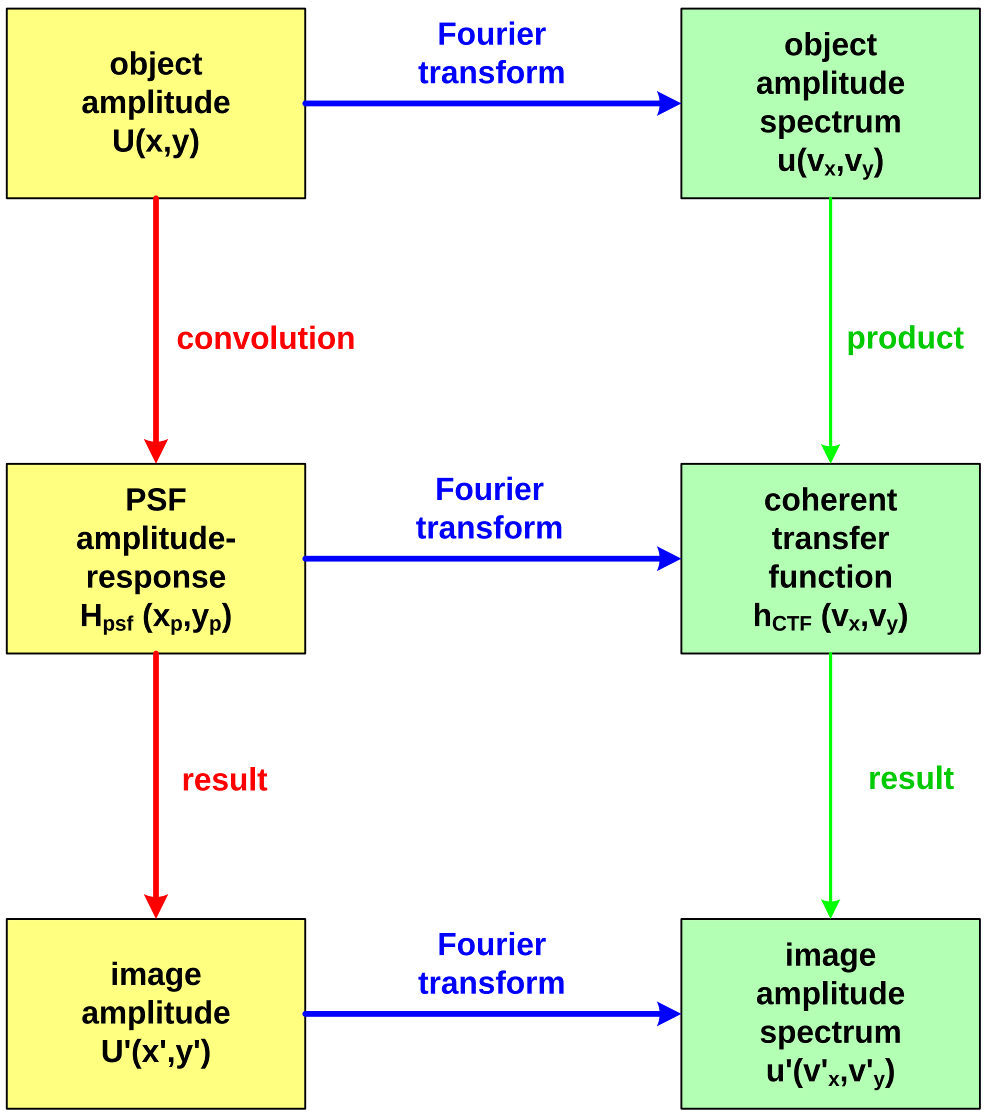
$$E(x',y') = \iint g_{psf}\left(x-x',y-y'\right)\cdot E(x,y)\,dx\,dy$$

$$E(x',y') = g_{psf}\left(x,y\right) * E(x,y)$$

- Complex fields are additive



9 Imaging
Fourier Theory of Coherent Image Formation



9 Imaging
Fourier Theory of Incoherent Image Formation

- Transfer of an extended object distribution I(x,y)

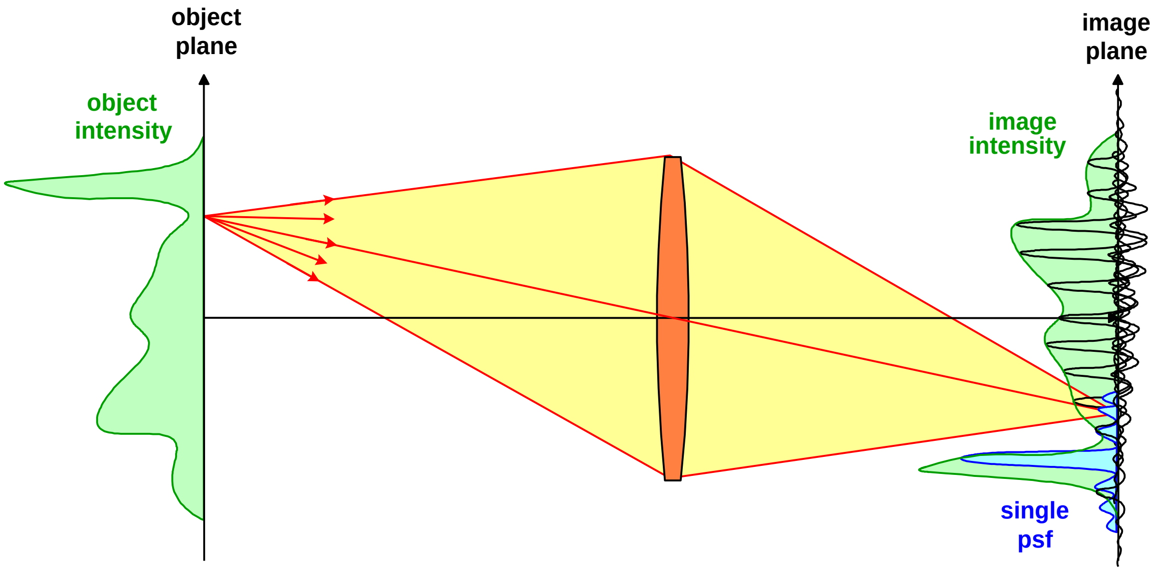
$$I_{inc}(x',y') = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left| g_{psf}(x',x,y',y) \right|^2 \cdot I(x,y)\,dxdy$$

- In the case of shift invariance (isoplanasie): incoherent convolution

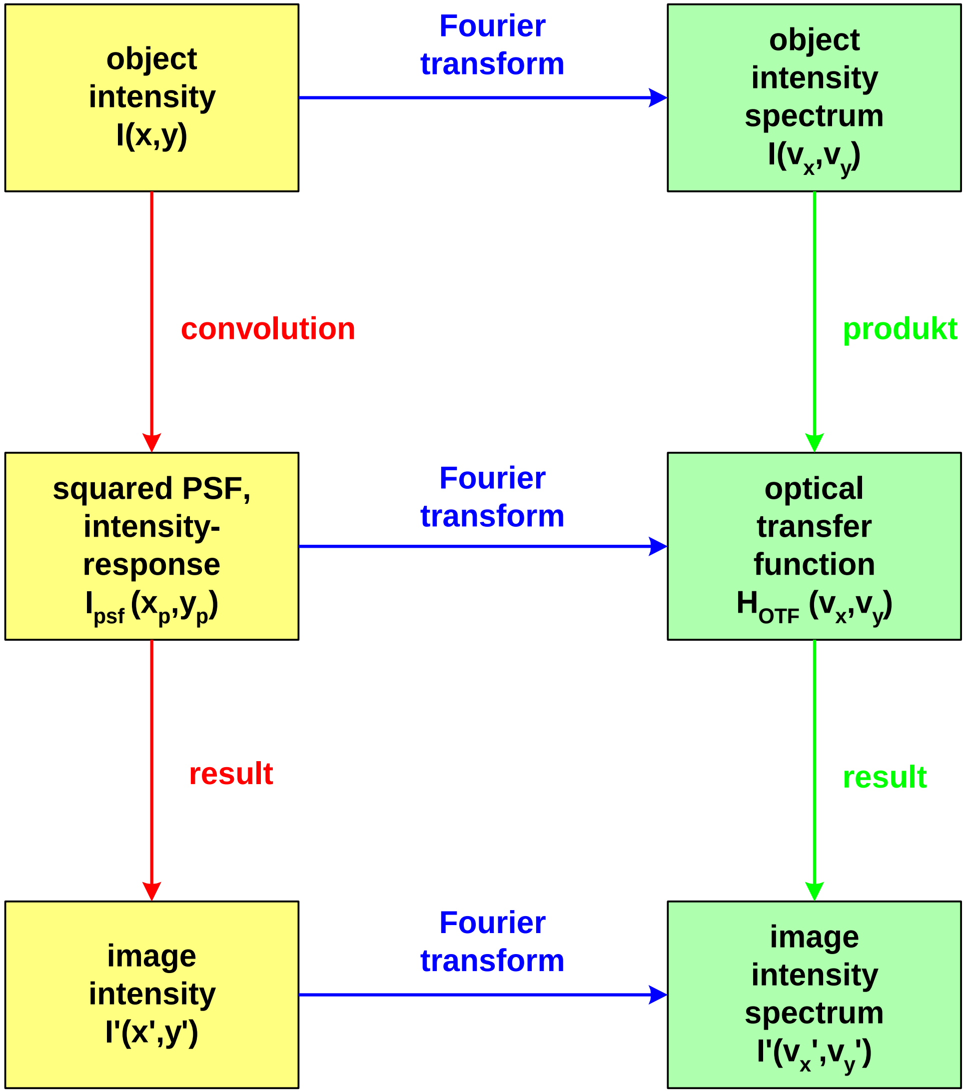
$$I_{inc}(x',y') = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left| g_{psf}(x'-x,y'-y) \right|^2 \cdot I(x,y)\,dxdy$$

- Intensities are additive

$$I_{image}(x',y') = I_{psf}(x,y) * I_{obj}(x,y)$$



9 Imaging
Fourier Theory of Incoherent Image Formation



9 Imaging
Incoherent Image Formation in Frequency Space

- Incoherent illumination:
  No correlation between neighbouring object points
  Superposition of intensity in the image

$$I_{inc}(x',y') = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left| g_{psf}(x',x,y',y) \right|^2 \cdot I(x,y)\,dxdy$$

- In the case of shift invariance (isoplanasie):
  Incoherent imaging with convolution

$$I_{inc}(x',y') = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left| g_{psf}(x'-x,y'-y) \right|^2 \cdot I(x,y)\,dxdy$$

$$I_{image}(x',y') = I_{psf}(x,y) * I_{obj}(x,y)$$

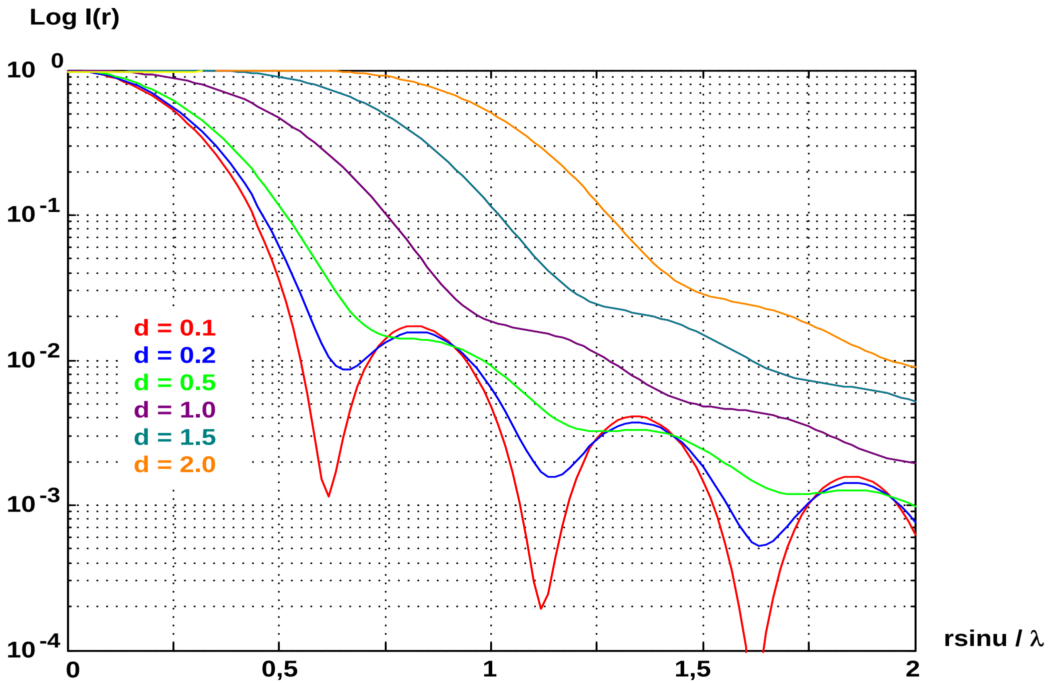
- In frequency space:
  Product of spectra, linear transfer theory
  The spectrum of the psf works as low pass filter onto the object spectrum
  Optical transfer function

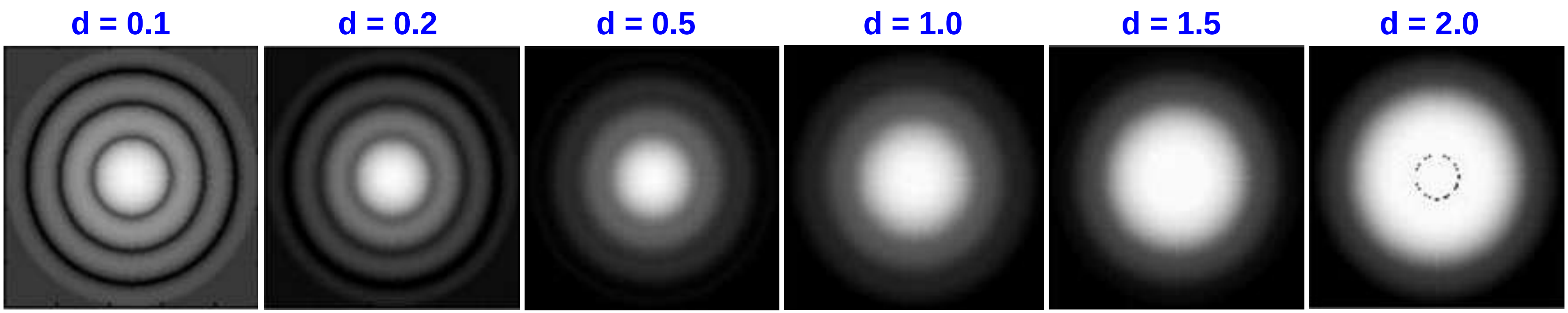
$$H_{otf}(v_x,v_y) = FT\left[I_{PSF}(x,y) \right]$$

$$I_{image}(v_x,v_y) = H_{otf}(v_x,v_y) \cdot I_{obj}(v_x,v_y)$$

Incoherent Image of a Circular Disc

- Circular disc with diameter
  $D = d \times D_{airy}$
- Small d << 1 : Airy disc
- Increasing d:
  Diffraction ripple disappear

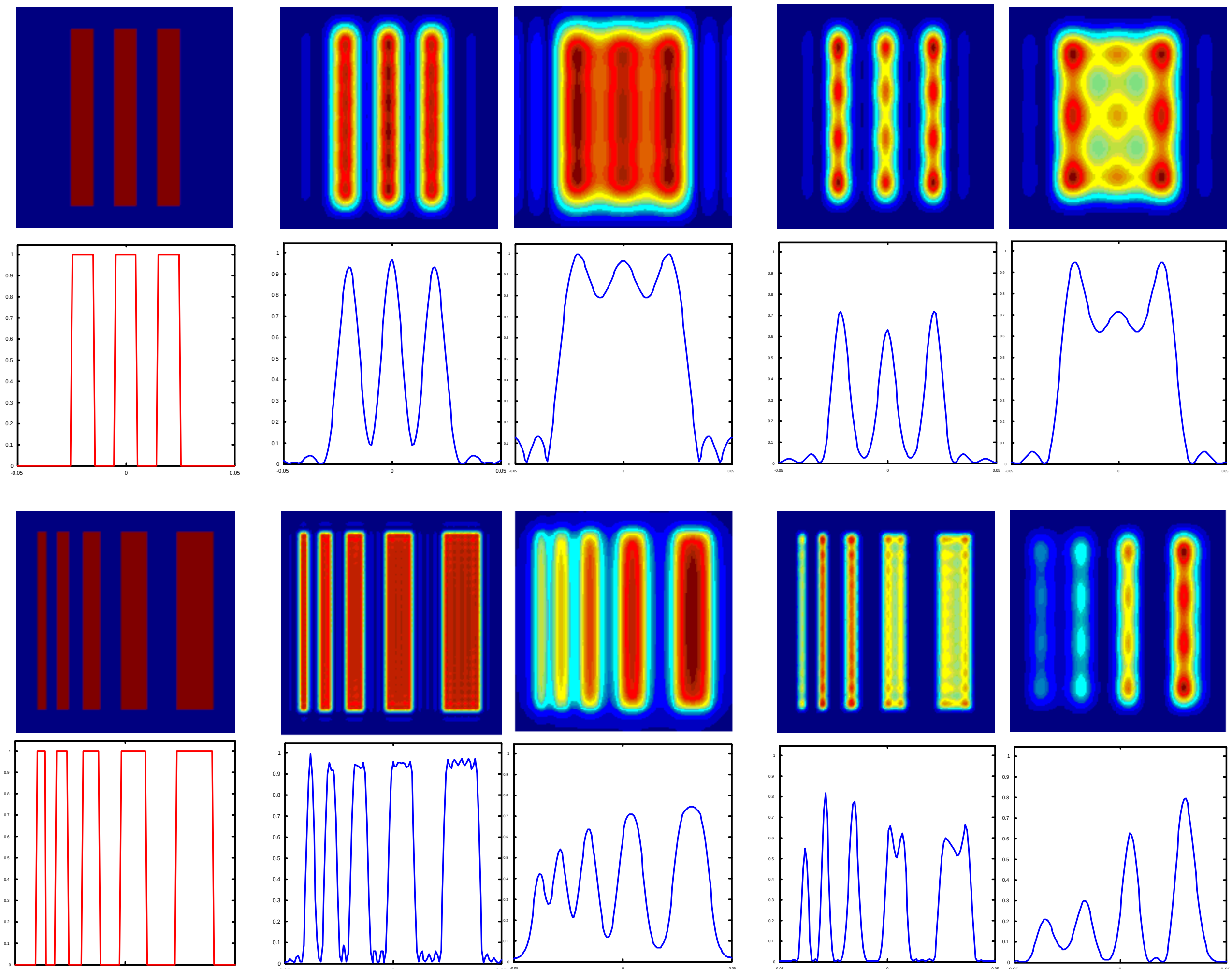




9 Imaging
Comparison Coherent – Incoherent Images

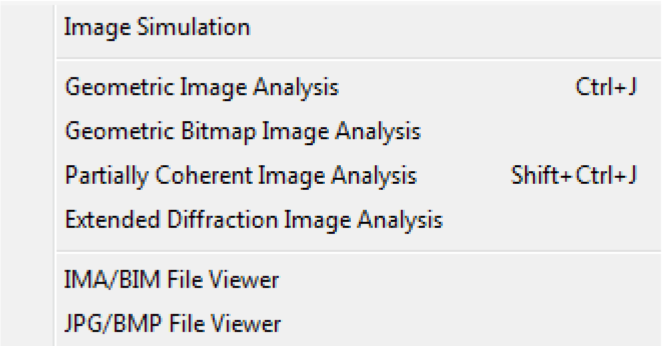
object **incoherent** **coherent**

bars resolved **bars not resolved** **bars resolved** **bars not resolved**



Possible options in Zemax:

- Convolution of image with Psf
  1. geometrical
  2. with diffraction
- Geometrical raytrace analysis
  1. simple geometrical shapes (IMA-files)
  2. bitmaps
- Diffraction imaging:
  1. partial coherent
  2. extended with variable PSF
- Structure of options in Zemax not clear
- Redundance
- Field definition and size scaling not good
- Numerical conditions and algorithms partially not stable



9 Imaging
General Image Simulation

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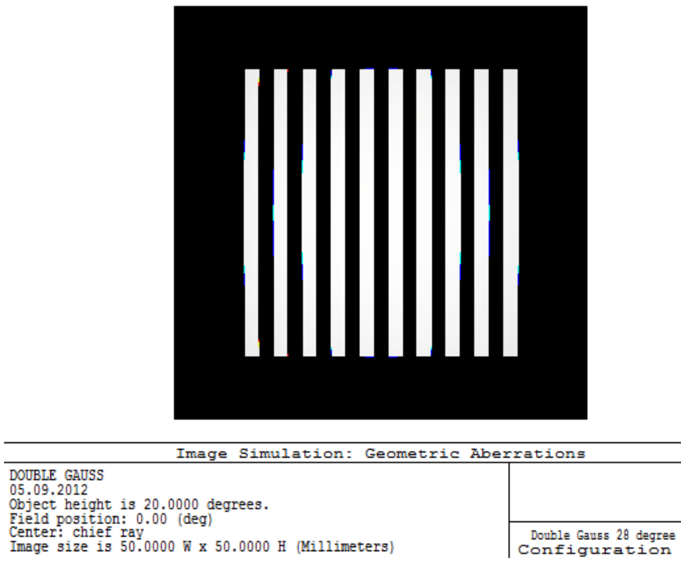
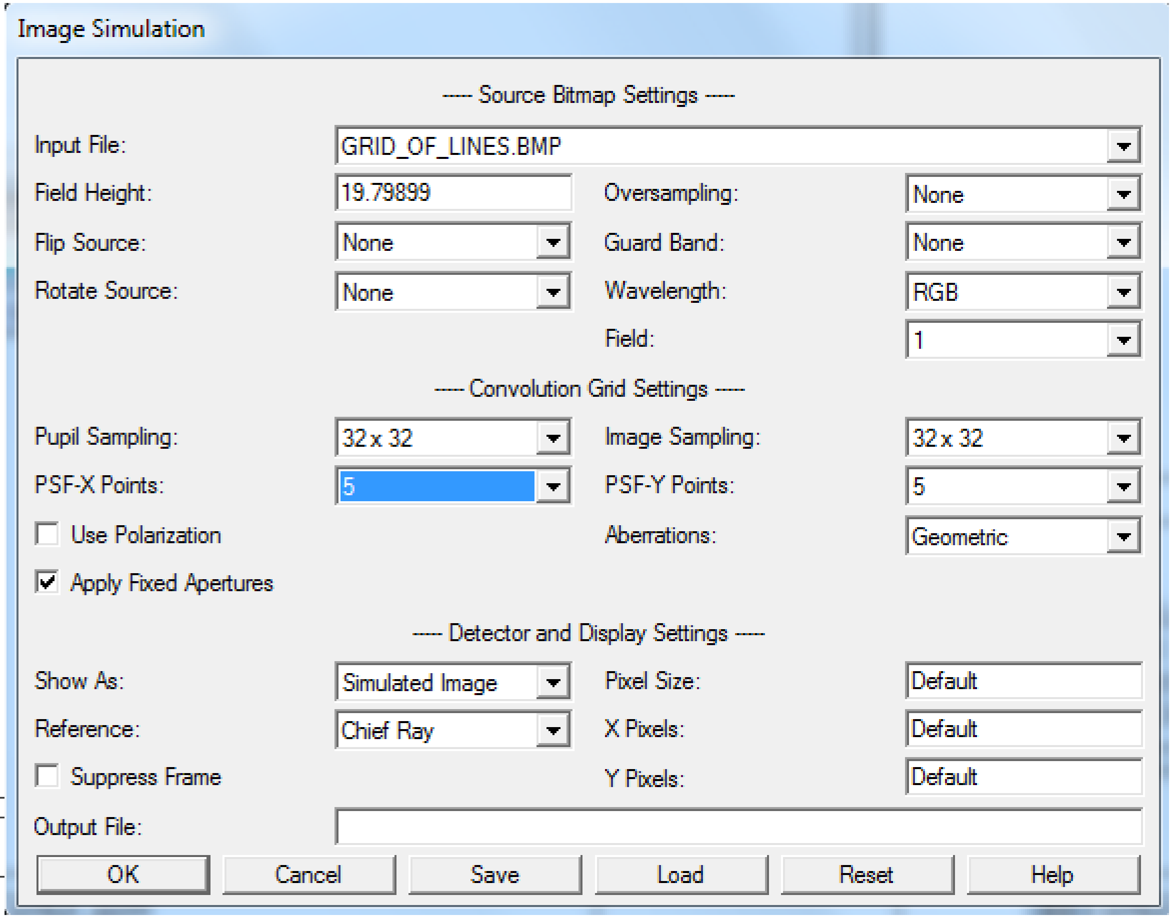
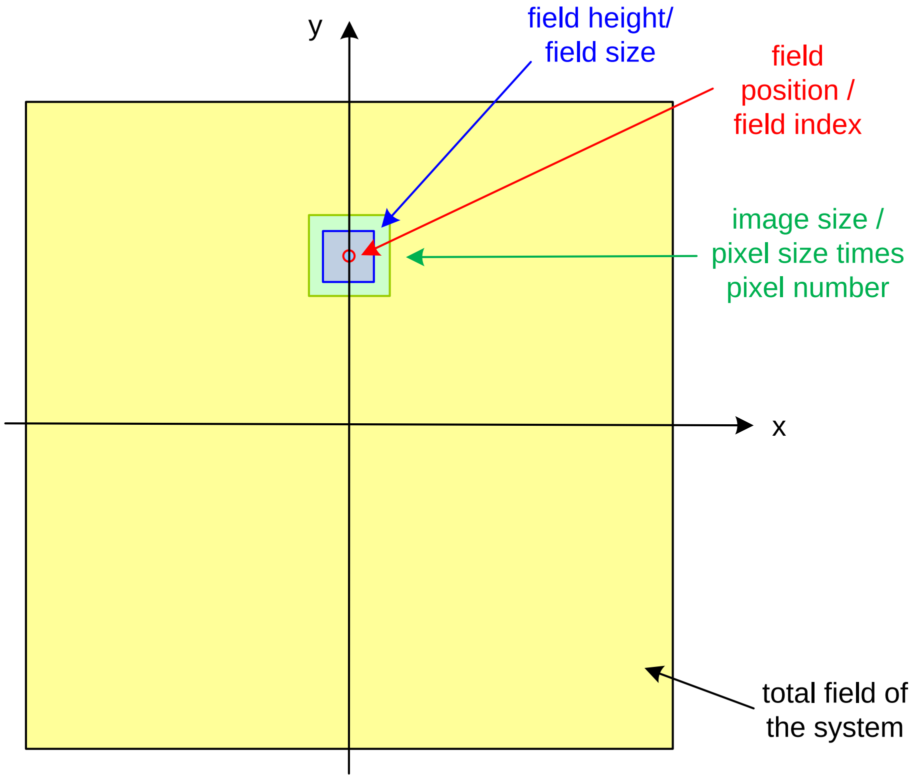
- Field height: location of object in the specific coordinates of the system
  - zero padding included (not: size = diameter)
  - image size shon is product of pixel number x pixel size
  - can be full field or centre of local extracted part of the field
- PSF-X/Y points: number of field points to incorporate the changes of the PSF, interpolation between this coarse grid
- Object: bitmap
- PSF: geometrical or diffraction





- Total field size: defined by system
- Field height/size: reduced field corresponding to the structure as considered in the imaging calculation
- Field position: reference point of the considered reduced field (center) in the total field
- Image size: size of the represented field size, should be a little larger as field size to clearly see the boundary
  In some tools calculated as product of pixel number and pixel size



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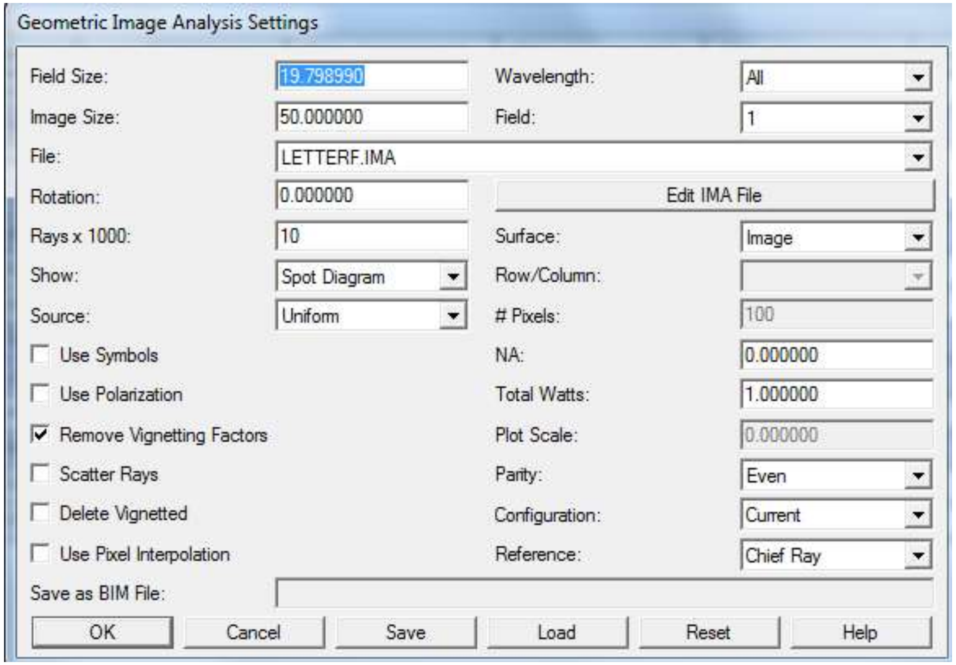
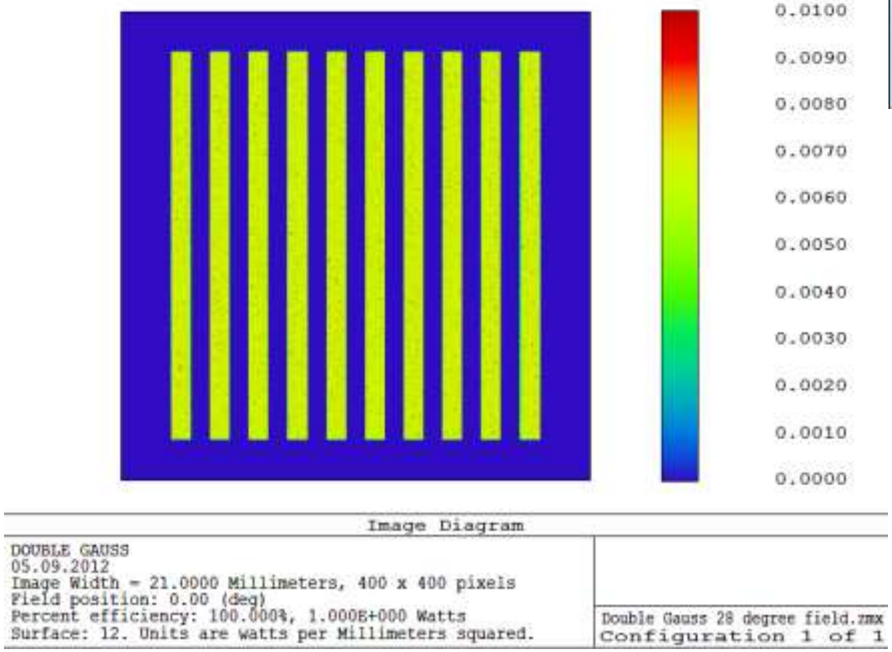
Geometrical imaging by raytrace

- Binary IMA-files with geometrical shapes
- Choice of:
  - field size
  - image size,
  - wavelengths
  - number of rays
- Interpolation possible

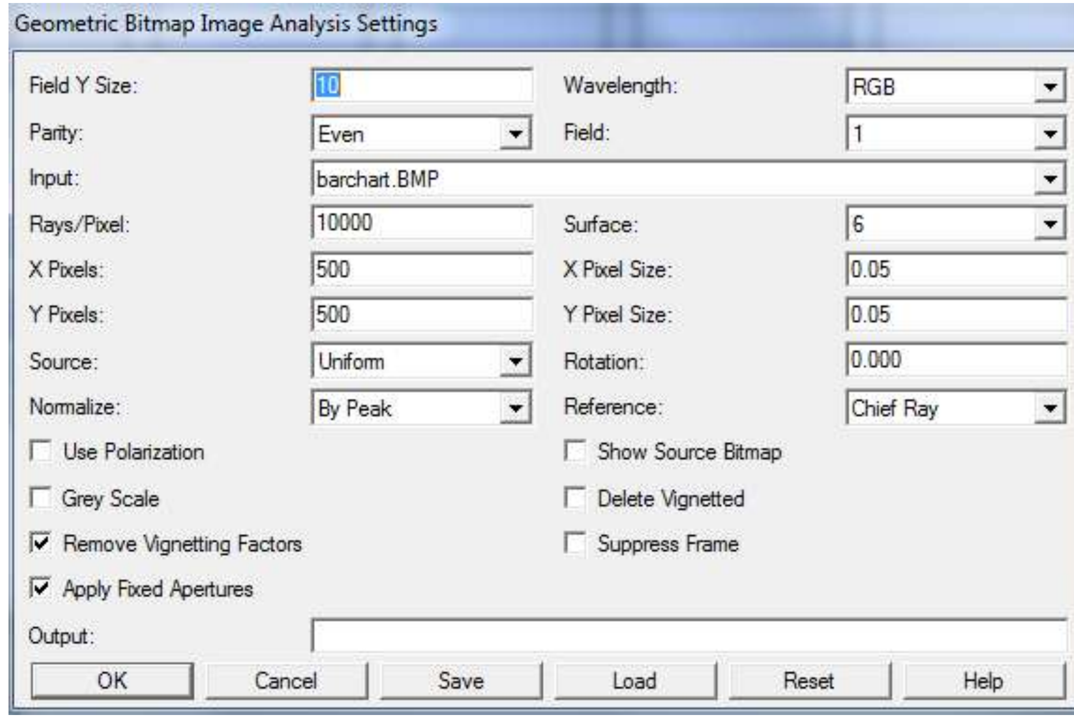


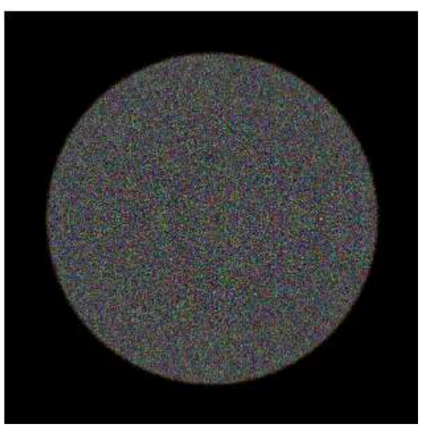


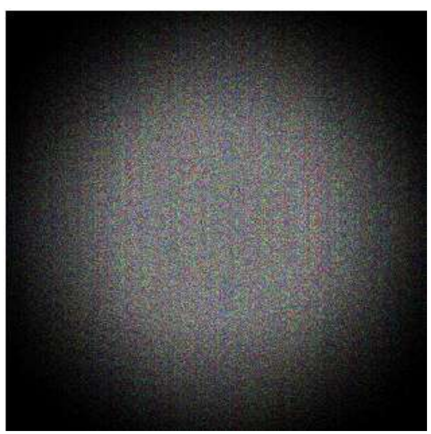
9 Imaging
Geometric Imaging II

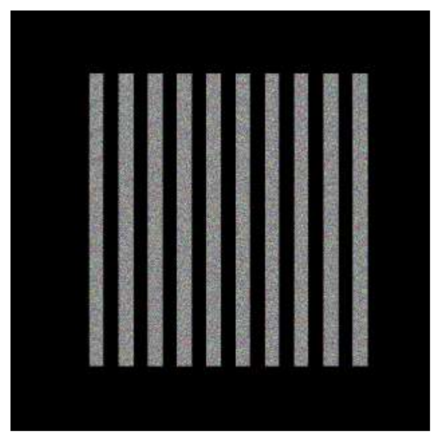
Geometrical imaging by raytrace of bitmaps

- Extension of 1st option: can be calculated at any surface
- If full field is used, this corresponds to a footprint with all rays
- Example: light distribution in pupil, at last surface, in image









9 Imaging
Patial Coherent Imaging

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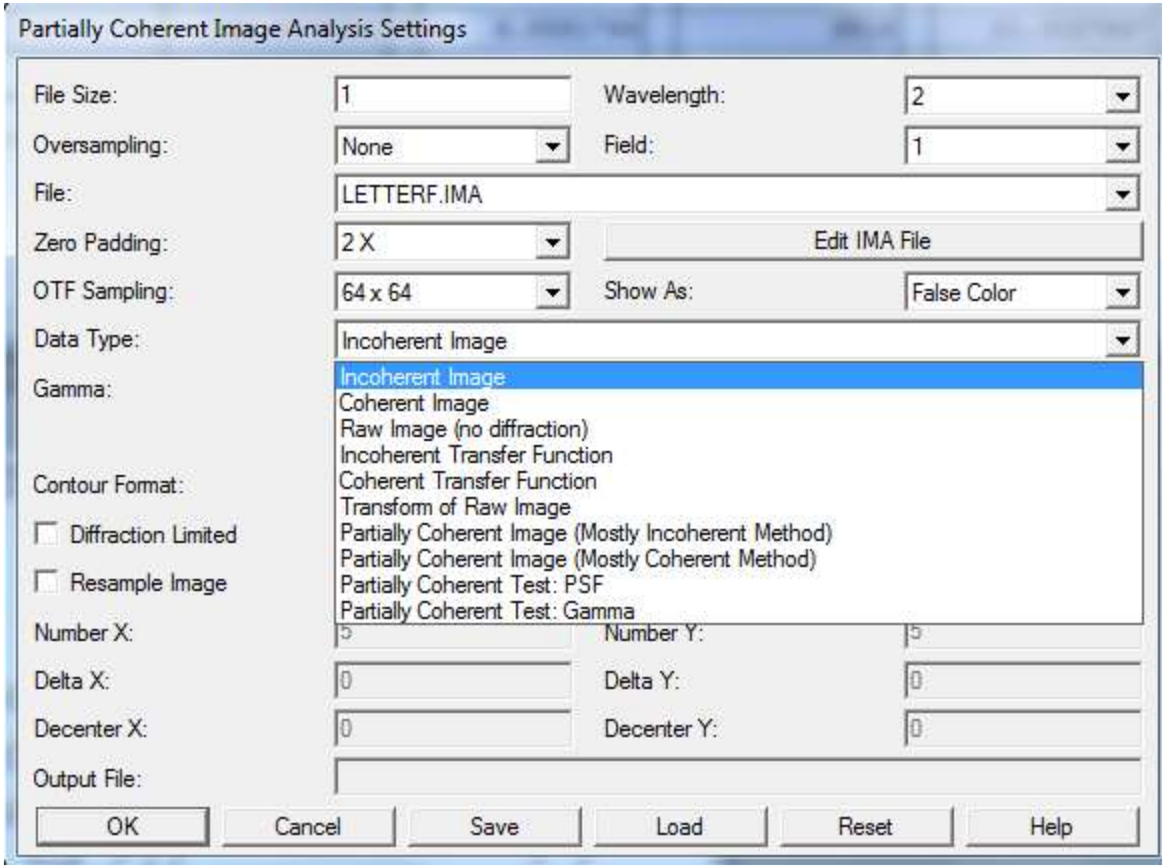
- Different types of partial coherent model algorithms possible
- Only IMA-Files can be used as objects
- α describes the coherence factor (relative pupil filling)
- Control and algorithms not clear, not stable



9 Imaging
Extended Diffraction

- Classical convolution of psf with pixels of IMA-File
- Coherent and incoherent model possible
- PSF may vary over field position