



**Institute of  
Applied Physics**

Friedrich-Schiller-Universität Jena

# Optical Design with Zemax

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Lecture 7: Optimization I

2012-12-11

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# 7 Optimization I

## Time schedule

1	16.10.	Introduction	Introduction, Zemax interface, menus, file handling, preferences, Editors, updates, windows, Coordinate systems and notations, System description, Component reversal, system insertion, scaling, 3D geometry, aperture, field, wavelength
2	23.10.	Properties of optical systems I	Diameters, stop and pupil, vignetting, Layouts, Materials, Glass catalogs, Raytrace, Ray fans and sampling, Footprints
3	30.10.	Properties of optical systems II	Types of surfaces, Aspheres, Gratings and diffractive surfaces, Gradient media, Cardinal elements, Lens properties, Imaging, magnification, paraxial approximation and modelling
4	06.11.	Aberrations I	Representation of geometrical aberrations, Spot diagram, Transverse aberration diagrams, Aberration expansions, Primary aberrations,
5	13.+27.11.	Aberrations II	Wave aberrations, Zernike polynomials, Point spread function, Optical transfer function
6	04.12.	Advanced handling	Telecentricity, infinity object distance and afocal image, Local/global coordinates, Add fold mirror, Vignetting, Diameter types, Ray aiming, Material index fit, Universal plot, Slider, IO of data, Multiconfiguration, Macro language, Lens catalogs
7	11.12.	Optimization I	Principles of nonlinear optimization, Optimization in optical design, Global optimization methods, Solves and pickups, variables, Sensitivity of variables in optical systems
8	18.12.	Optimization II	Systematic methods and optimization process, Starting points, Optimization in Zemax
9	08.01	Imaging	Fundamentals of Fourier optics, Physical optical image formation, Imaging in Zemax
10	15.01.	Illumination	Introduction in illumination, Simple photometry of optical systems, Non-sequential raytrace, Illumination in Zemax
11	22.01.	Correction I	Symmetry principle, Lens bending, Correcting spherical aberration, Coma, stop position, Astigmatism, Field flattening, Chromatical correction, Retrofocus and telephoto setup, Design method
12	29.01.	Correction II	Field lenses, Stop position influence, Aspheres and higher orders, Principles of glass selection, Sensitivity of a system correction, Microscopic objective lens, Zoom system
13	05.02.	Physical optical modelling	Gaussian beams, POP propagation, polarization raytrace, coatings

# 7 Optimization I

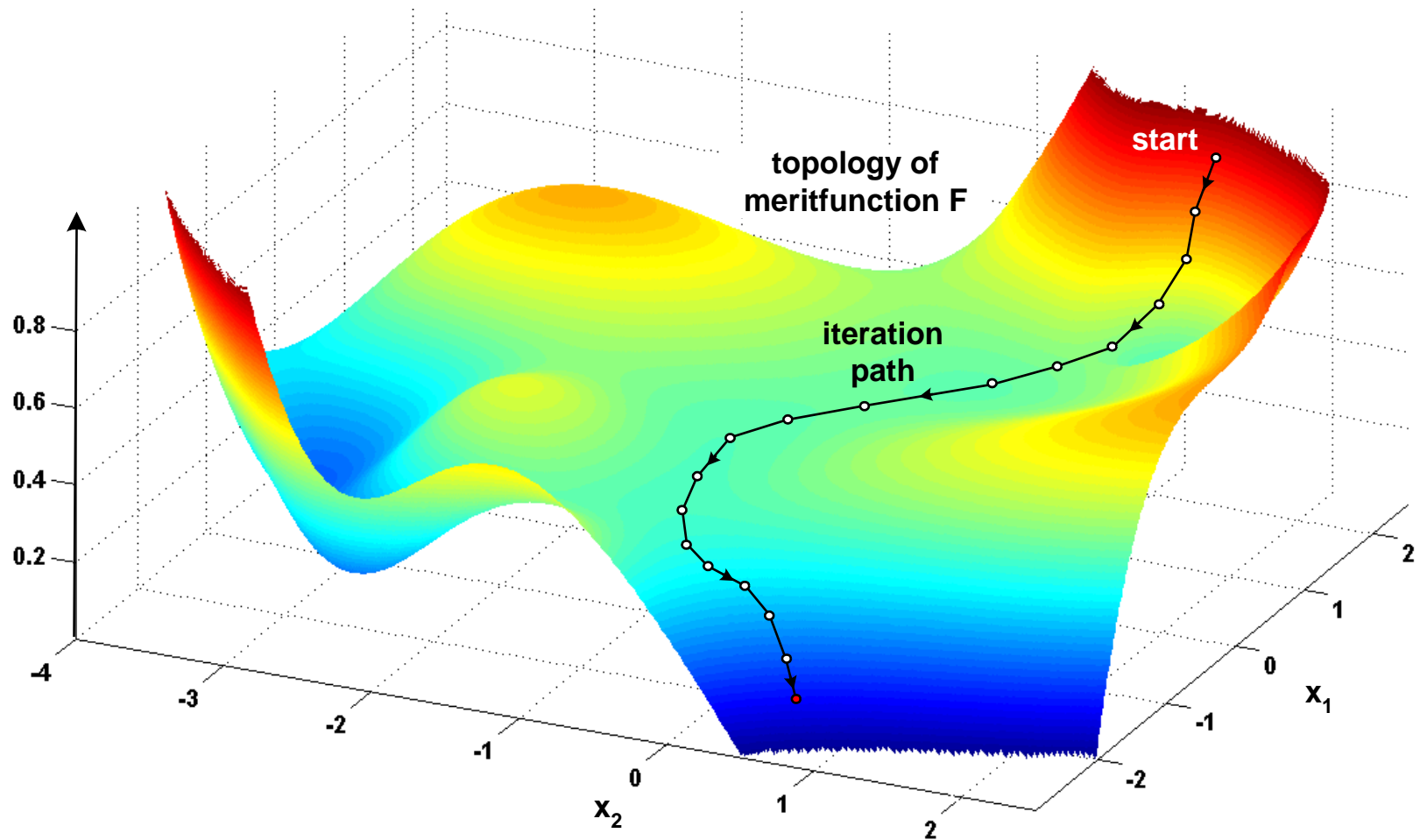
## Contents

1. Principles of nonlinear optimization
2. Optimization in optical design
3. Global optimization methods
4. Sensitivity of variables in optical systems
5. Systematic methods and optimization process
6. Optimization in Zemax

# 7 Optimization I

## Basic Idea of Optimization

- Topology of the merit function in 2 dimensions
- Iterative down climbing in the topology



Mathematical description of the problem:

- n variable parameters
- m target values
- Jacobi system matrix of derivatives, Influence of a parameter change on the various target values, sensitivity function
- Scalar merit function
- Gradient vector of topology
- Hesse matrix of 2nd derivatives

$$\vec{x}$$

$$\vec{f}(\vec{x})$$

$$J_{ij} = \frac{\partial f_i}{\partial x_j}$$

$$F(\vec{x}) = \sum_{i=1}^m w_i \cdot [y_i - f(\vec{x})]^2$$

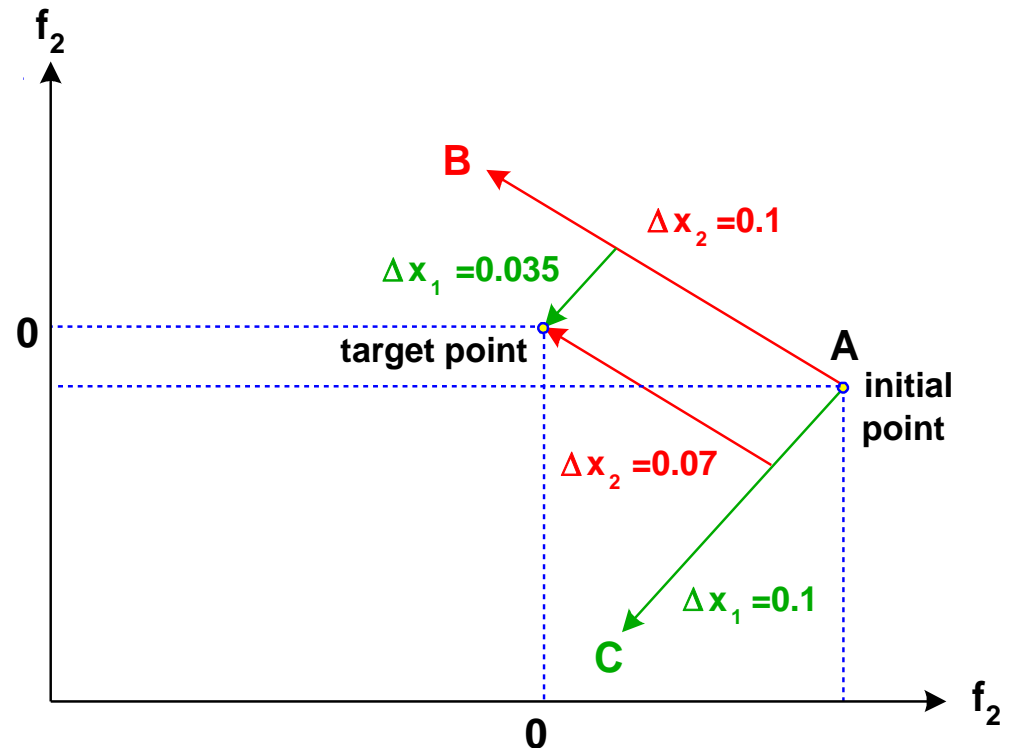
$$g_j = \frac{\partial F}{\partial x_j}$$

$$H_{jk} = \frac{\partial^2 F}{\partial x_j \partial x_k}$$

# 7 Optimization I

## Optimization Principle for 2 Degrees of Freedom

- Aberration depends on two parameters
- Linearization of sensitivity, Jacobian matrix  
Independent variation of parameters
- Vectorial nature of changes:  
Size and direction of change
- Vectorial decomposition of an ideal step of improvement,  
linear interpolation
- Due to non-linearity:  
change of Jacobian matrix,  
next iteration gives better result



- Linearized environment around working point  
Taylor expansion of the target function

$$\vec{f} = \vec{f}_0 + \underline{J} \cdot \vec{x}$$

- Quadratical approximation of the merit function

$$F(\vec{x}) = F(\vec{x}_0) + \underline{J} \cdot \Delta\vec{x} + \frac{1}{2} \cdot \Delta\vec{x} \cdot \underline{H} \cdot \Delta\vec{x}$$

- Solution by lineare Algebra  
system matrix  $\underline{A}$   
cases depending on the numbers  
of  $n / m$

$$\underline{A}^+ = \begin{cases} \underline{A}^{-1} & \text{if } m = n \\ \left(\underline{A}^T \underline{A}\right)^{-1} \cdot \underline{A}^T & \text{if } m > n \text{ (under determined)} \\ \underline{A}^T \cdot \left(\underline{A}\underline{A}^T\right)^{-1} & \text{if } m < n \text{ (over determined)} \end{cases}$$

- Iterative numerical solution:  
Strategy: optimization of
  - direction of improvement step
  - size of improvement step

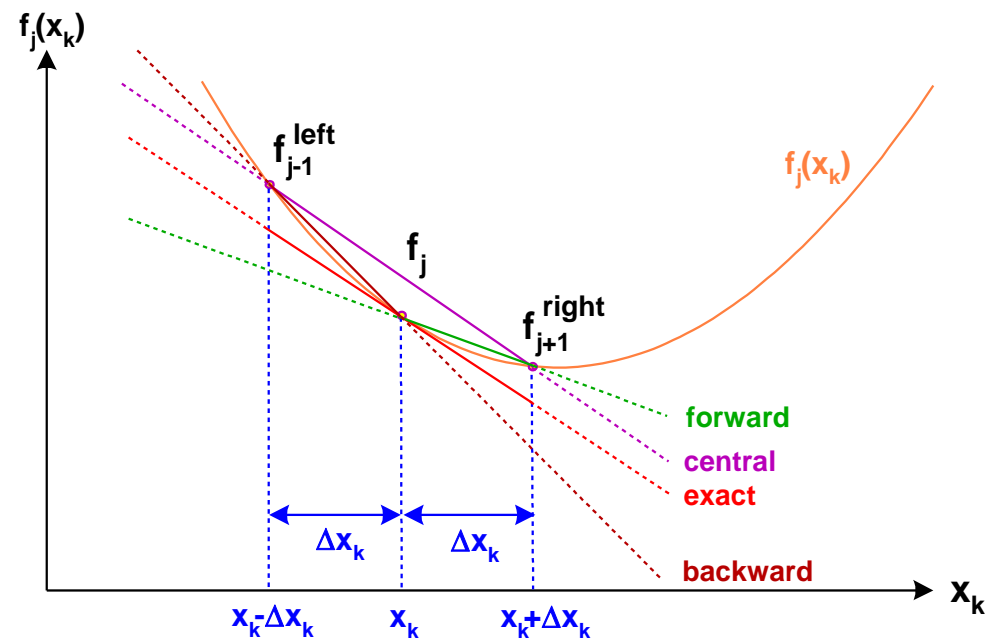
# 7 Optimization I

## Calculation of Derivatives

- Derivative vector in merit function topology:  
Necessary for gradient-based methods
- Numerical calculation by finite differences
- Possibilities and accuracy

$$g_{jk} = \frac{\partial f_j(\vec{x})}{\partial x_k} = \nabla_{x_k} f_j(\vec{x})$$

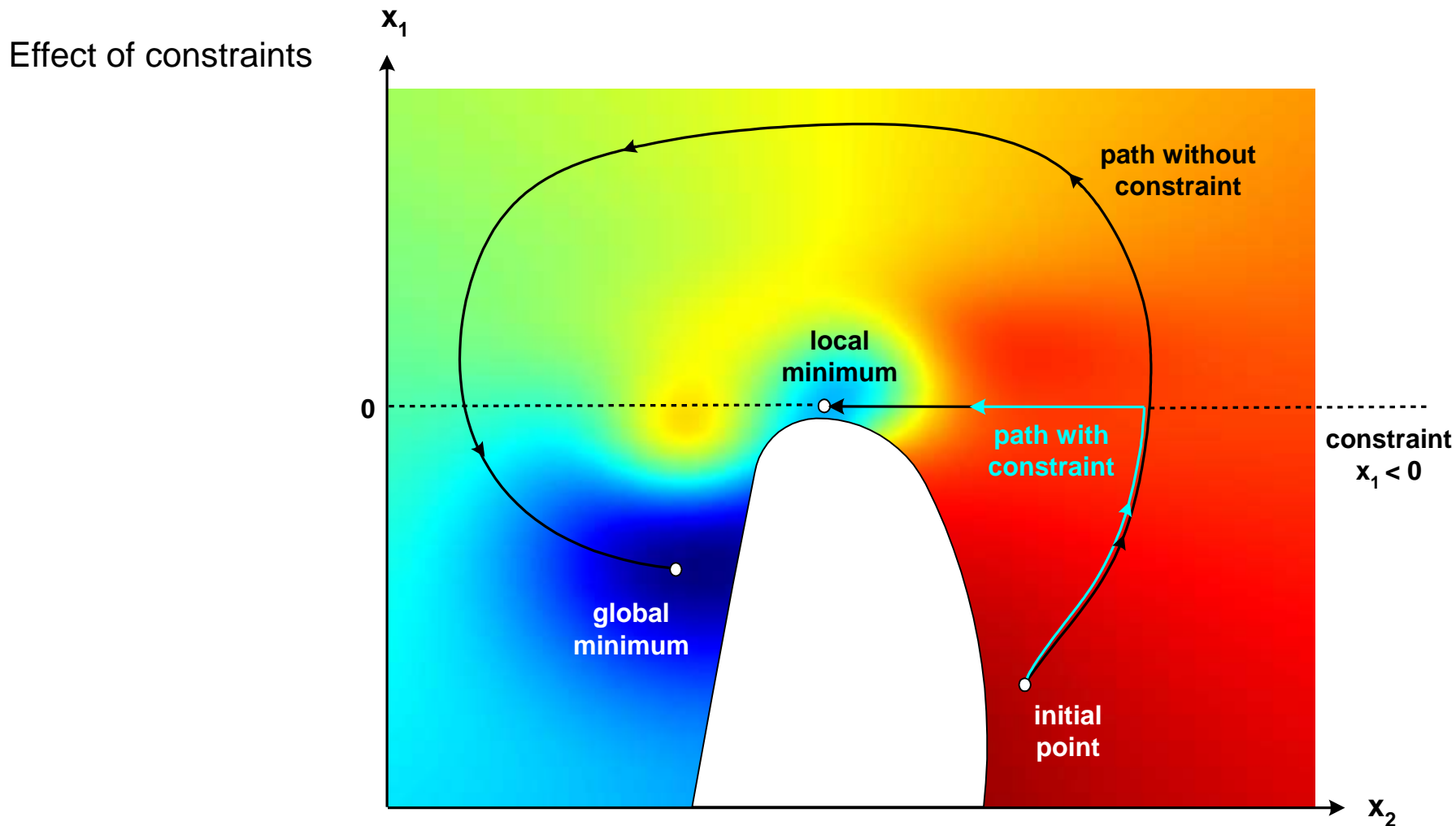
$$g_{jk} = \frac{f_j^{\text{right}} - f_j}{\Delta x_k}$$





# 7 Optimization I

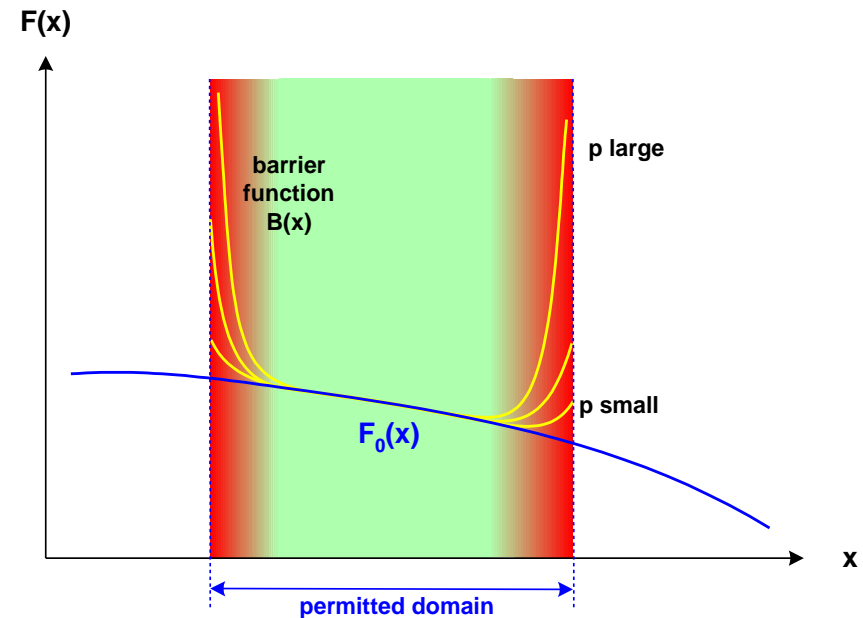
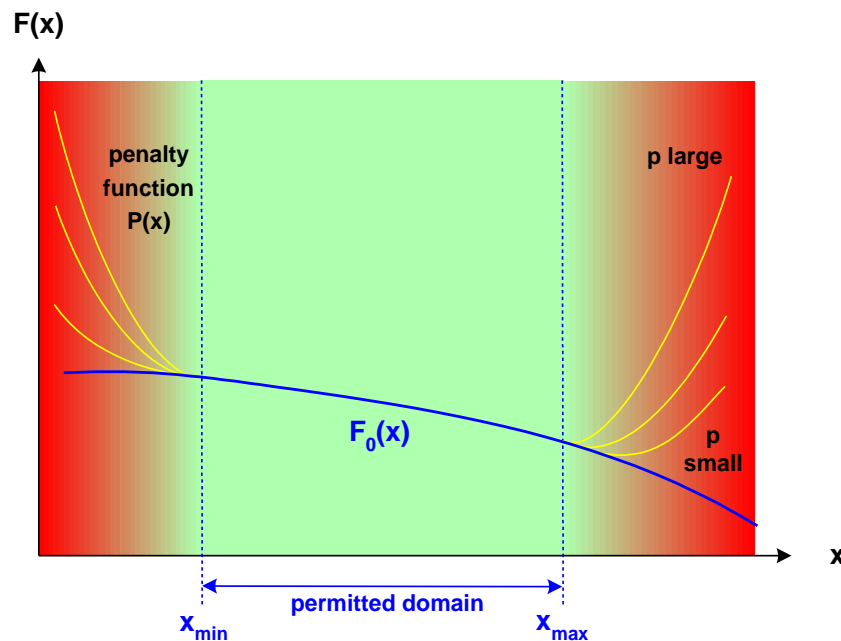
## Effect of Constraints on Optimization



# 7 Optimization I

## Boundary Conditions and Constraints

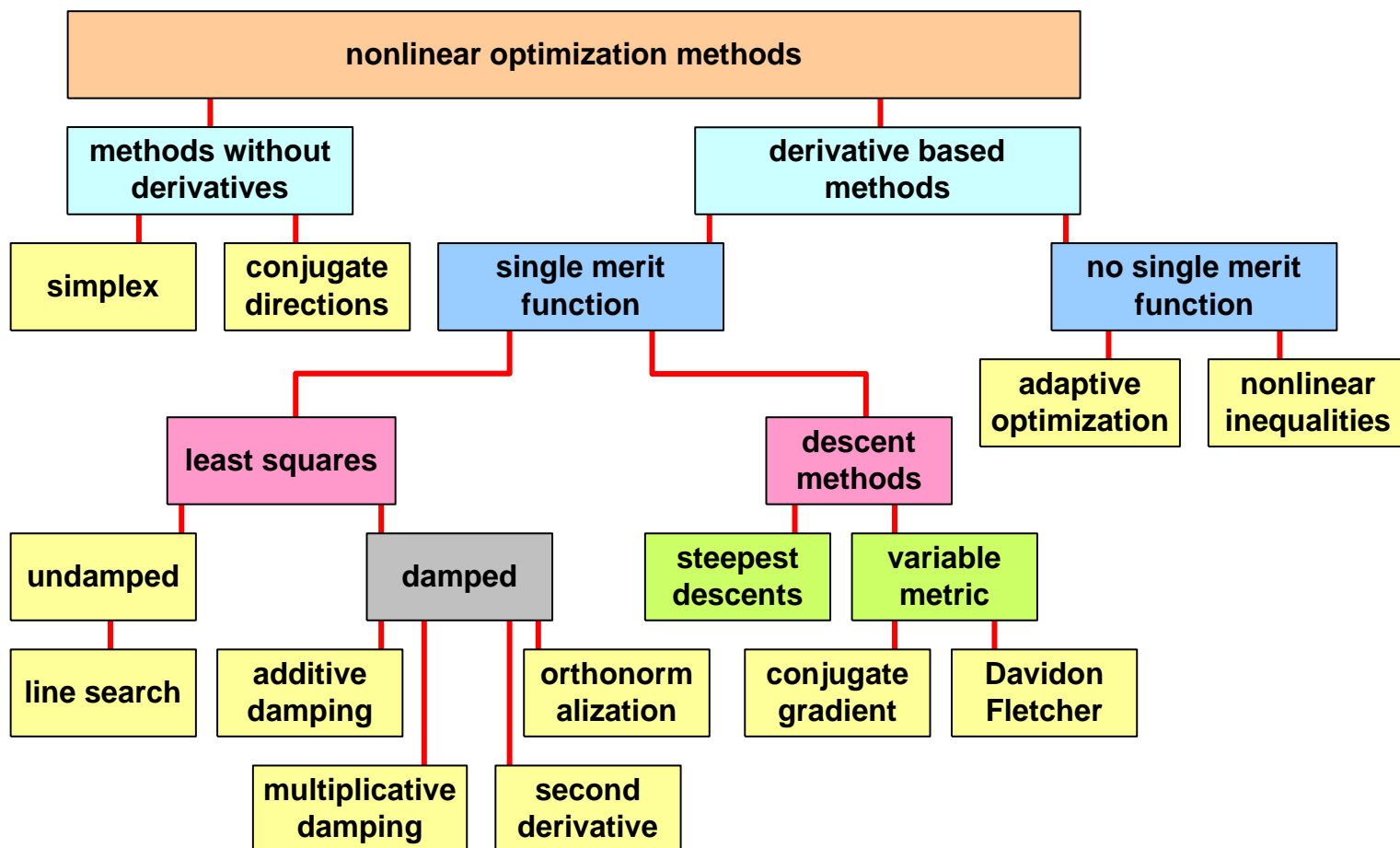
- Types of constraints
  - Equation, rigid coupling, pick up
  - One-sided limitation, inequality
  - Double-sided limitation, interval
- Numerical realizations :
  - Lagrange multiplier
  - Penalty function
  - Barriere function
  - Regular variable, soft-constraint



# 7 Optimization I

## Optimization Algorithms in Optical Design

Local working optimization algorithms



- Gauss-Newton method  
Normal equations

System matrix

$$\Delta \vec{x} = -(\underline{J}^T \cdot \underline{J})^{-1} \cdot \underline{J}^T \cdot \Delta \vec{f}$$

$$\underline{A} = (\underline{J}^T \cdot \underline{J})^{-1} \cdot \underline{J}^T$$

- Damped least squares method (DLS)  
Damping reduces step size, better convergence  
without oscillations

$$\Delta x_j = \left( J_{ij}^T \cdot J_{ij} + \lambda^2 \cdot I_{ij} \right)^{-1} \cdot J_{ij}^T \cdot \Delta f_i$$

- ACM method according to E.Glatzel  
Special algorithm with reduced error vector

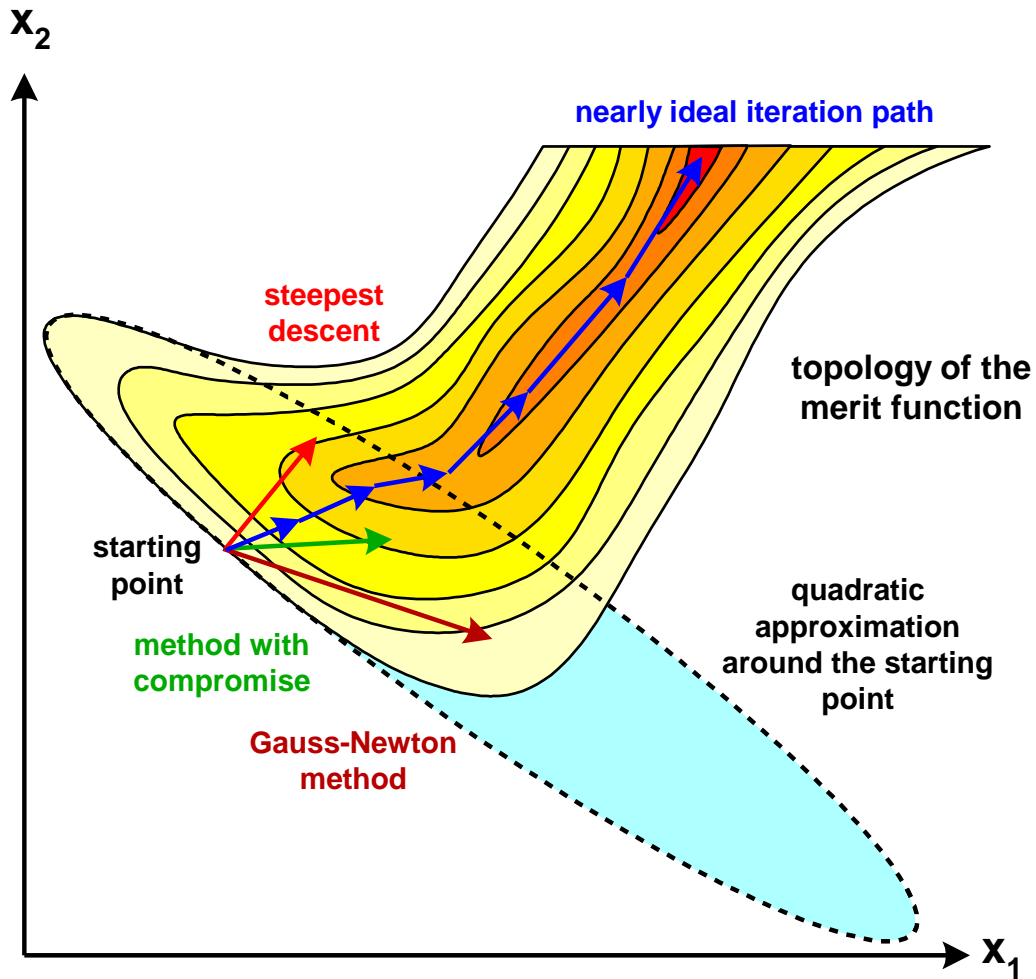
$$\Delta x_j = J_{ij}^T \cdot \left( J_{ij} \cdot J_{ij}^T \right)^{-1} \cdot \Delta f_i$$

- Conjugate gradient method  
Reduction of oscillations

# 7 Optimization I

## Optimization Minimum Search

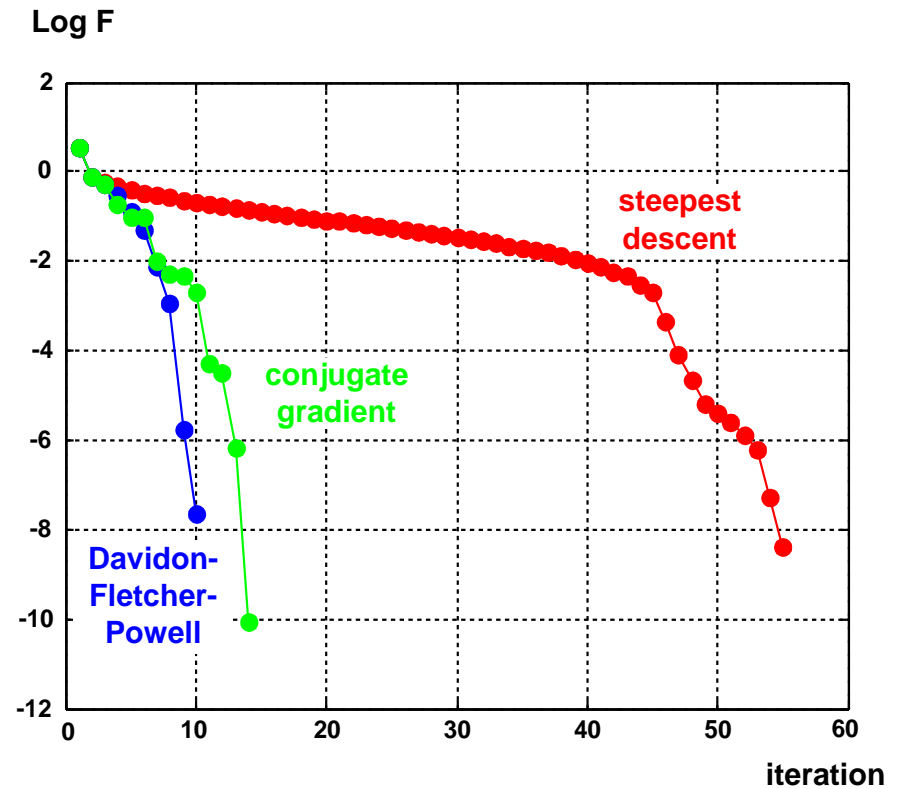
Principle of searching the local minimum



# 7 Optimization I

## Optimization: Convergence

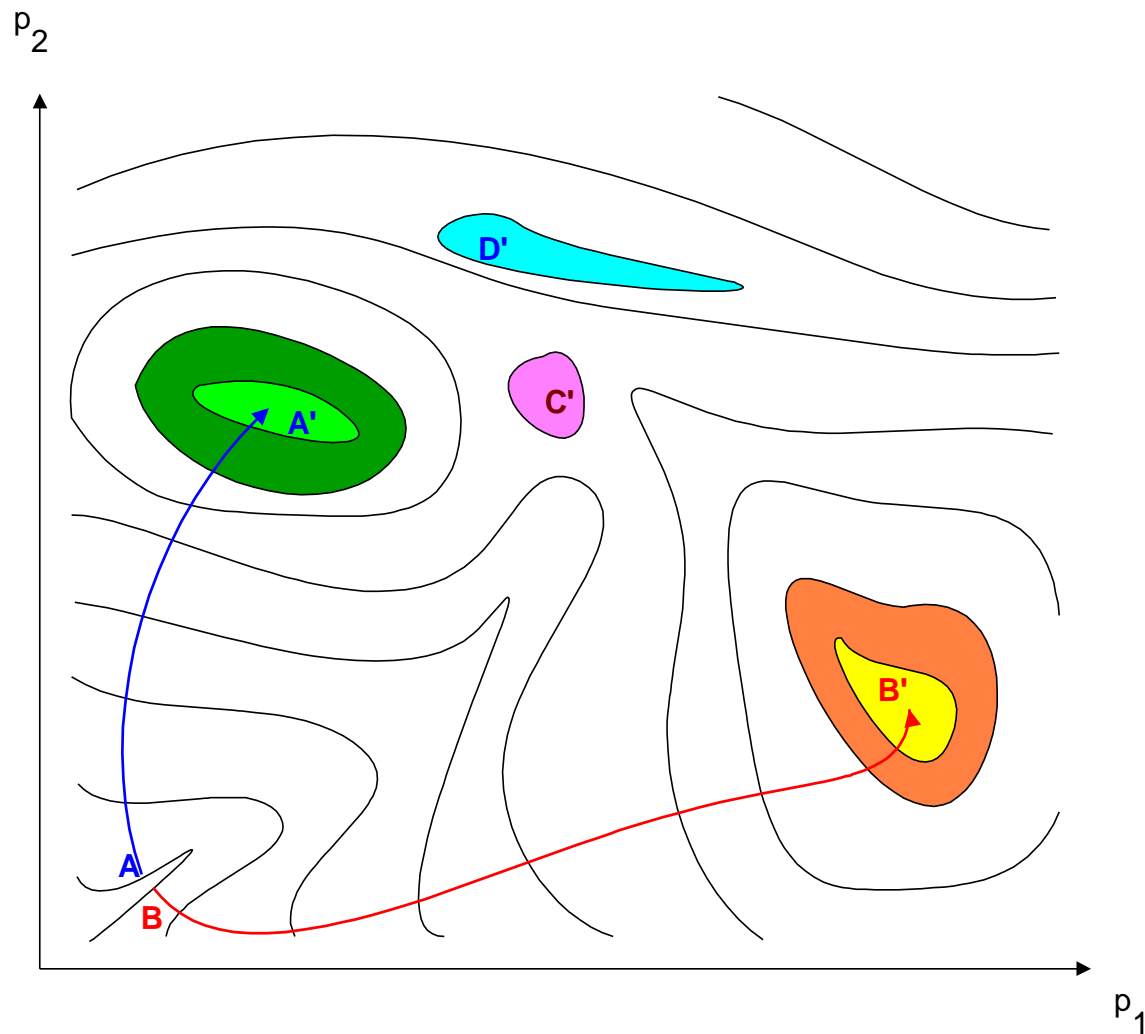
- Adaptation of direction and length of steps:  
rate of convergence
- Gradient method:  
slow due to zig-zag



# 7 Optimization I

## Optimization and Starting Point

- The initial starting point determines the final result
- Only the next located solution without hill-climbing is found



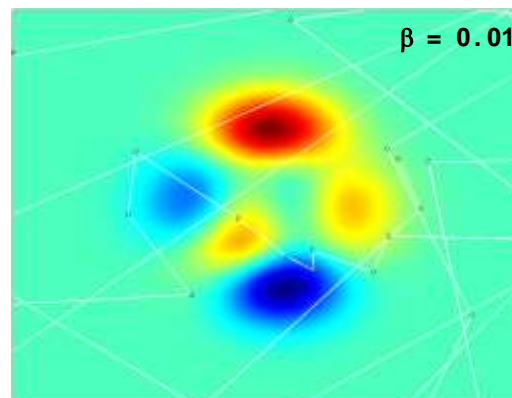
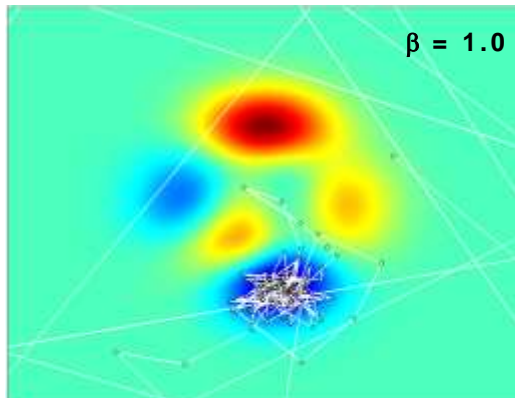
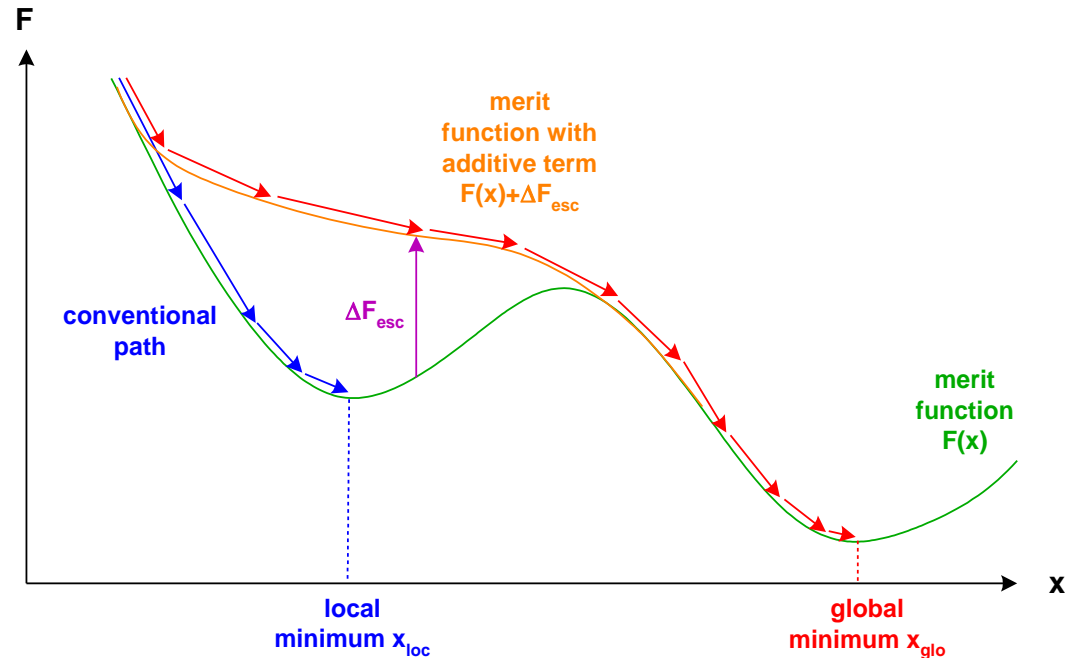
# 7 Optimization I

## Global Optimization: Simulated Annealing

- Simulated Annealing: temporarily added term to overcome local minimum

$$\Delta F_{esc}(\vec{x}) = \Delta F_0 \cdot e^{-\beta \cdot (F(\vec{x}) - F_0)^2}$$

- Optimization and adaptation of annealing parameters





- Goal of optimization:  
Find the system layout which meets the required performance targets according of the specification
  
- Formulation of performance criteria must be done for:
  - Apertur rays
  - Field points
  - Wavelengths
  - Optional several zoom or scan positions
  
- Selection of performance criteria depends on the application:
  - Ray aberrations
  - Spot diameter
  - Wavefornt description by Zernike coefficients, rms value
  - Strehl ratio, Point spread function
  - Contrast values for selected spatial frequencies
  - Uniformity of illumination
  
- Usual scenario:  
Number of requirements and targets quite larger than degrees od freedom,  
Therefore only solution with compromize possible

- Merit function:  
Weighted sum of deviations from target values
- Formulation of target values:
  1. fixed numbers
  2. one-sided interval (e.g. maximum value)
  3. interval
- Problems:
  1. linear dependence of variables
  2. internal contradiction of requirements
  3. initial value far off from final solution
- Types of constraints:
  1. exact condition (hard requirements)
  2. soft constraints: weighted target
- Finding initial system setup:
  1. modification of similar known solution
  2. Literature and patents
  3. Intuition and experience

$$\Phi = \sum_{j=1,m} g_j \cdot (f_j^{ist} - f_j^{soll})^2$$

- Characterization and description of the system delivers free variable parameters of the system:
  - Radii
  - Thickness of lenses, air distances
  - Tilt and decenter
  - Free diameter of components
  - Material parameter, refractive indices and dispersion
  - Aspherical coefficients
  - Parameter of diffractive components
  - Coefficients of gradient media
  
- General experience:
  - Radii as parameter very effective
  - Benefit of thickness and distances only weak
  - Material parameter can only be changes discrete

# 7 Optimization I

## Constraints in Optical Systems

Constraints in the optimization of optical systems:

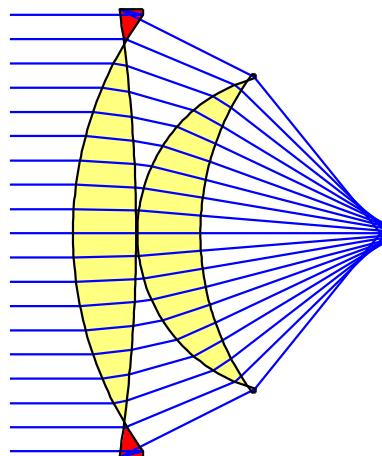
1. Discrete standardized radii (tools, metrology)
2. Total track
3. Discrete choice of glasses
4. Edge thickness of lenses (handling)
5. Center thickness of lenses (stability)
6. Coupling of distances (zoom systems, forced symmetry,...)
7. Focal length, magnification, working distance
8. Image location, pupil location
9. Avoiding ghost images (no concentric surfaces)
10. Use of given components (vendor catalog, availability, costs)

# 7 Optimization I

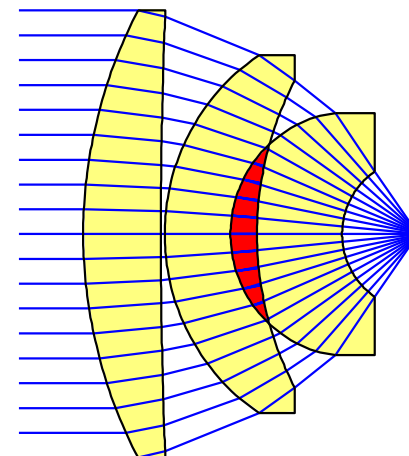
## Lack of Constraints in Optimization

Illustration of not useful results due to non-sufficient constraints

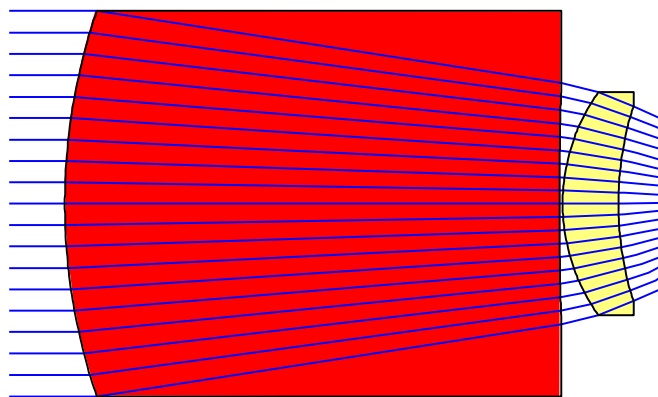
**negative edge thickness**



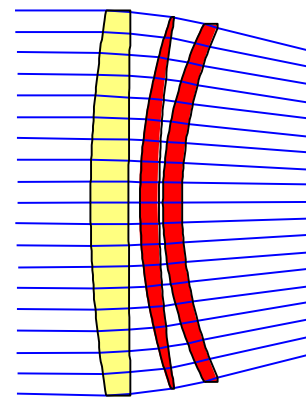
**negative air distance**



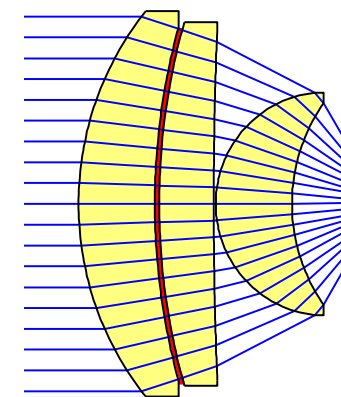
**lens thickness to large**



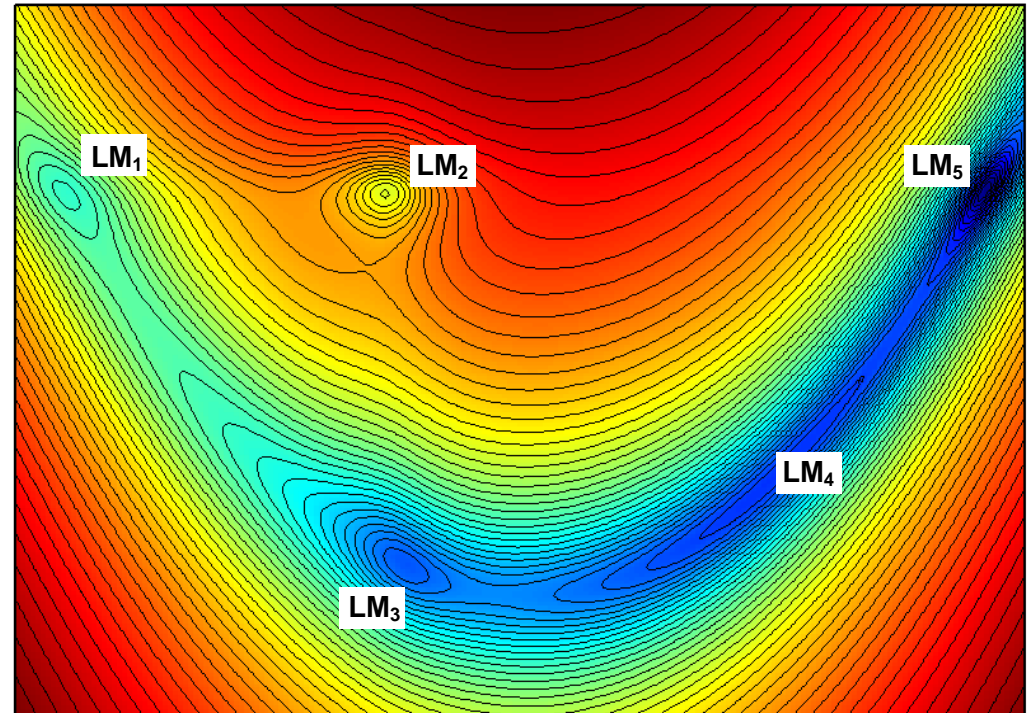
**lens stability to small**



**air space to small**



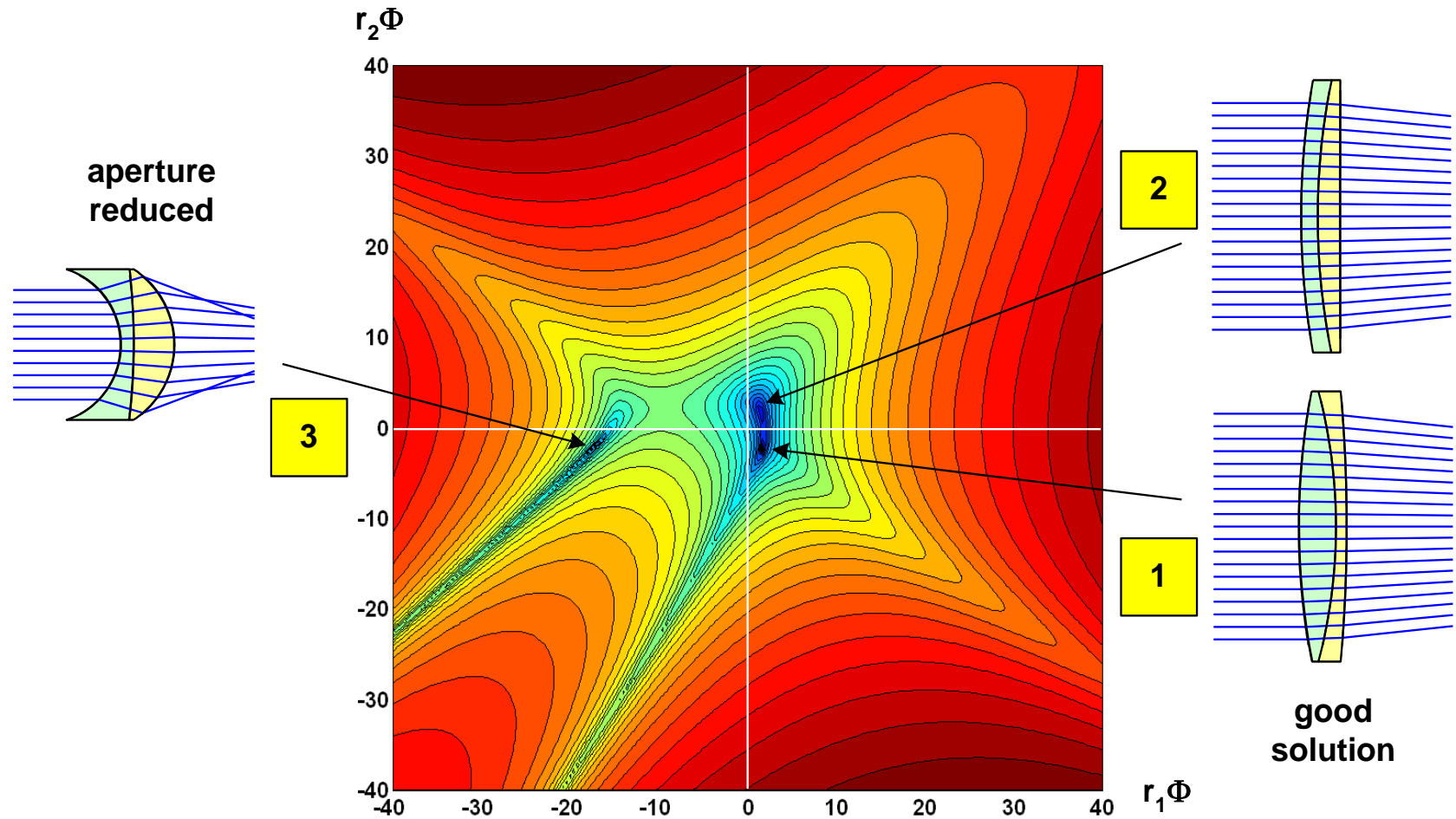
- Typical in optics:  
Twisted valleys in the topology
- Selection of local minima



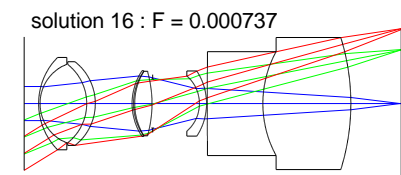
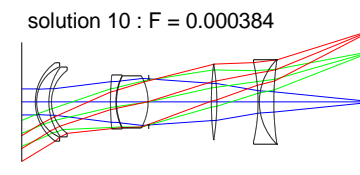
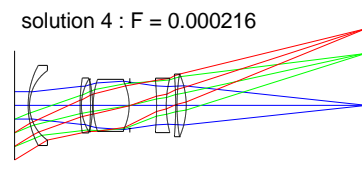
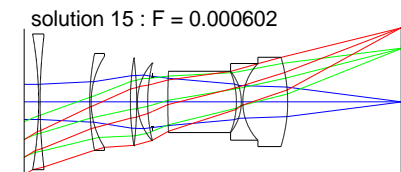
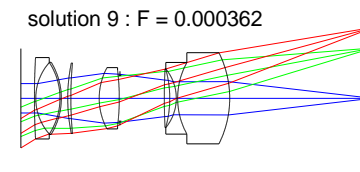
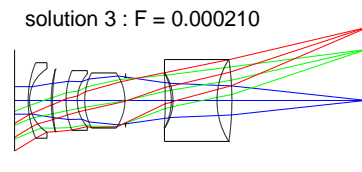
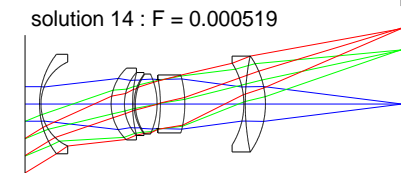
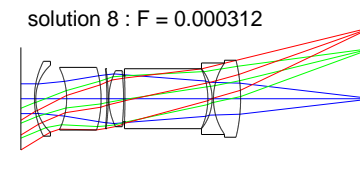
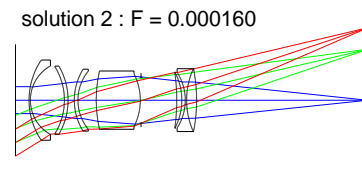
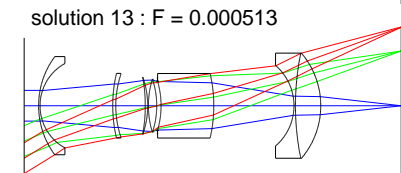
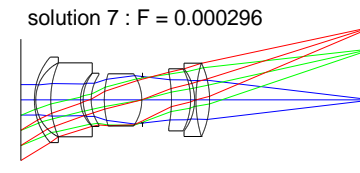
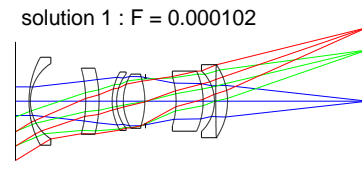
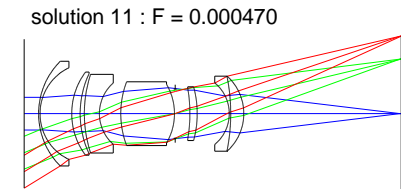
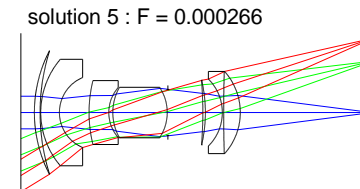
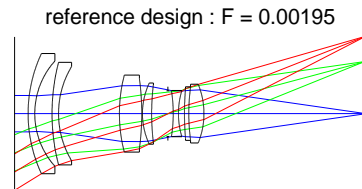
# 7 Optimization I

## Optimization Landscape of an Achromate

- Typical merit function of an achromate
- Three solutions, only two are useful



- No unique solution
- Constraints not sufficient fixed:  
unwanted lens shapes
- Many local minima with nearly the same performance

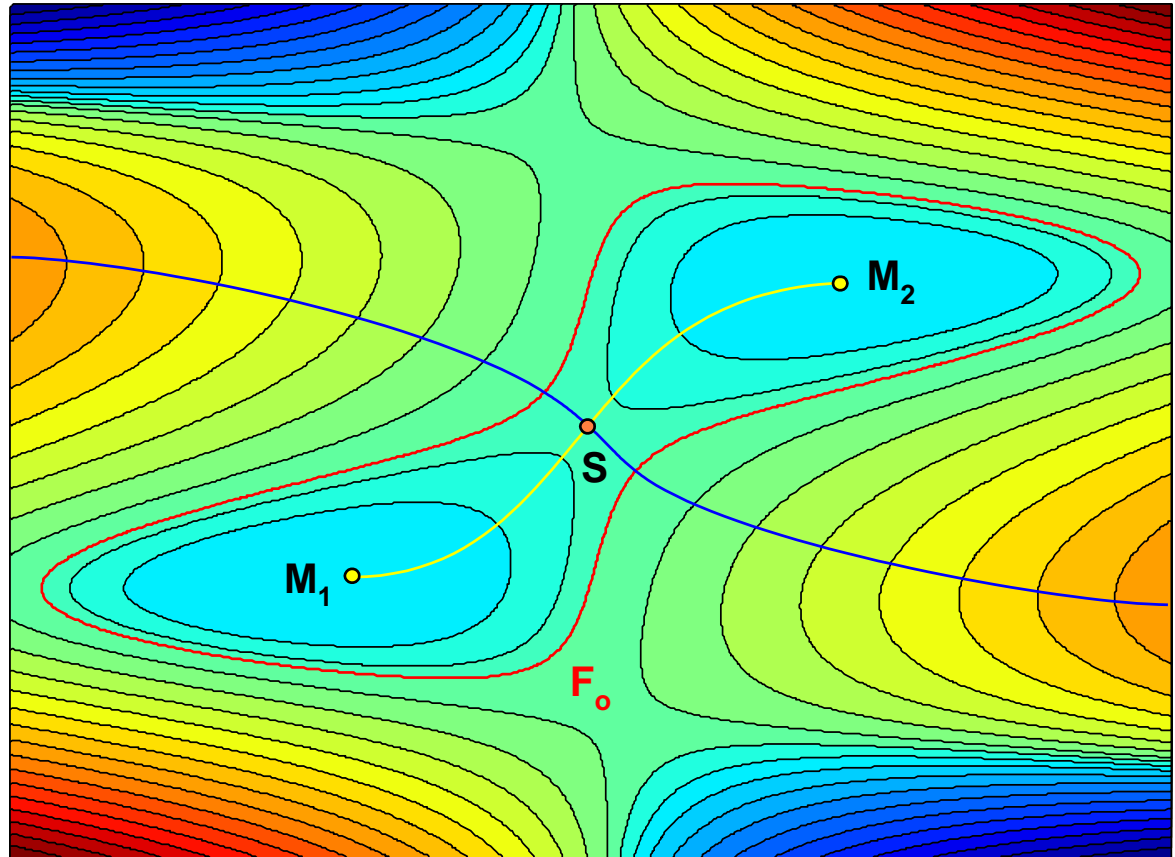




# 7 Optimization I

## Saddel Point Method

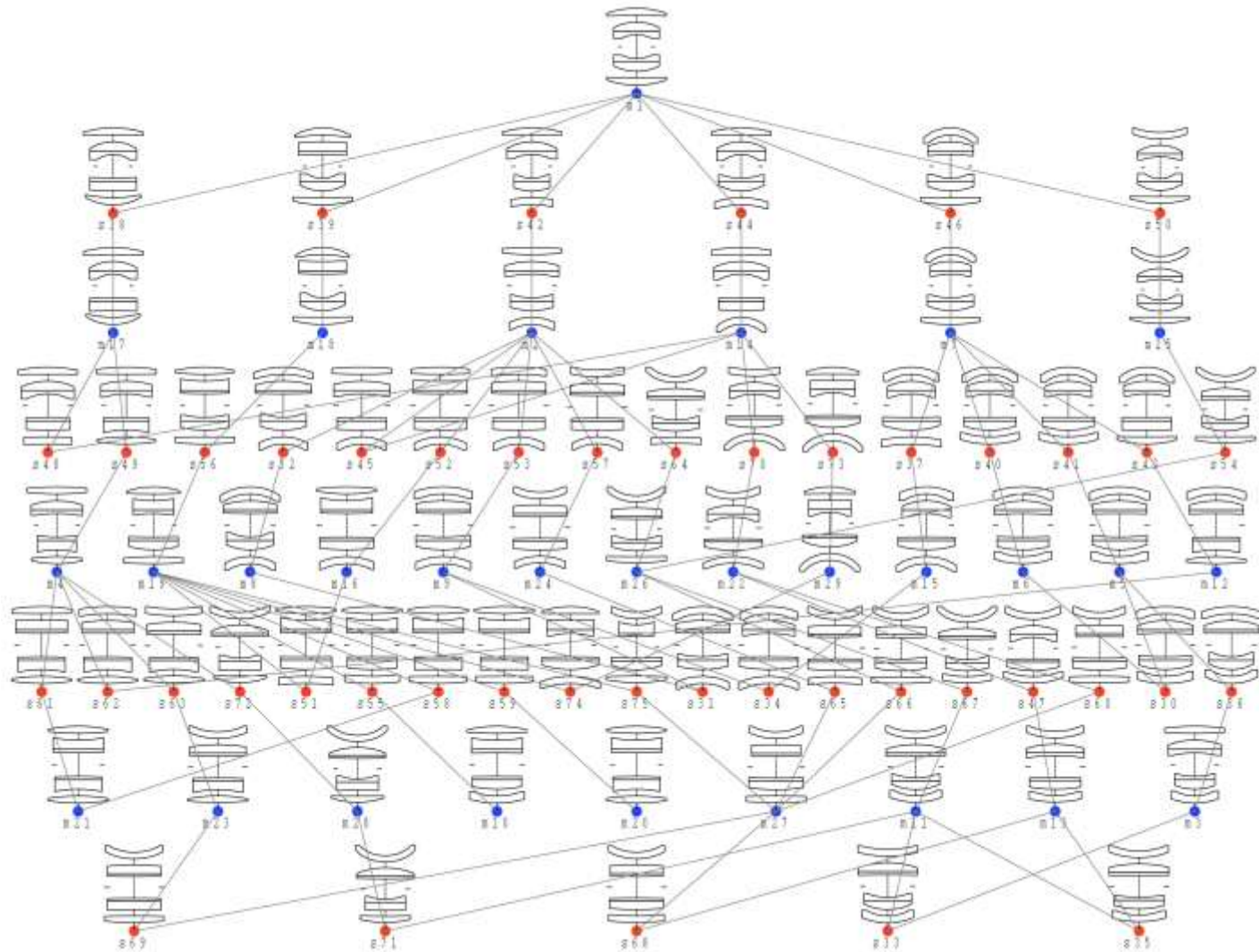
- Saddle points in the merit function topology
- Systematic search of adjacent local minima is possible
- Exploration of the complete network of local minima via saddelpoints



# 7 Optimization I

## Saddle Point Method

- Example Double Gauss lens of system network with saddelpoints



# 7 Optimization I

## Optimization: Discrete Materials

- Special problem in glass optimization: finite area of definition with discrete parameters  $n$ ,  $v$
- Restricted permitted area as one possible constraint
- Model glass with continuous values of  $n$ ,  $v$  in a pre-phase of glass selection, freezing to the next adjacent glass

