



**Institute of
Applied Physics**

Friedrich-Schiller-Universität Jena

Optical Design with Zemax

Lecture 4: Aberrations I

2012-11-06

Herbert Gross

4 Aberrations I

Time schedule

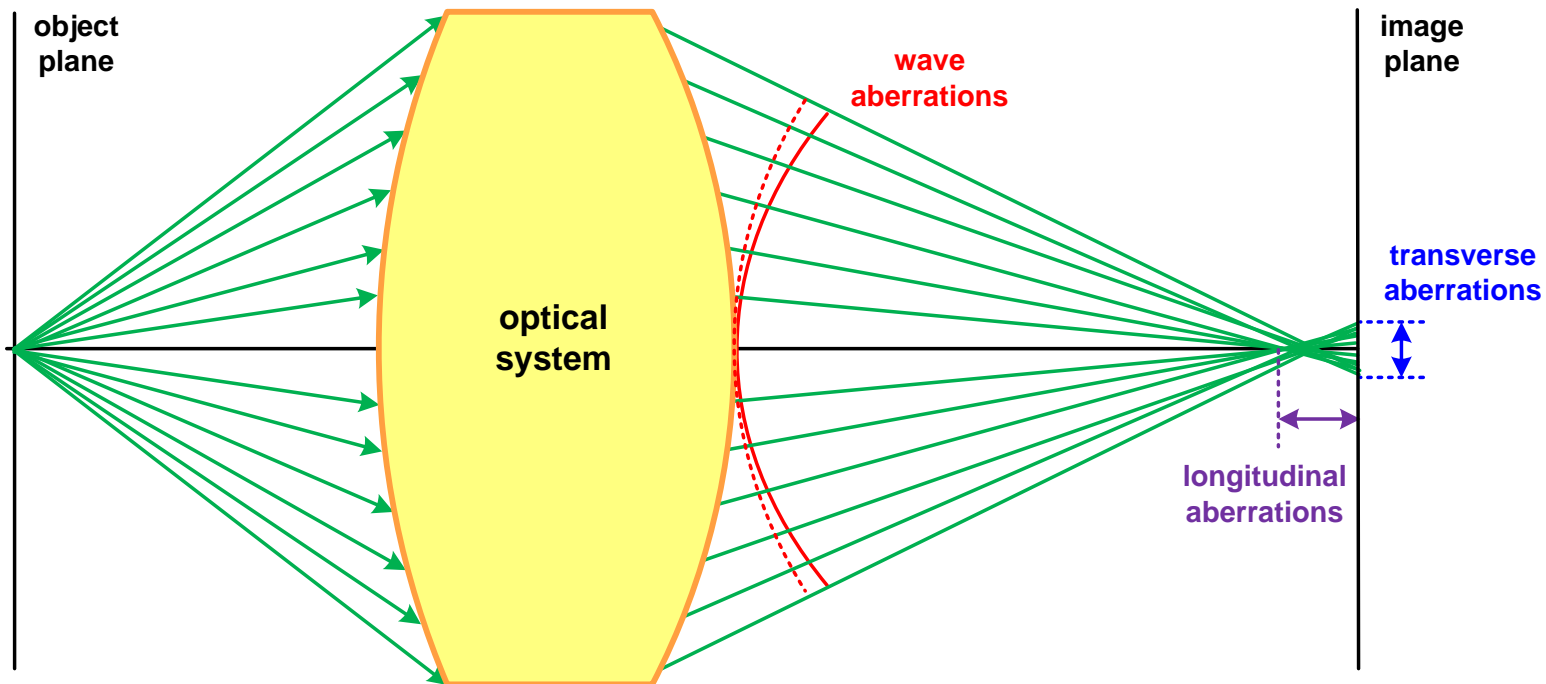
1	16.10.	Introduction	Introduction, Zemax interface, menus, file handling, preferences, Editors, updates, windows, Coordinate systems and notations, System description, Component reversal, system insertion, scaling, 3D geometry, aperture, field, wavelength
2	23.10.	Properties of optical systems I	Diameters, stop and pupil, vignetting, Layouts, Materials, Glass catalogs, Raytrace, Ray fans and sampling, Footprints
3	30.10.	Properties of optical systems II	Types of surfaces, Aspheres, Gratings and diffractive surfaces, Gradient media, Cardinal elements, Lens properties, Imaging, magnification, paraxial approximation and modelling
4	06.11.	Aberrations I	Representation of geometrical aberrations, Spot diagram, Transverse aberration diagrams, Aberration expansions, Primary aberrations,
5	13.11.	Aberrations II	Wave aberrations, Zernike polynomials, Point spread function, Optical transfer function
6	20.11.	Optimization I	Principles of nonlinear optimization, Optimization in optical design, Global optimization methods, Solves and pickups, variables, Sensitivity of variables in optical systems
7	27.11.	Optimization II	Systematic methods and optimization process, Starting points, Optimization in Zemax
8	04.12	Imaging	Fundamentals of Fourier optics, Physical optical image formation, Imaging in Zemax
9	11.12.	Illumination	Introduction in illumination, Simple photometry of optical systems, Non-sequential raytrace, Illumination in Zemax
10	18.12.	Advanced handling I	Telecentricity, infinity object distance and afocal image, Local/global coordinates, Add fold mirror, Scale system, Make double pass, Vignetting, Diameter types, Ray aiming, Material index fit
11	08.01.	Advanced handling II	Report graphics, Universal plot, Slider, Visual optimization, IO of data, Multiconfiguration, Fiber coupling, Macro language, Lens catalogs
12	15.01.	Correction I	Symmetry principle, Lens bending, Correcting spherical aberration, Coma, stop position, Astigmatism, Field flattening, Chromatical correction, Retrofocus and telephoto setup, Design method
13	22.01.	Correction II	Field lenses, Stop position influence, Aspheres and higher orders, Principles of glass selection, Sensitivity of a system correction, Microscopic objective lens, Zoom system
14	29.01.	Physical optical modelling I	Gaussian beams, POP propagation, polarization raytrace, polarization transmission, polarization aberrations
15	05.02.	Physical optical modelling II	coatings, representations, transmission and phase effects, ghost imaging, general straylight with BRDF

4 Aberrations I

Contents

1. Representation of geometrical aberrations
2. Spot diagram
3. Transverse aberration diagrams
4. Aberration expansions
5. Primary aberrations

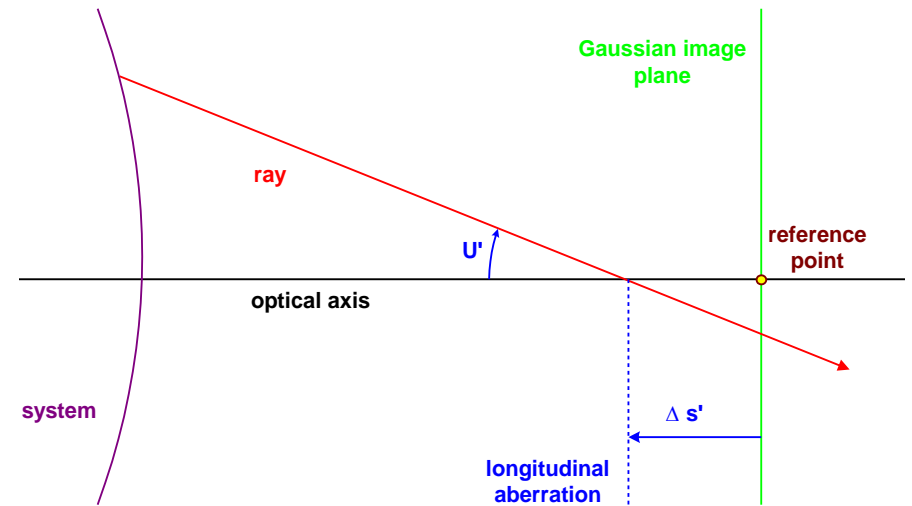
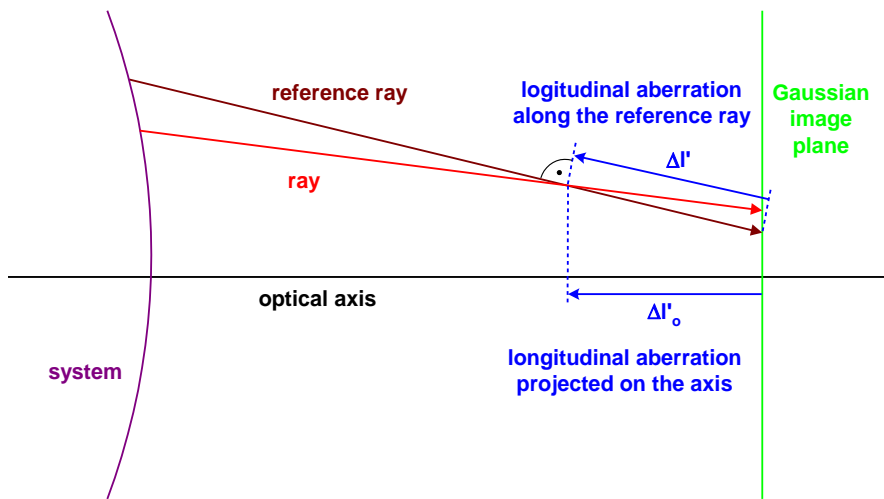
- Perfect optical image:
All rays coming from one object point intersect in one image point
- Real system with aberrations:
 1. transverse aberrations in the image plane
 2. longitudinal aberrations from the image plane
 3. wave aberrations in the exit pupil



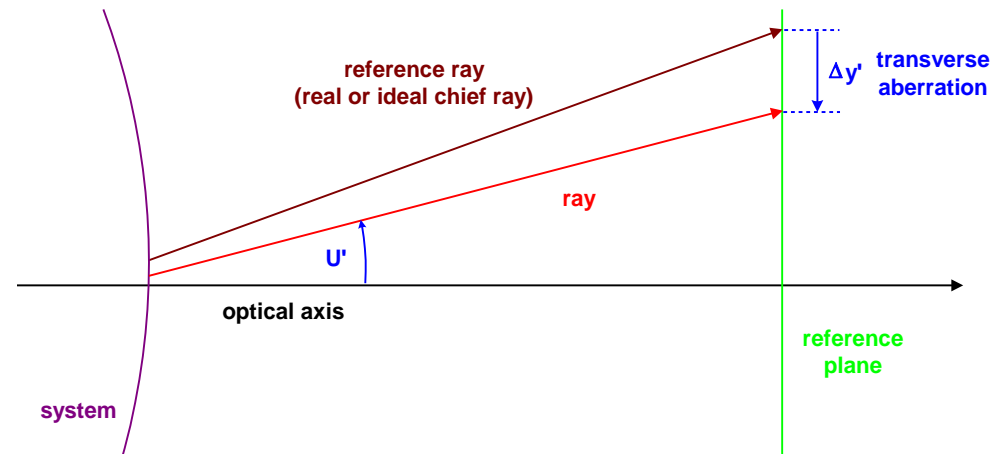
4 Aberrations I

Representation of Geometrical Aberrations

- Longitudinal aberrations Δs



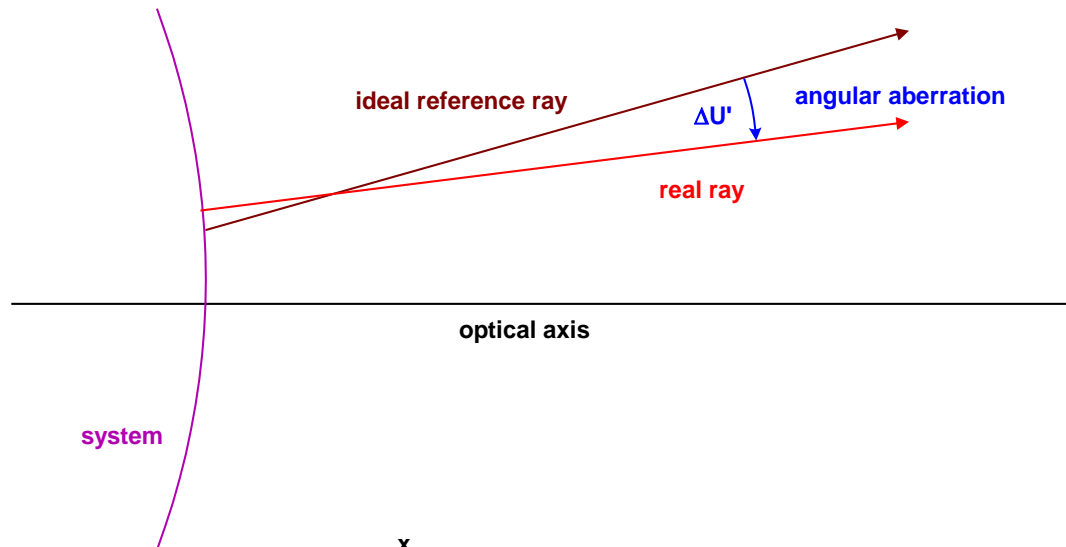
- Transverse aberrations Δy



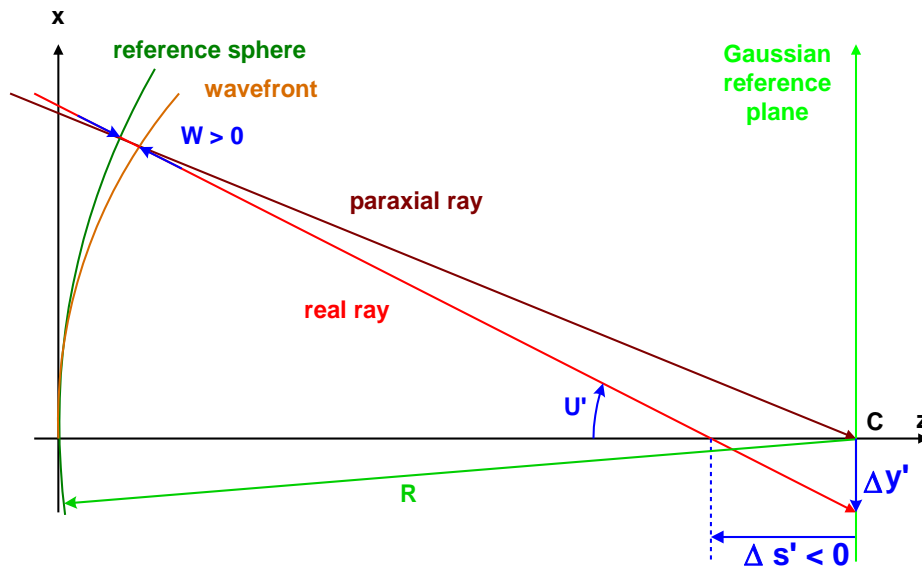
4 Aberrations I

Representation of Geometrical Aberrations

- Angle aberrations Δu

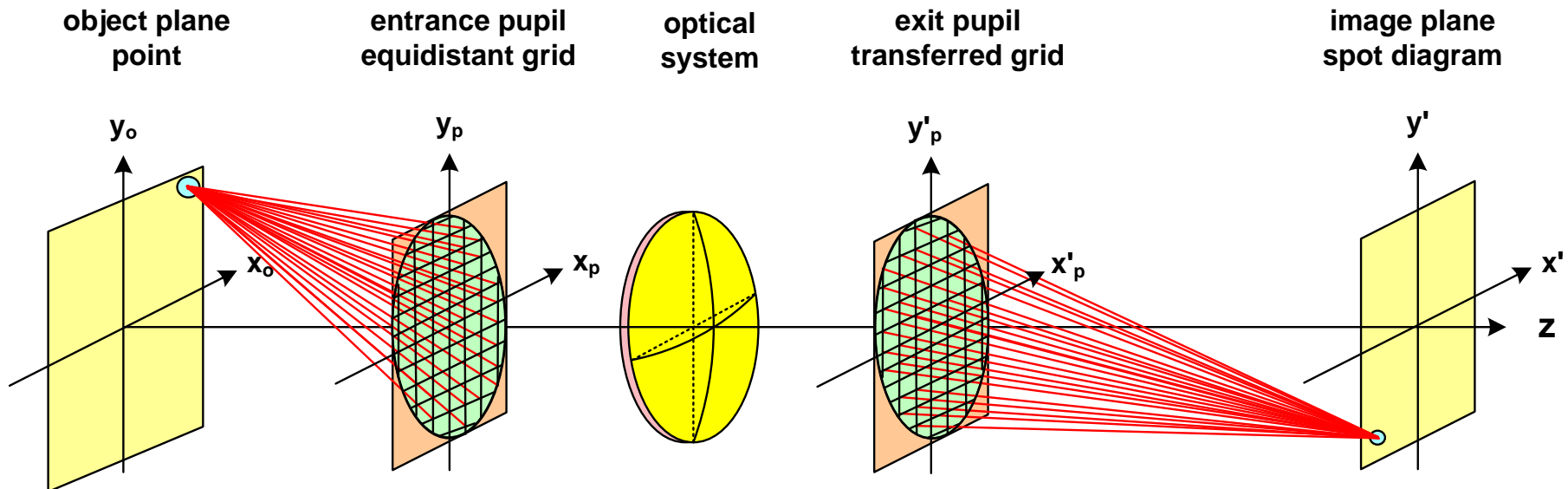
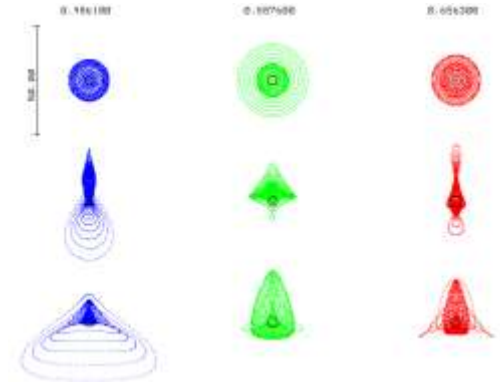
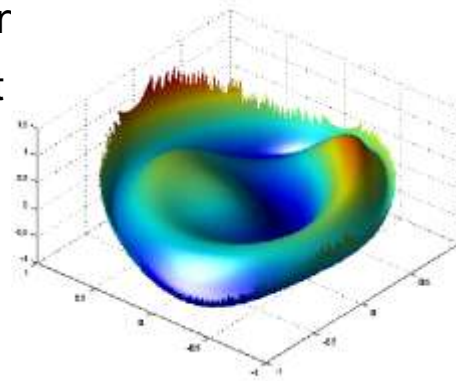


- Wave aberrations ΔW



4 Aberrations I Spot Diagram

- All rays start in one point in the object plane
- The entrance pupil is sampled equidistant
- In the exit pupil, the transferred grid may be distorted
- In the image plane a spreaded spot diagram is generated



4 Aberrations I

Spot Diagram

- Table with various values of:

1. Field size

2. Color

- Small circle:

Airy diameter for **axis** comparison

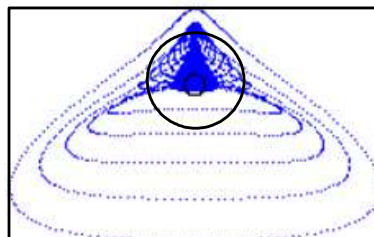
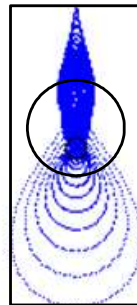
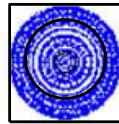
- Large circle:

Gaussian moment

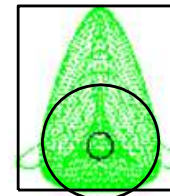
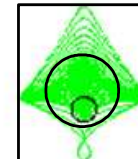
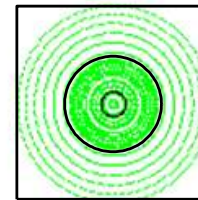
field zone

full field

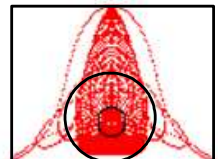
486 nm



546 nm



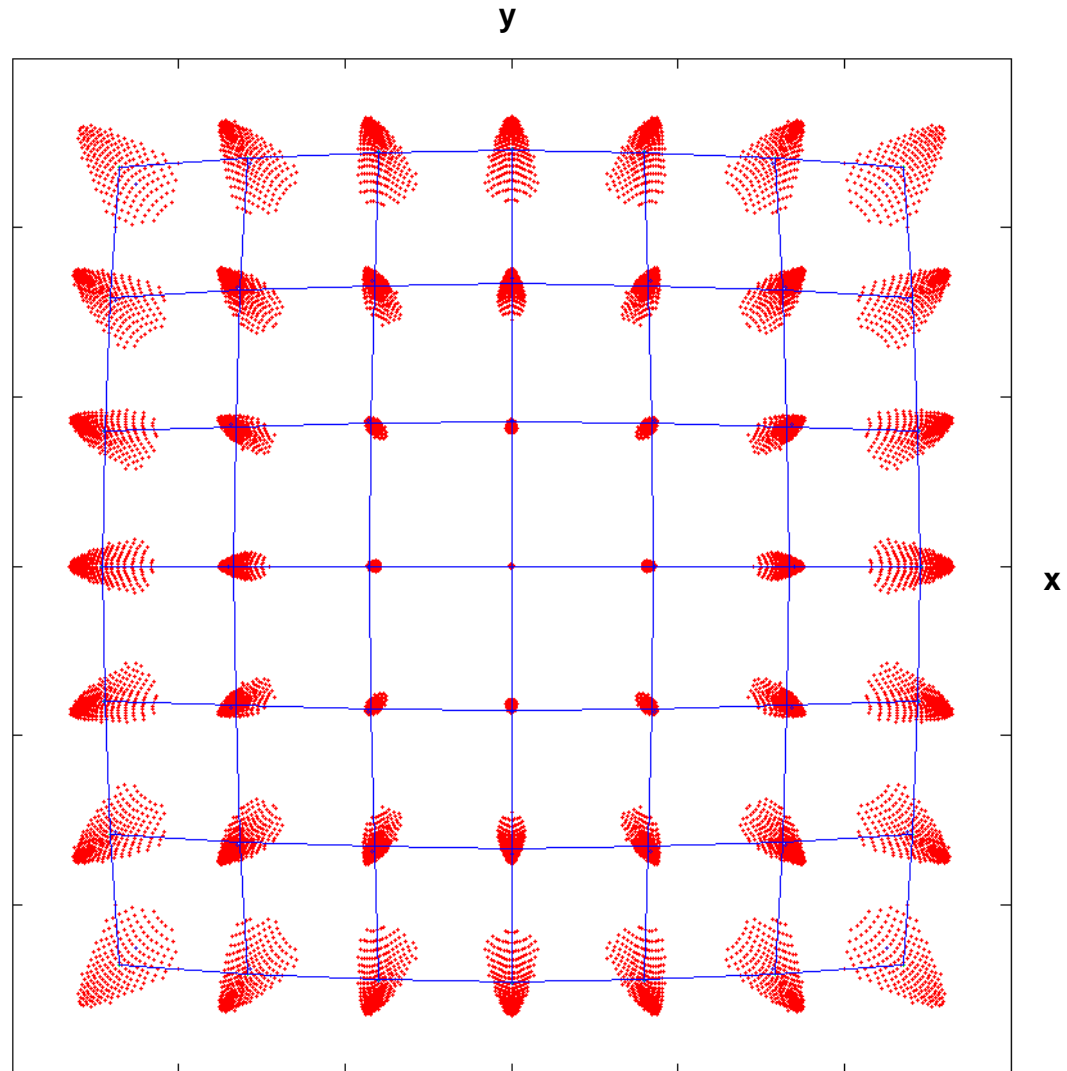
656 nm



4 Aberrations I

Aberrations of a Single Lens

- Single plane-convex lens,
BK7, $f = 100 \text{ mm}$, $\lambda = 500 \text{ nm}$
- Spot as a function of
field position
- Coma shape rotates according
to circular symmetry
- Decrease of performance with
the distance to the axis

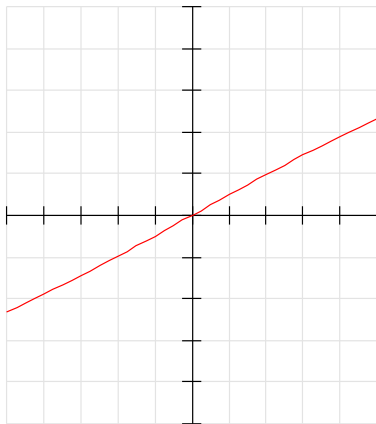


4 Aberrations I

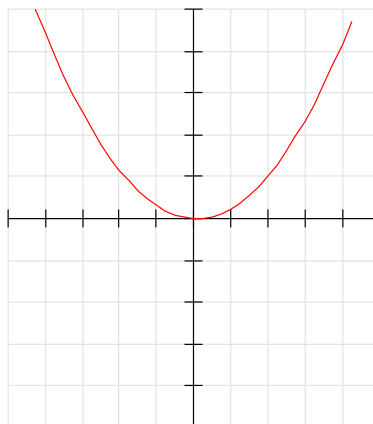
Transverse Aberrations

- Typical low order polynomial contributions for:
Defocus, coma, spherical, lateral color
- This allows a quick classification of real curves

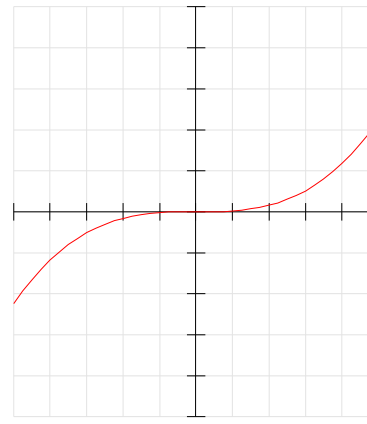
**linear:
defocus**



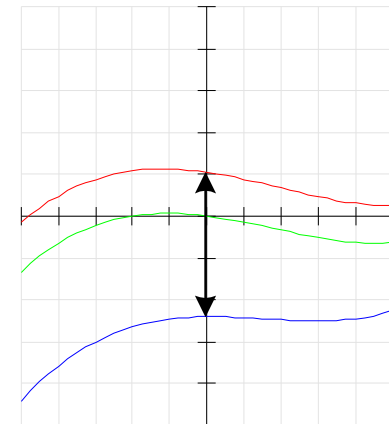
**quadratic:
coma**



**cubic:
spherical**



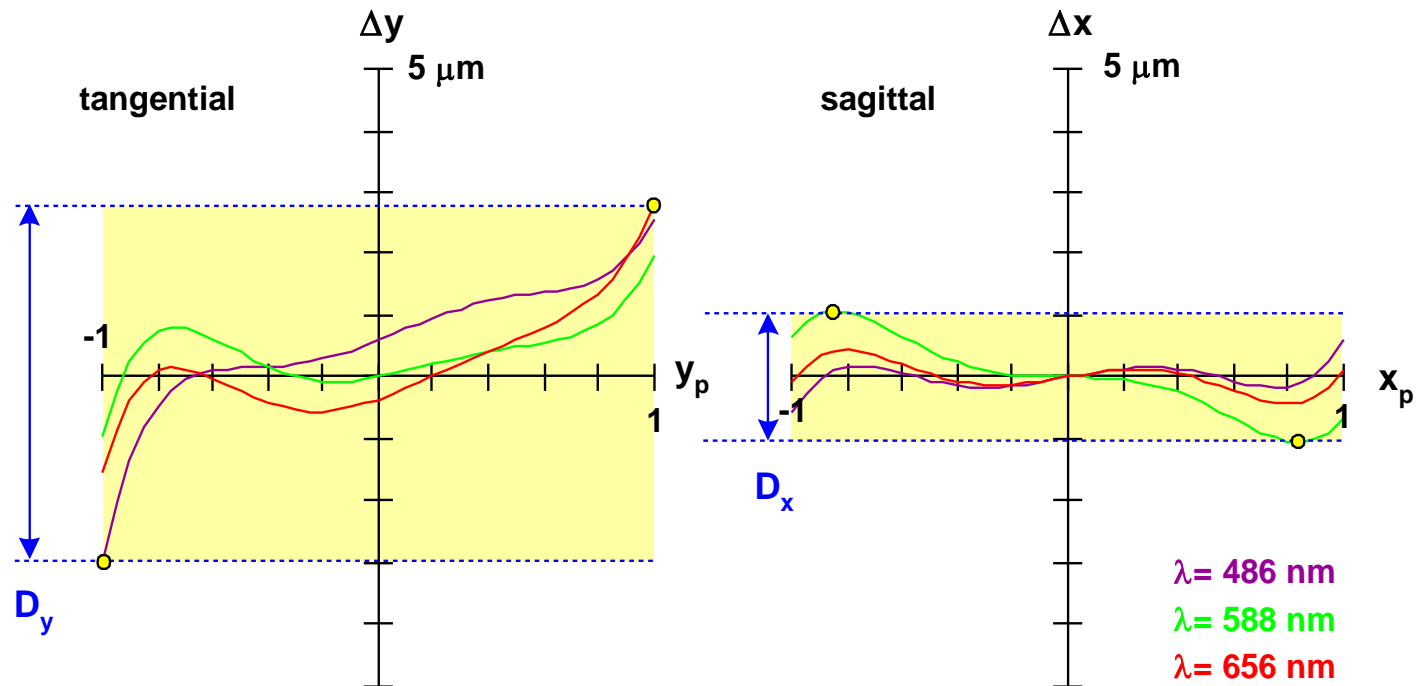
**offset:
lateral color**



4 Aberrations I

Transverse Aberrations

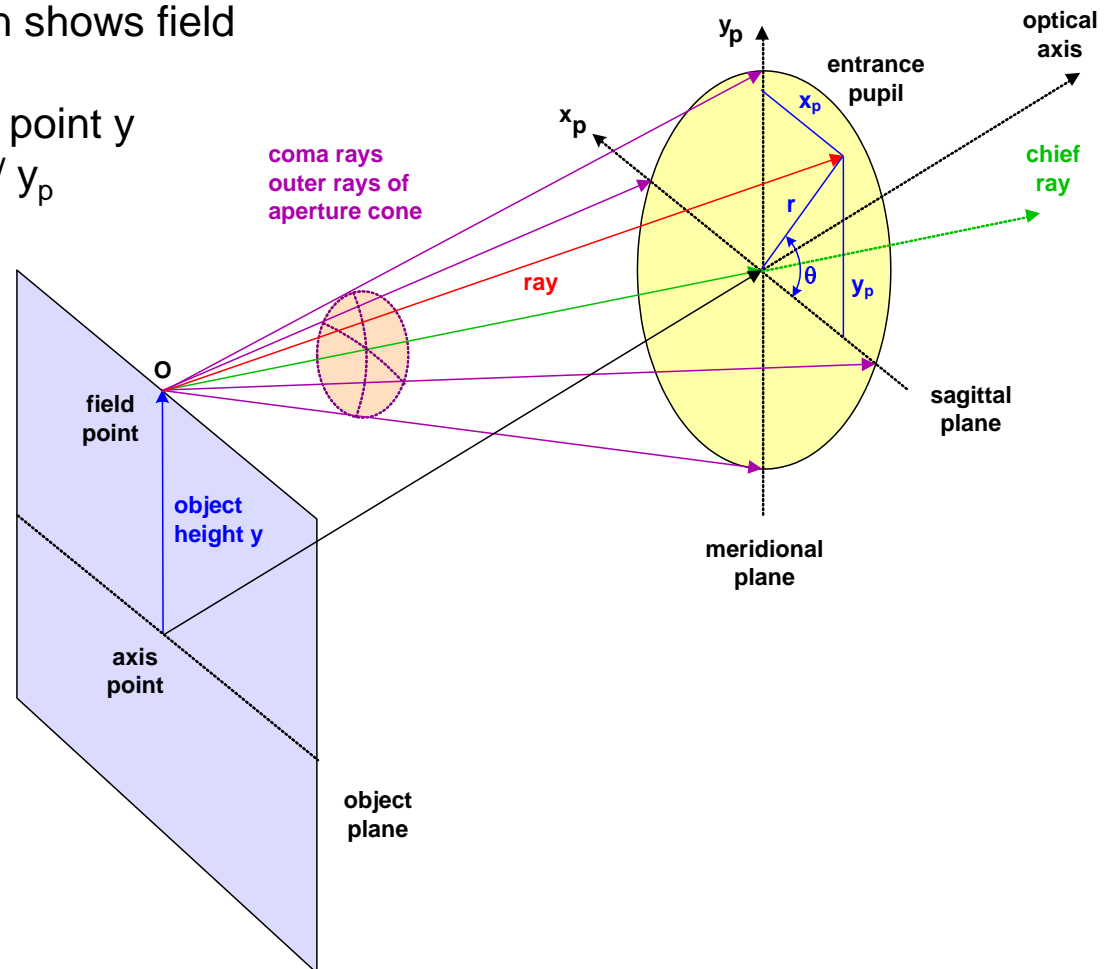
- Classical aberration curves
- Strong relation to spot diagram
- Usually only linear sampling along the x-, y-axis
no information in the quadrant of the aperture



4 Aberrations I

Polynomial Expansion of the Aberrations

- Paraxial optics: small field and aperture angles
Aberrations occur for larger angle values
- Two-dimensional Taylor expansion shows field and aperture dependence
- Expansion for one meridional field point y
- Pupil: cartesian or polar grid in x_p / y_p



4 Aberrations I

Polynomial Expansion of Aberrations

- Taylor expansion of the deviation:

y' Image height index k
 r_p Pupil height index l
 θ Pupil azimuth angle index m

$$\Delta y(y', r_p, \theta) = \sum_{k,l,m} a_{klm} \cdot y'^k \cdot r_p^l \cdot \cos^m \theta$$

- Symmetry invariance: selection of special combinations of exponent terms

- Number of terms: sum of indices in the exponent i_{sum}

i_{sum}	number of terms	Type of aberration
2	2	image location
4	5	primary aberrations, 3rd/4th order
6	9	secondary aberrations, 5th/6th order
8	14	higher order

- The order of the aperture function depends on the aberration type used:
 primary aberrations:
 - 3rd order in transverse aberration Δy
 - 4th order in wave aberration W
 Since the coupling relation

$$\Delta y = -R \cdot \frac{\partial W}{\partial x_p}$$

changes the order by 1

4 Aberrations I

Polynomial Expansion of Aberrations



- Representation of 2-dimensional Taylor series vs field y and aperture r
- Selection rules: checkerboard filling of the matrix
- Constant sum of exponents according to the order

		Field y →						
		y^0	y^1	y^2	y^3	y^4	y^5	
Aperture r ↓	Distortion	r^0		$y \cos\theta$ Tilt		$y^3 \cos\theta$ Distortion primary		$y^5 \cos\theta$ Distortion secondary
		r^1	r^1 Defocus		$y^2 r^1 \cos^2\theta$ $y^2 r^1$ Astig./Curvat.		$y^4 r^1 \cos^2\theta$ $y^4 r^1$	
		r^2		$y r^2 \cos\theta$ Coma primary		$y^3 r^2 \cos^3\theta$ $y^3 r^2 \cos\theta$		
		r^3	r^3 Spherical primary		$y^2 r^3 \cos^2\theta$ $y^2 r^3$			
		r^4		$y r^4 \cos\theta$ Coma secondary				
		r^5	r^5 Spherical secondary					

Image location (points to y^3 column)

Primary aberrations / Seidel (points to y^3 column)

Secondary aberrations (points to y^5 column)

- Expansion of the transverse aberration Δy on image height y and pupil height r
- Lowest order 3 of real aberrations: primary or Seidel aberrations

- Spherical aberration: S

- no dependence on field, valid on axis
- depends in 3rd order on apertur

$$\Delta y = r^3 \cdot S + r^2 \cdot y \cdot r^2 \cdot \cos \theta \cdot C$$
$$+ y^2 \cdot r \cdot \cos^2 \theta \cdot A + y^2 \cdot r \cdot P$$

- Coma: C

- linear function of field y
- depends in 2nd order on apertur with azimuthal variation

$$+ y^3 \cdot D$$

- Astigmatism: A

- linear function of apertur with azimuthal variation
- quadratic function of field size

- Image curvature (Petzval): P

- linear dependence on apertur
- quadratic function of field size

- Distortion: D

- No dependence on apertur
- depends in 3rd order on the field size

4 Aberrations I

Transverse Aberrations of Seidel



- Transverse deviations
- Sum of surface contributions

$$S' = \sum_{j=1}^k S_j$$

$$C' = \sum_{j=1}^k C_j$$

$$A' = \sum_{j=1}^k A_j$$

$$P' = \sum_{j=1}^k P_j$$

$$D' = \sum_{j=1}^k D_j$$

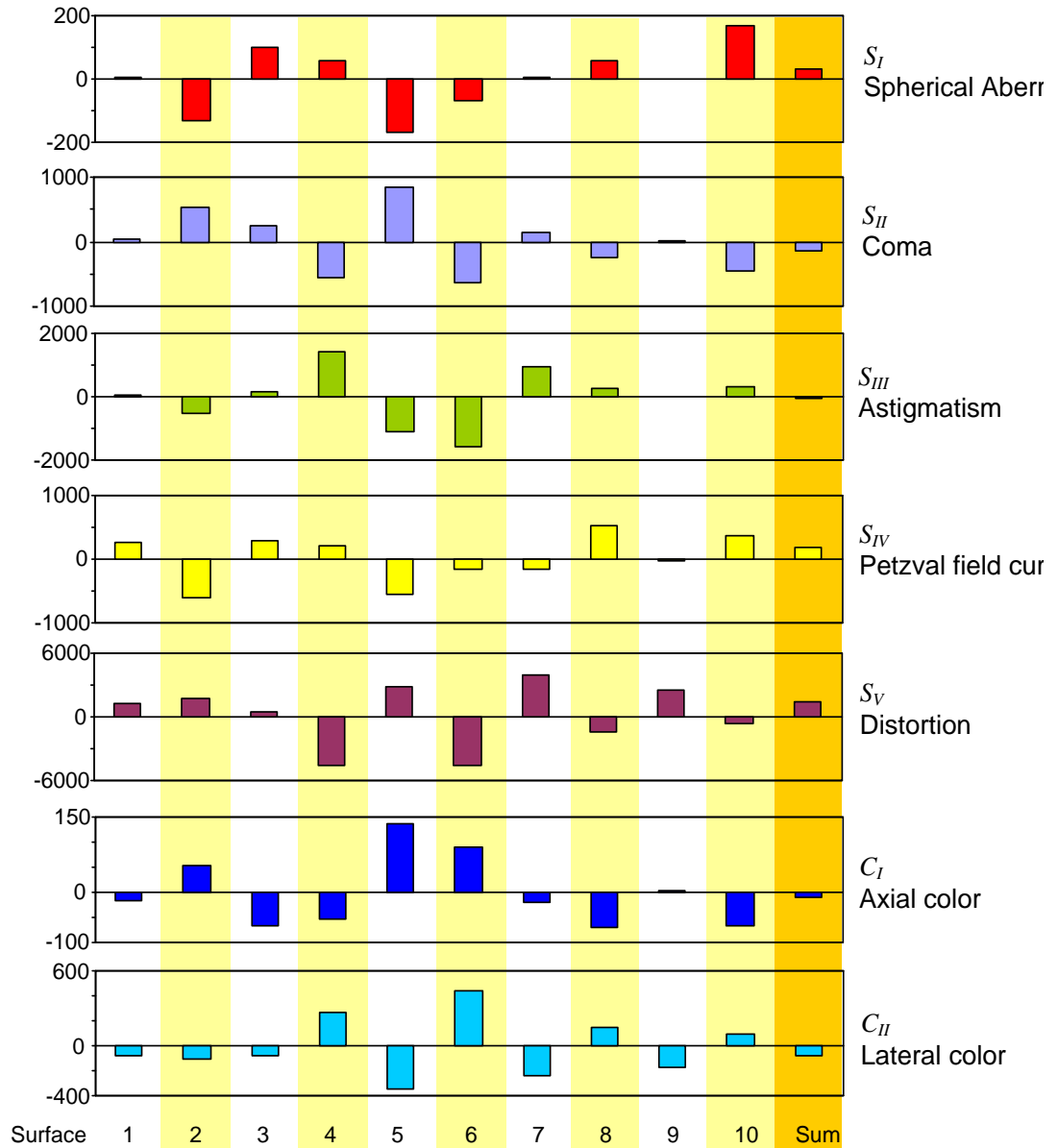
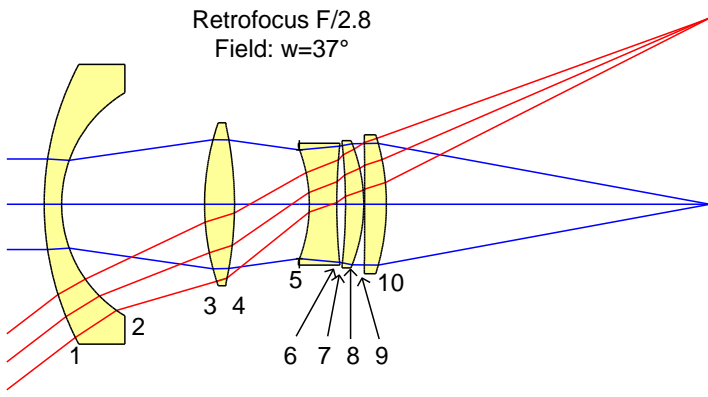
$$\Delta x' = \frac{x'_p (x_p'^2 + y_p'^2) s'^4}{2n' R_p'^3} S' - \frac{[2x'_p (x' x'_p + y' y'_p) + x' (x_p'^2 + y_p'^2)] s'^3 s'_p}{2n' R_p'^3} C' + \frac{x' (x' x'_p + y' y'_p) s'^2 s_p'^2}{n' R_p'^3} A' + \frac{x'_p (x_p'^2 + y_p'^2) s'^2 s_p'^2}{2n' R_p'^3} P' - \frac{x' (x'^2 + y'^2) s' s_p'^3}{2n' R_p'^3} D'$$

$$\Delta y' = \frac{y'_p (x_p'^2 + y_p'^2) s'^4}{2n' R_p'^3} S' - \frac{[2y'_p (x' x'_p + y' y'_p) + y' (x_p'^2 + y_p'^2)] s'^3 s'_p}{2n' R_p'^3} C' + \frac{y' (x' x'_p + y' y'_p) s'^2 s_p'^2}{n' R_p'^3} A' + \frac{y'_p (x_p'^2 + y_p'^2) s'^2 s_p'^2}{2n' R_p'^3} P' - \frac{y' (x'^2 + y'^2) s' s_p'^3}{2n' R_p'^3} D'$$

4 Aberrations I

Surface Contributions: Example

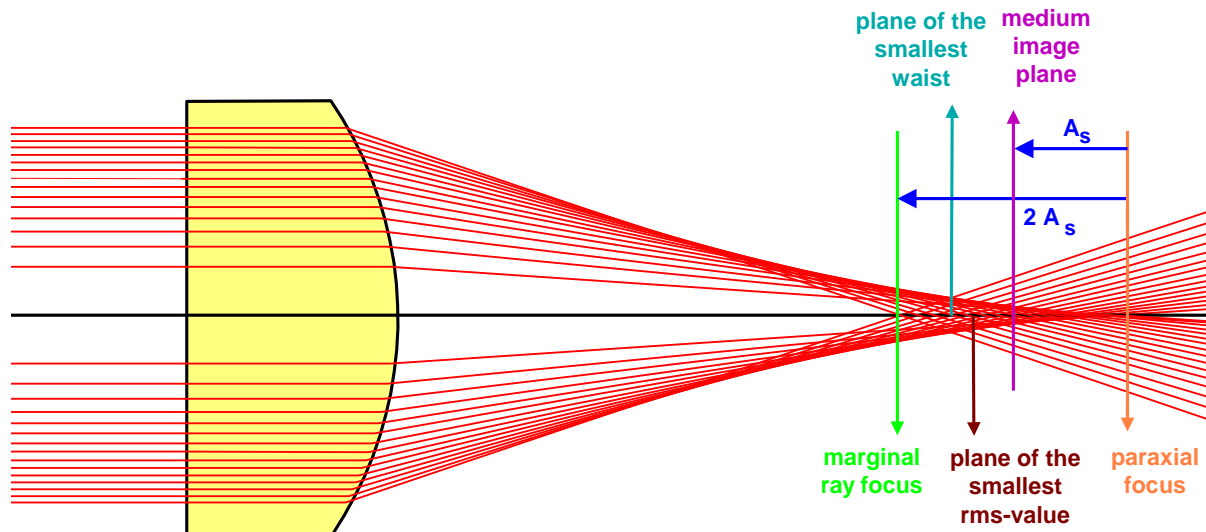
- Seidel aberrations: representation as sum of surface contributions possible
- Gives information on correction of a system
- Example: photographic lens



4 Aberrations I

Spherical Aberration

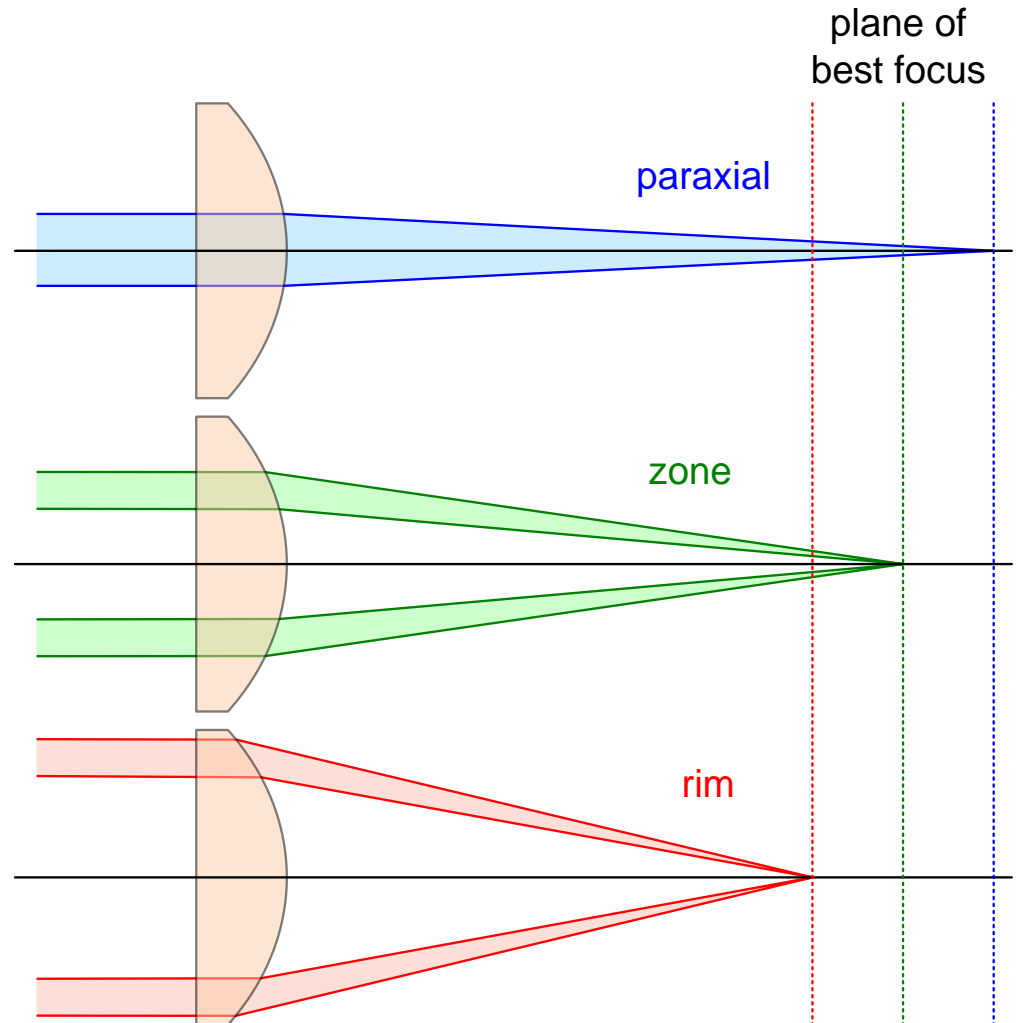
- Spherical aberration:
 - On axis, circular symmetry
- Perfect focussing near axis: paraxial focus
- Real marginal rays: shorter intersection length (for single positive lens)
- Optimal image plane: circle of least rms value



4 Aberrations I

Spherical Aberration

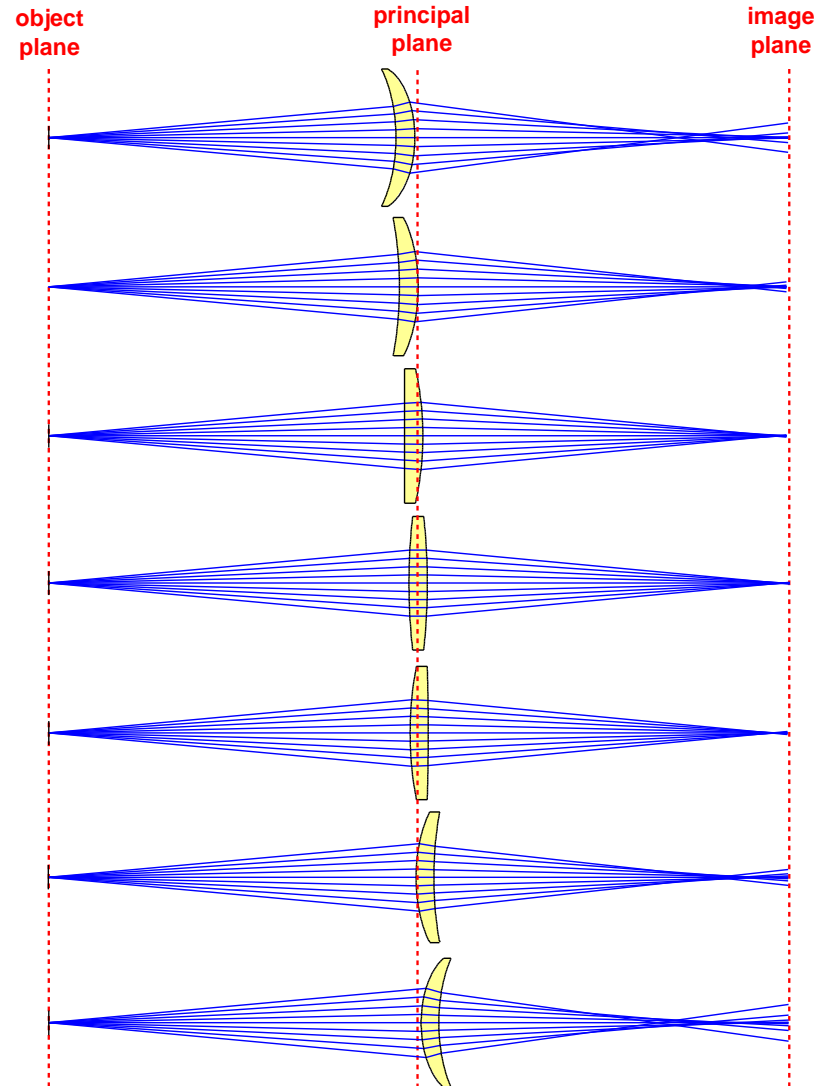
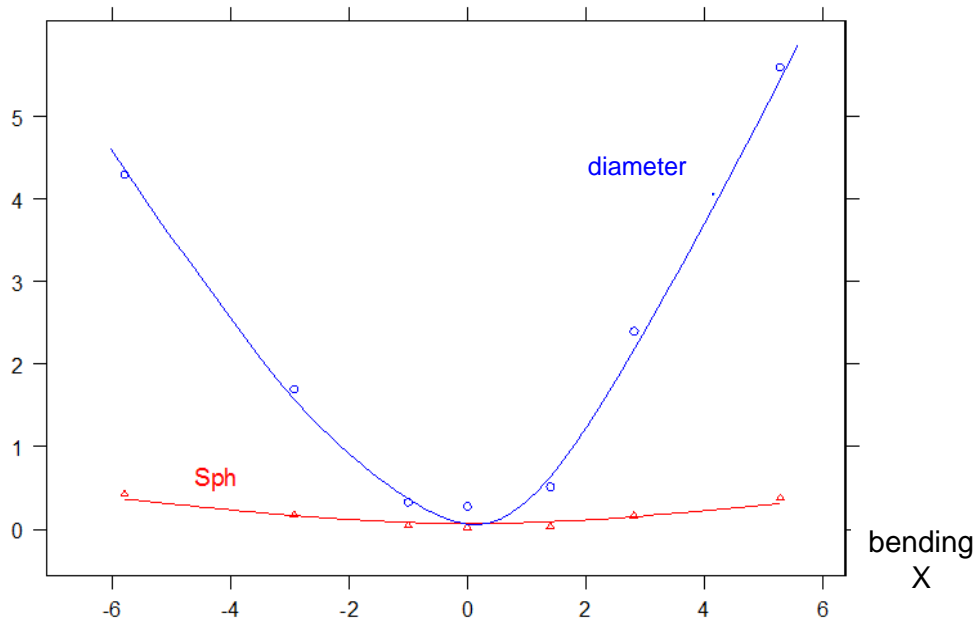
- Single positive lens
- Paraxial focal plane near axis, Largest intersection length
- Shorter intersection length for rim ray and outer aperture zones



4 Aberrations I

Spherical Aberration: Lens Bending

- Spherical aberration and focal spot diameter as a function of the lens bending (for $n=1.5$)
- Optimal bending for incidence averaged incidence angles
- Minimum larger than zero: usually no complete correction possible



4 Aberrations I

Aplanatic Surfaces

- Aplanatic surfaces: zero spherical aberration:

1. Ray through vertex $s' = s = 0$

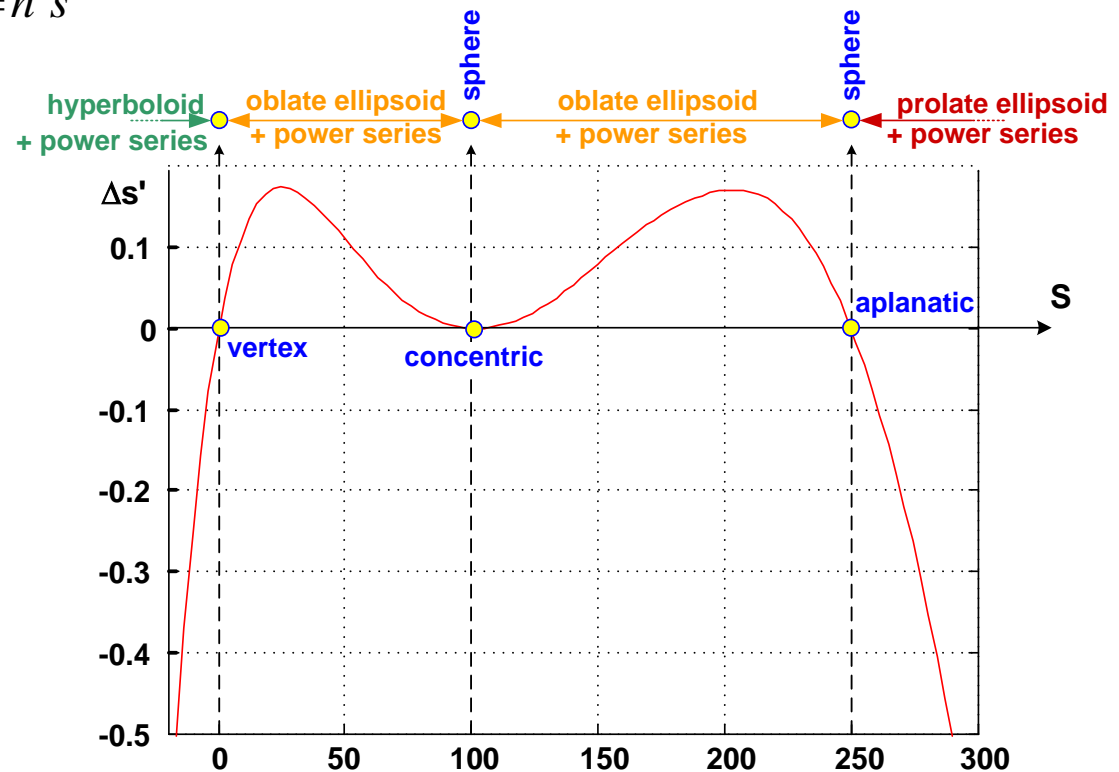
2. concentric $s' = s$ und $u = u'$

3. Aplanatic $ns = n' s'$

- Condition for aplanatic surface:

$$r = \frac{ns}{n + n'} = \frac{n' s'}{n + n'} = \frac{ss'}{s + s'}$$

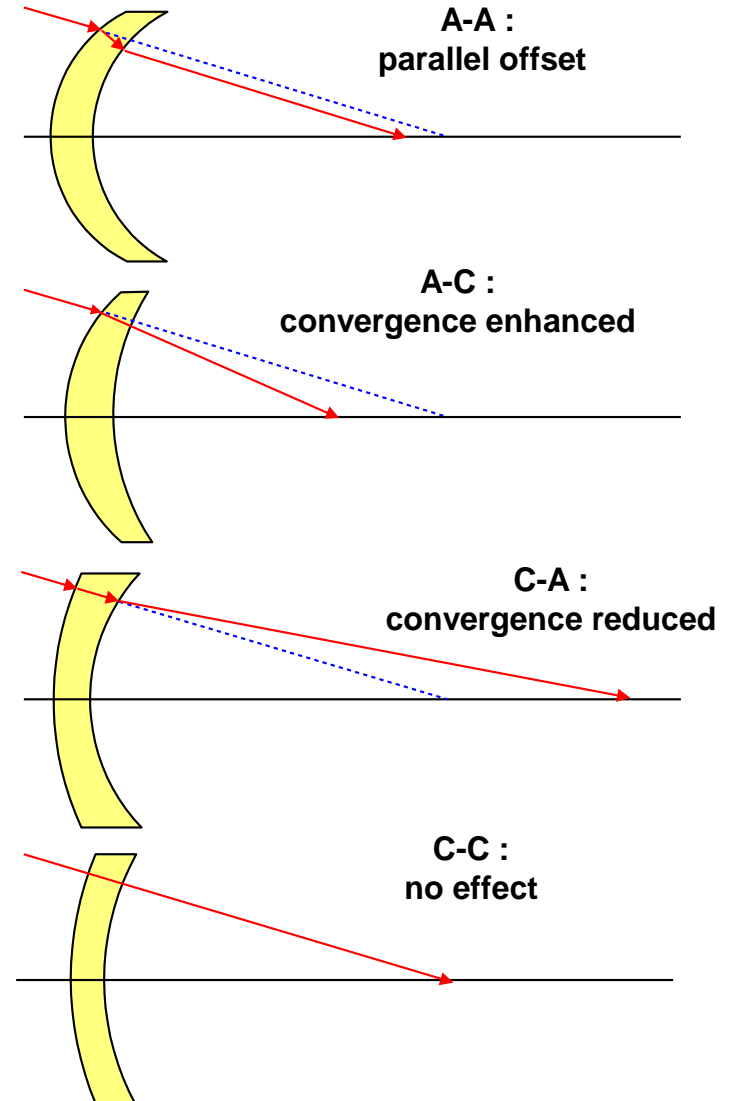
- Virtual image location
- Applications:
 - Microscopic objective lens
 - Interferometer objective lens



4 Aberrations I

Aplanatic Lenses

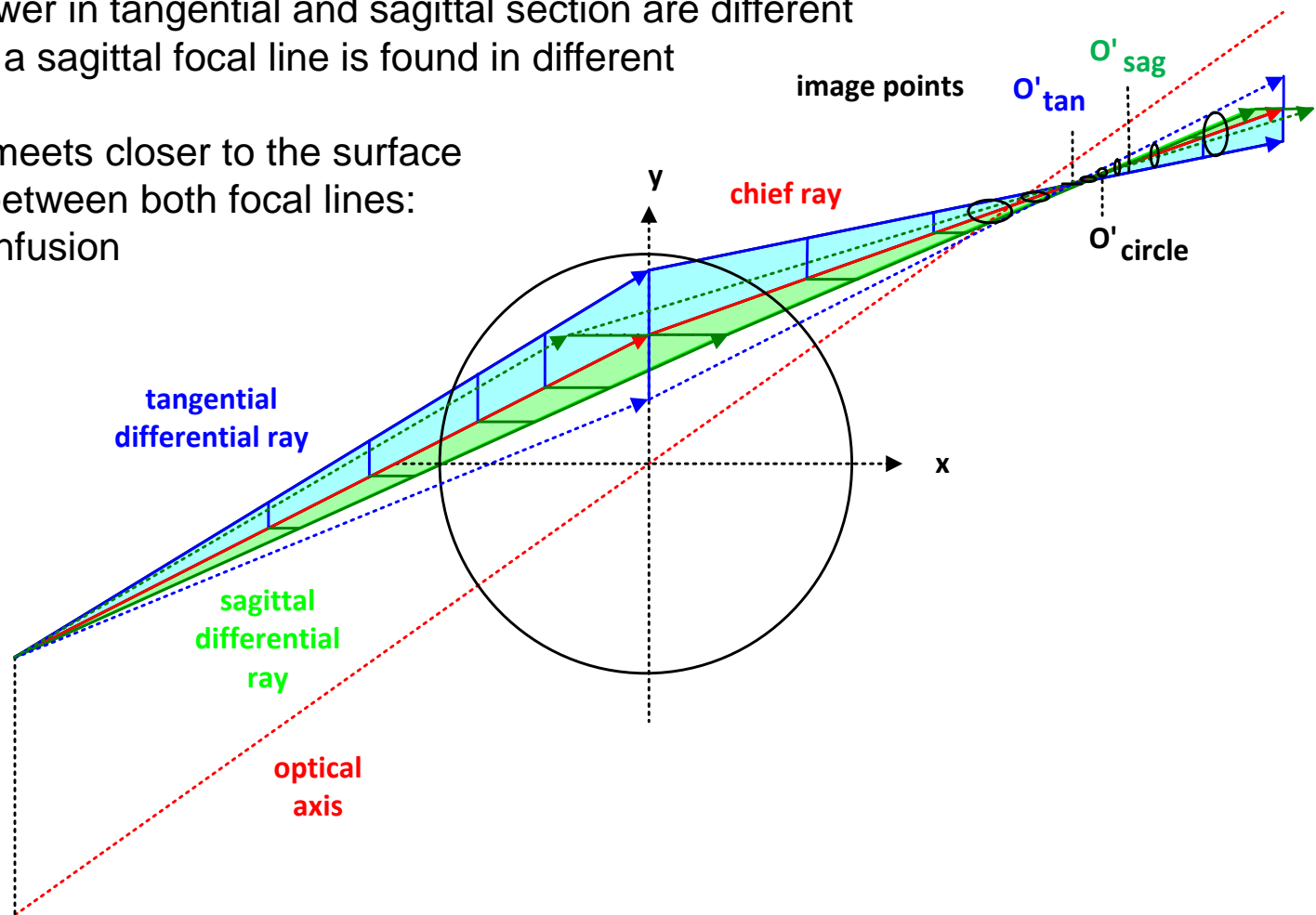
- Aplanatic lenses
- Combination of one concentric and one aplanatic surface:
zero contribution of the whole lens to spherical aberration
- Not useful:
 1. aplanatic-aplanatic
 2. concentric-concentricbended plane parallel plate,
nearly vanishing effect on rays



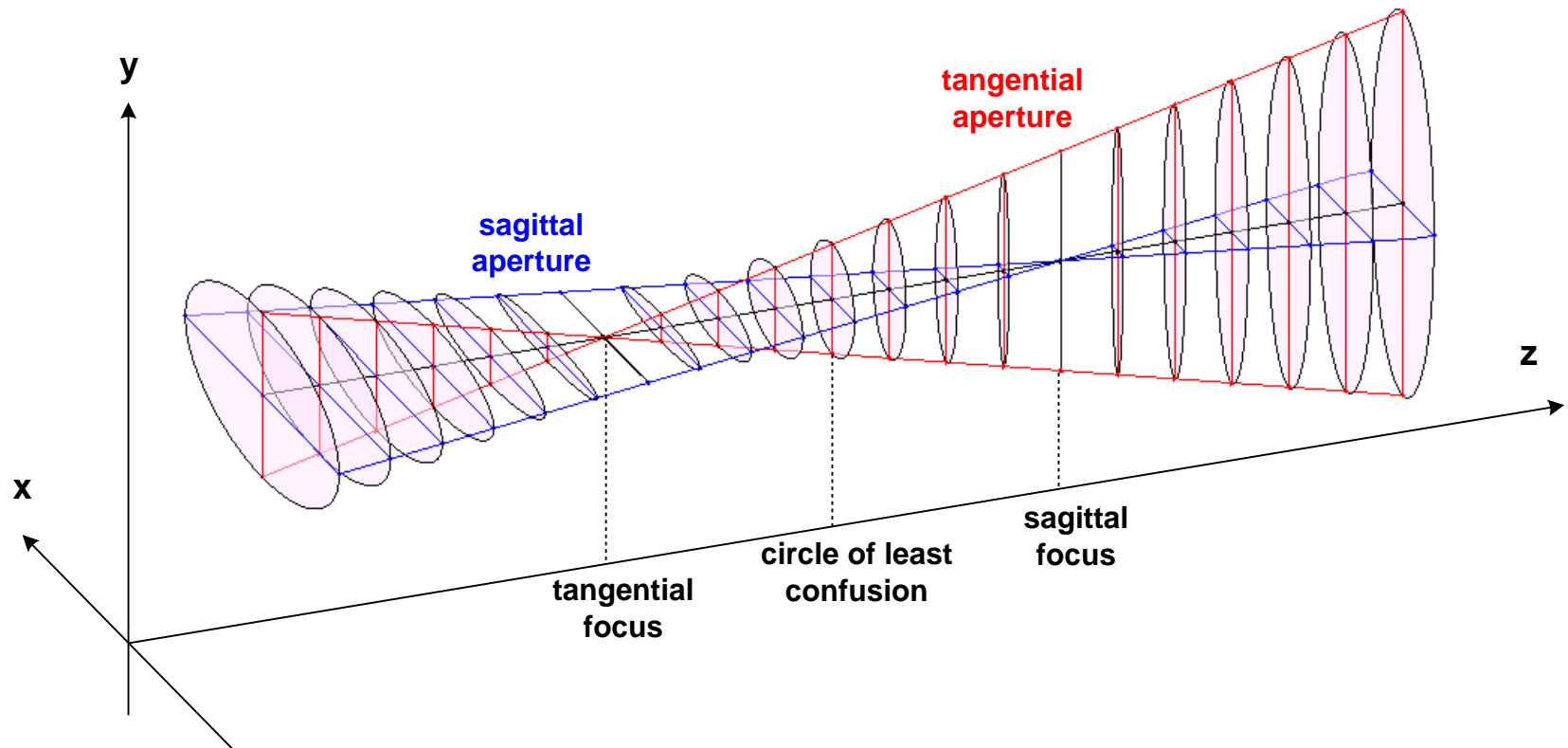
4 Aberrations I

Astigmatism

- Reason for astigmatism:
chief ray passes a surface under an oblique angle,
the refractive power in tangential and sagittal section are different
- A tangential and a sagittal focal line is found in different distances
- Tangential rays meet closer to the surface
- In the midpoint between both focal lines:
circle of least confusion



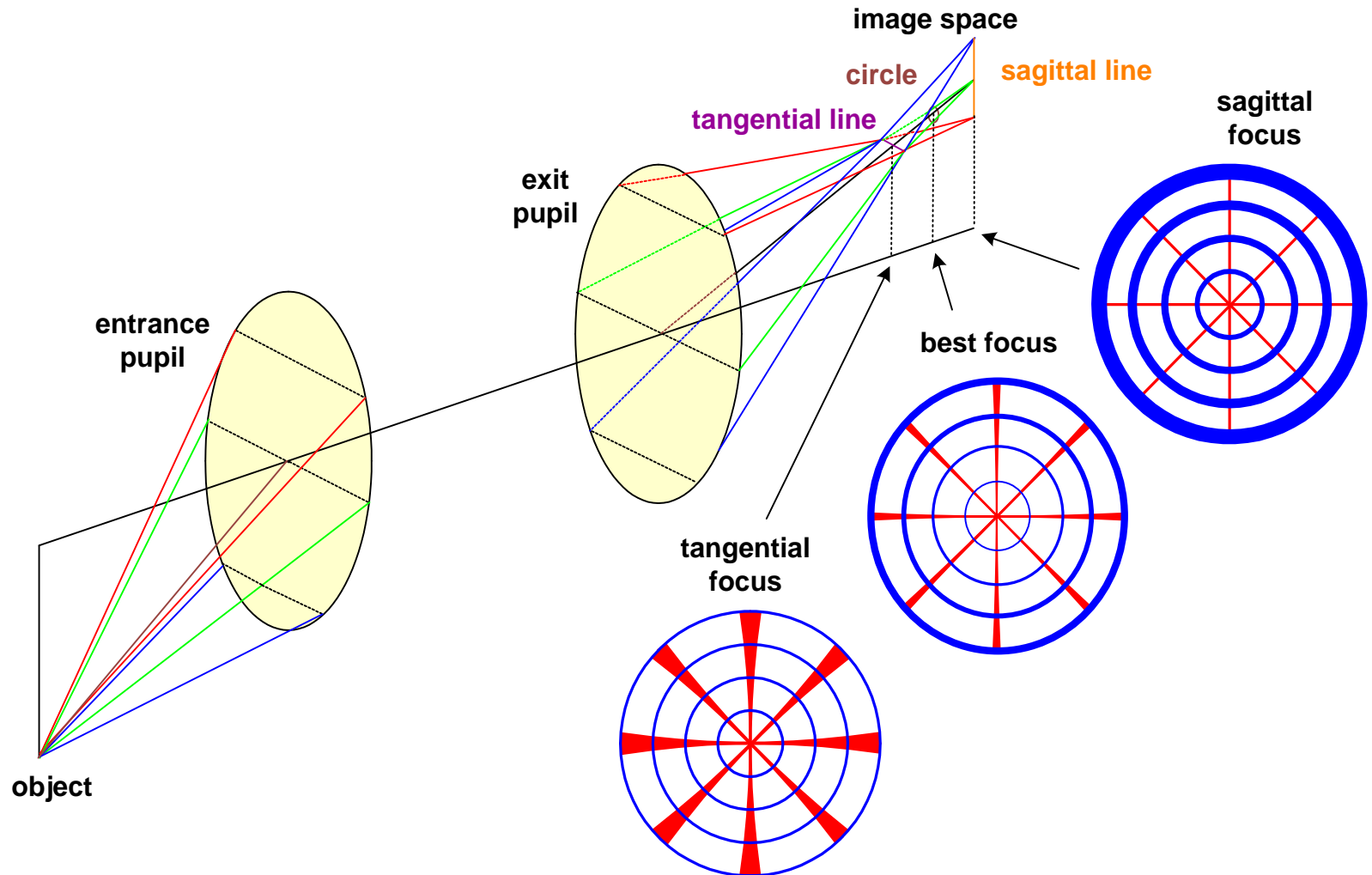
- Beam cross section in the case of astigmatism:
 - Elliptical shape transforms its aspect ratio
 - degenerate into focal lines in the focal plane distances
 - special case of a circle in the midpoint: smallest spot



4 Aberrations I

Astigmatism

Imaging of a polar grid in different planes



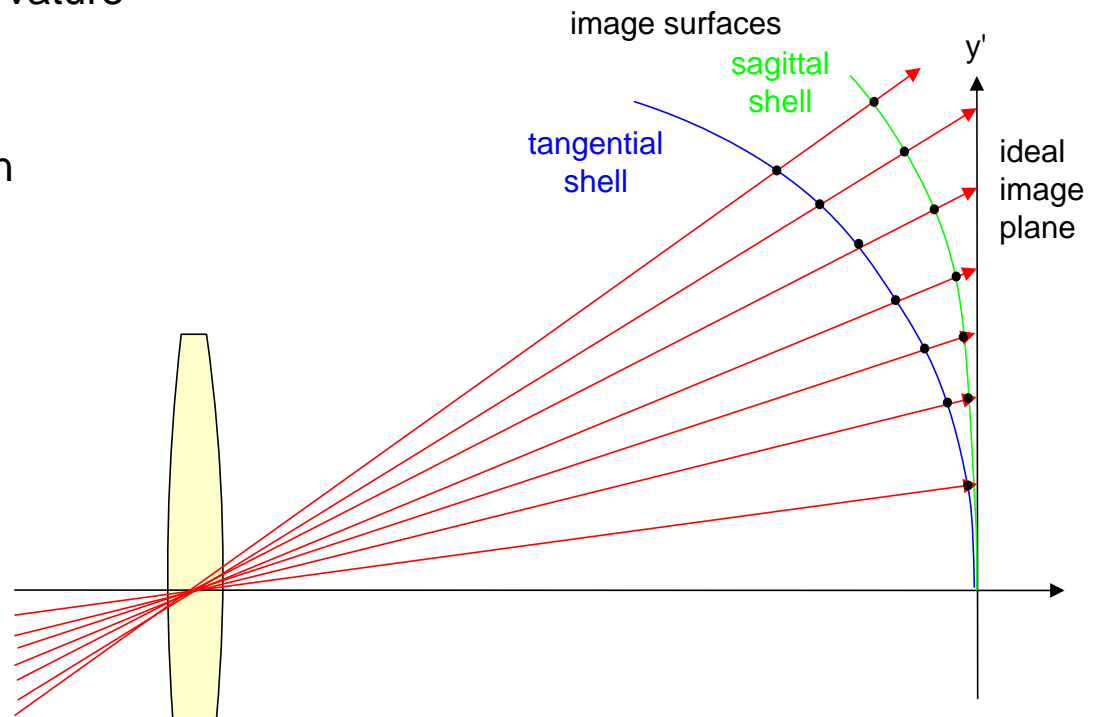
4 Aberrations I

Field Curvature and Image Shells

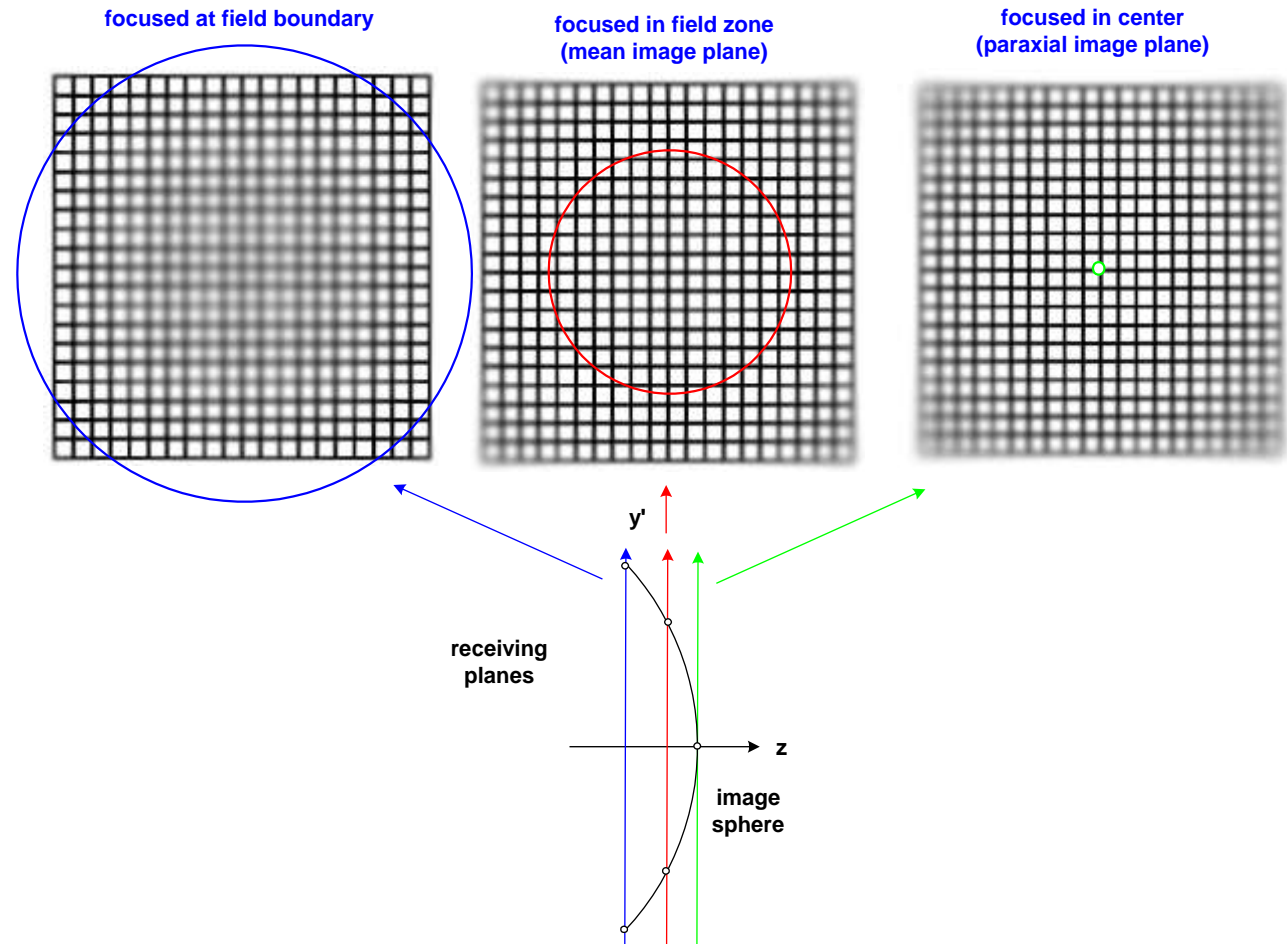
- Imaging with astigmatism:
Tangential and sagittal image shell depending on the azimuth
- Difference between the image shells: astigmatism
- Astigmatism corrected:
It remains a curved image shell,
Bended field: also called Petzval curvature
- System with astigmatism:
Petzval sphere is not an optimal
surface with good imaging resolution
- Law of Petzval: curvature given by:

$$\frac{1}{r_p} = -n' \cdot \sum_k \frac{1}{n_k \cdot f_k}$$

- No effect of bending on curvature,
important: distribution of lens
powers and indices



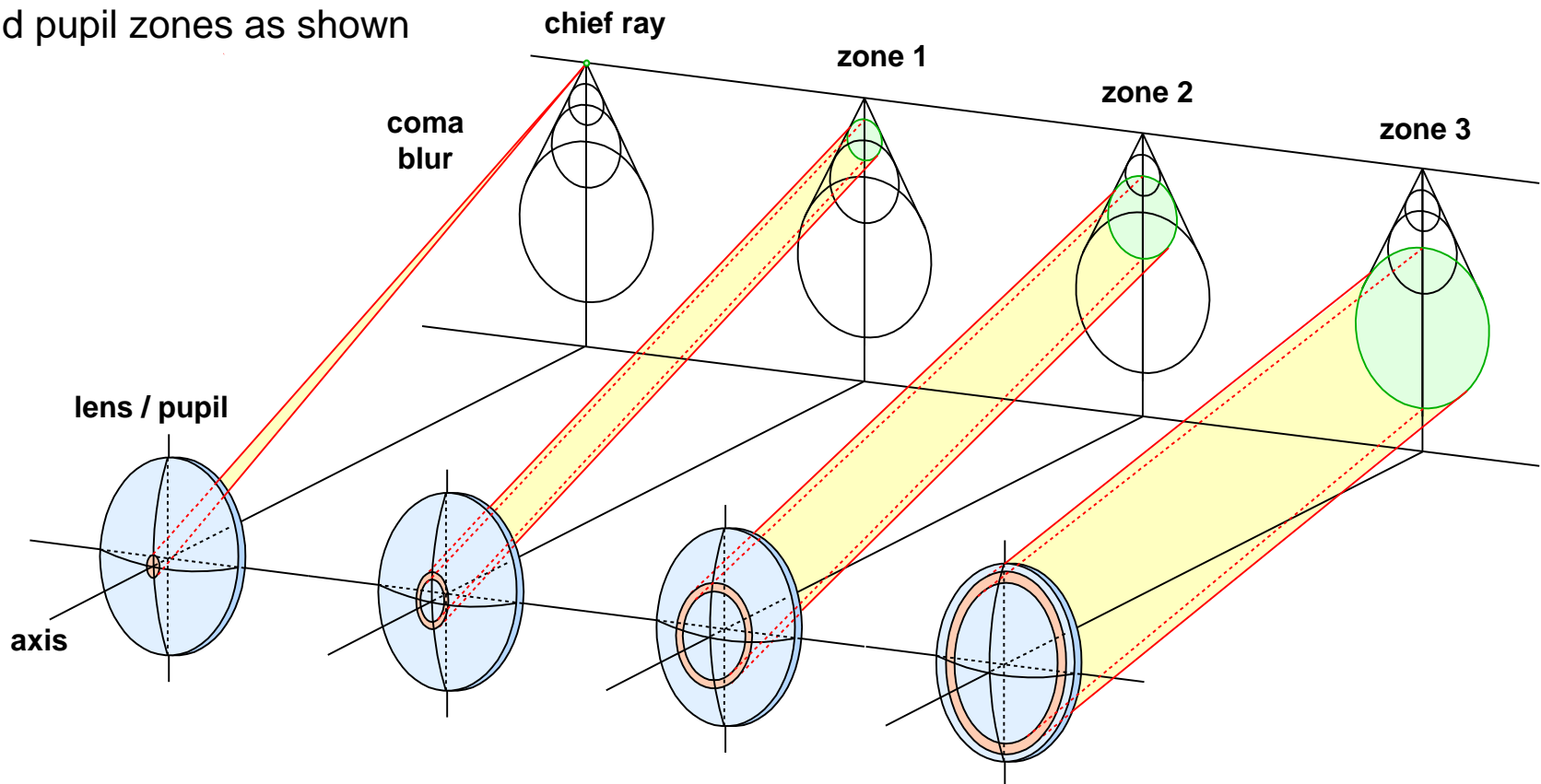
- Focussing into different planes of a system with field curvature
- Sharp imaged zone changes from centre to margin of the image field



4 Aberrations I

Blurred Coma Spot

- Coma aberration: for oblique bundles and finite aperture due to asymmetry
- Primary effect: coma grows linear with field size y
- Systems with large field of view: coma hard to correct
- Relation of spot circles and pupil zones as shown



4 Aberrations I

Distortion Example: 10%

- What is the type of degradation of this image ?
- Sharpness good everywhere !



Ref : H. Zügge

4 Aberrations I

Distortion Example: 10%

- Image with sharp but bended edges/lines
- No distortion along central directions



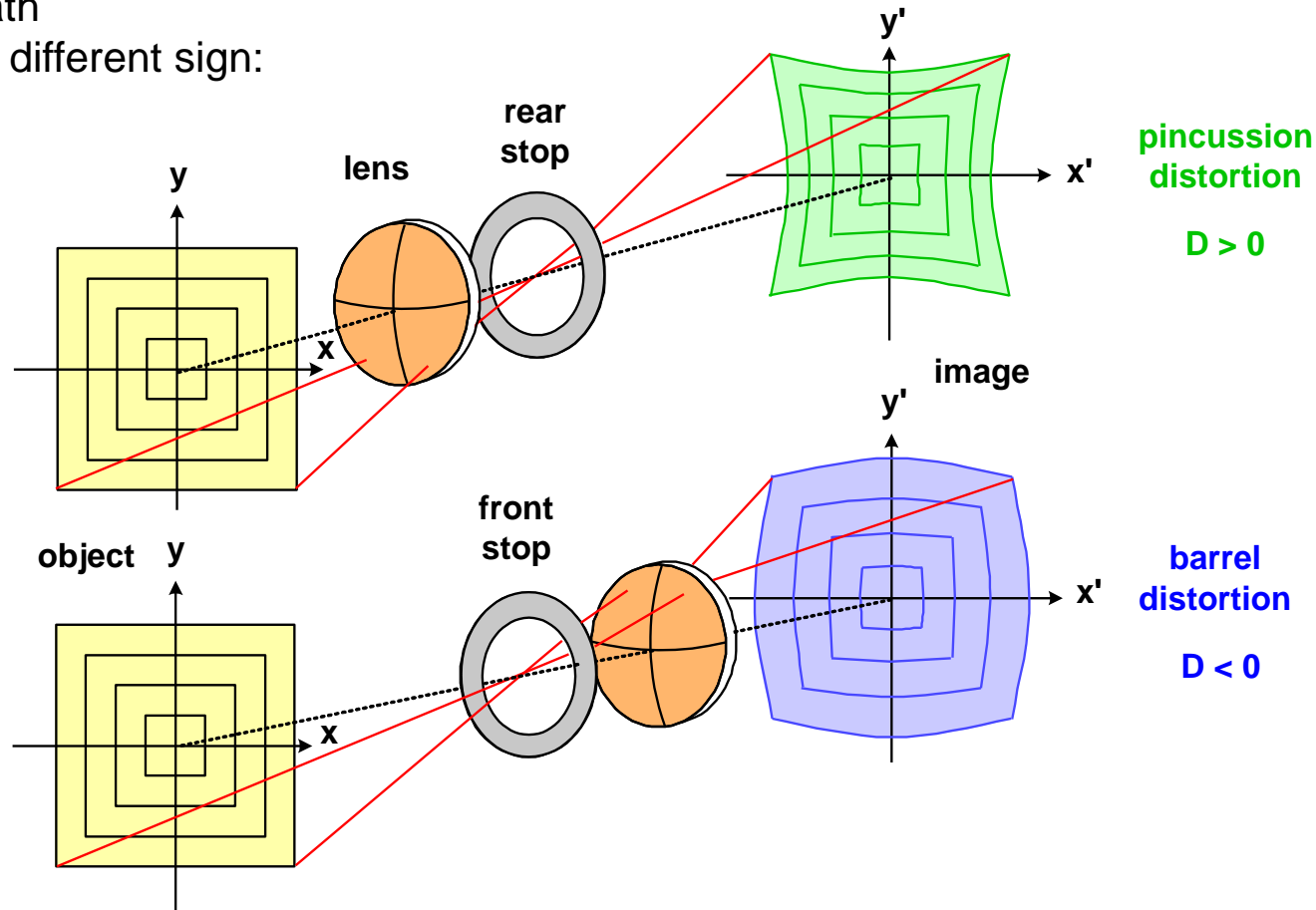
Ref : H. Zügge

4 Aberrations I

Distortion

- Purely geometrical deviations without any blurr
- Distortion corresponds to spherical aberration of the chief ray
- Important is the location of the stop: defines the chief ray path
- Two primary types with different sign:
 1. barrel, $D < 0$
front stop
 2. pincushion, $D > 0$
rear stop
- Definition of local magnification changes

$$D = \frac{y'_{real} - y'_{ideal}}{y'_{ideal}}$$



4 Aberrations I

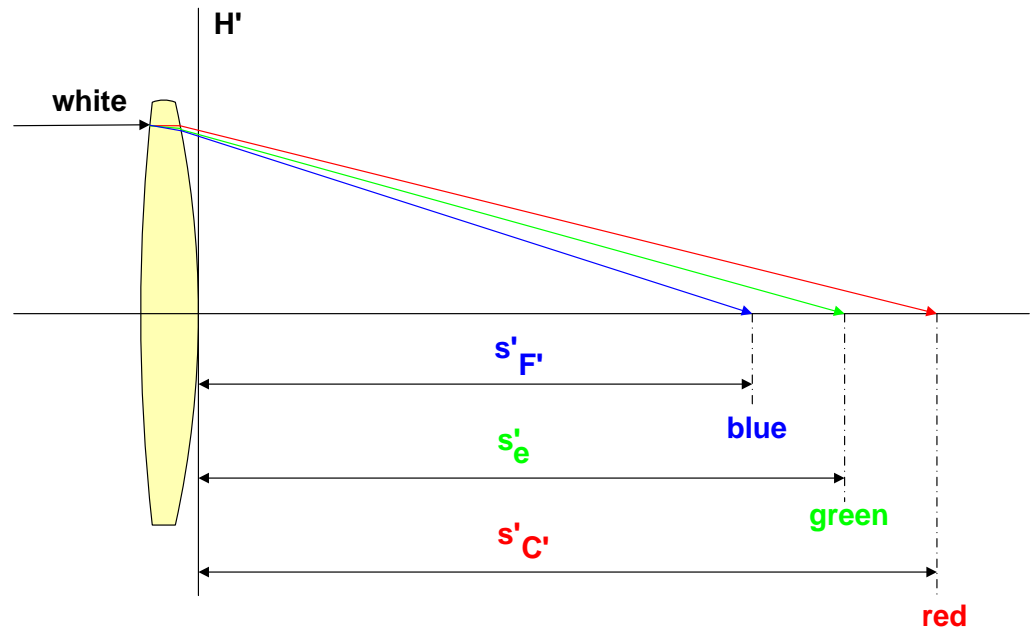
Axial Chromatical Aberration

- Axial chromatical aberration:

Higher refractive index in the blue results in a shorter intersection length for a single lens

- The colored images are defocussed along the axis
- Definition of the error: change in image location / intersection length
- Correction needs several glasses with different dispersion

$$\Delta s'_{CHL} = s'_{F'} - s'_{C'}$$



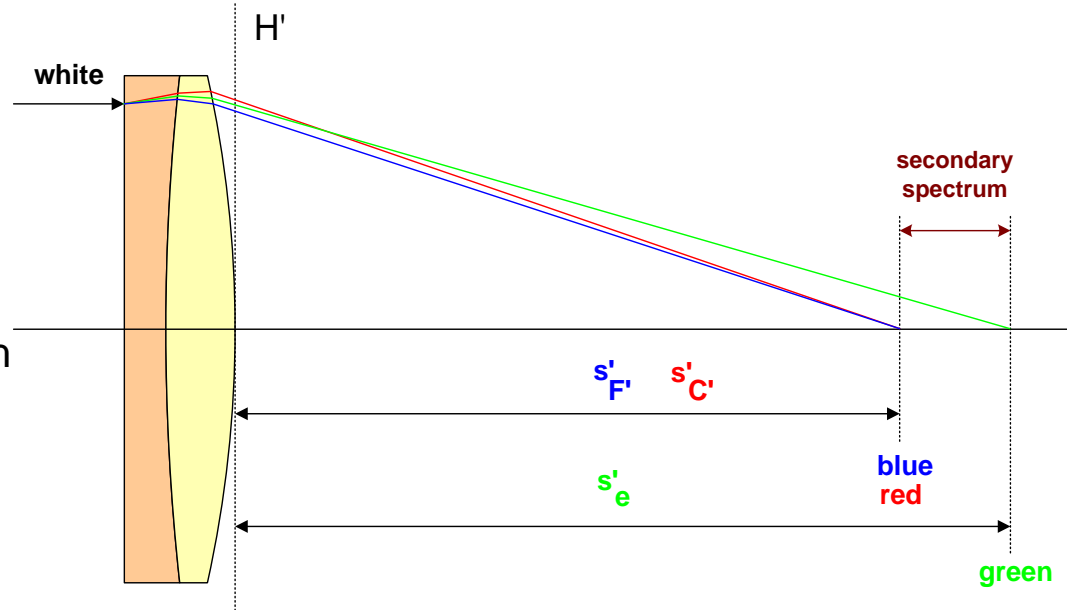
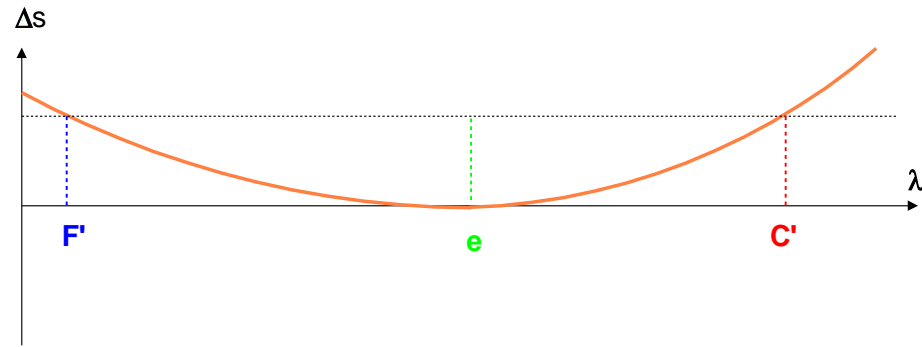
4 Aberrations I

Axial Chromatical Aberration

- Simple achromatization / first order correction:
 - two glasses with different dispersion
 - equal intersection length for outer wavelengths (blue F', red C')
 - residual deviation for middle wavelength (green e)

- Residual errors in image location: secondary spectrum

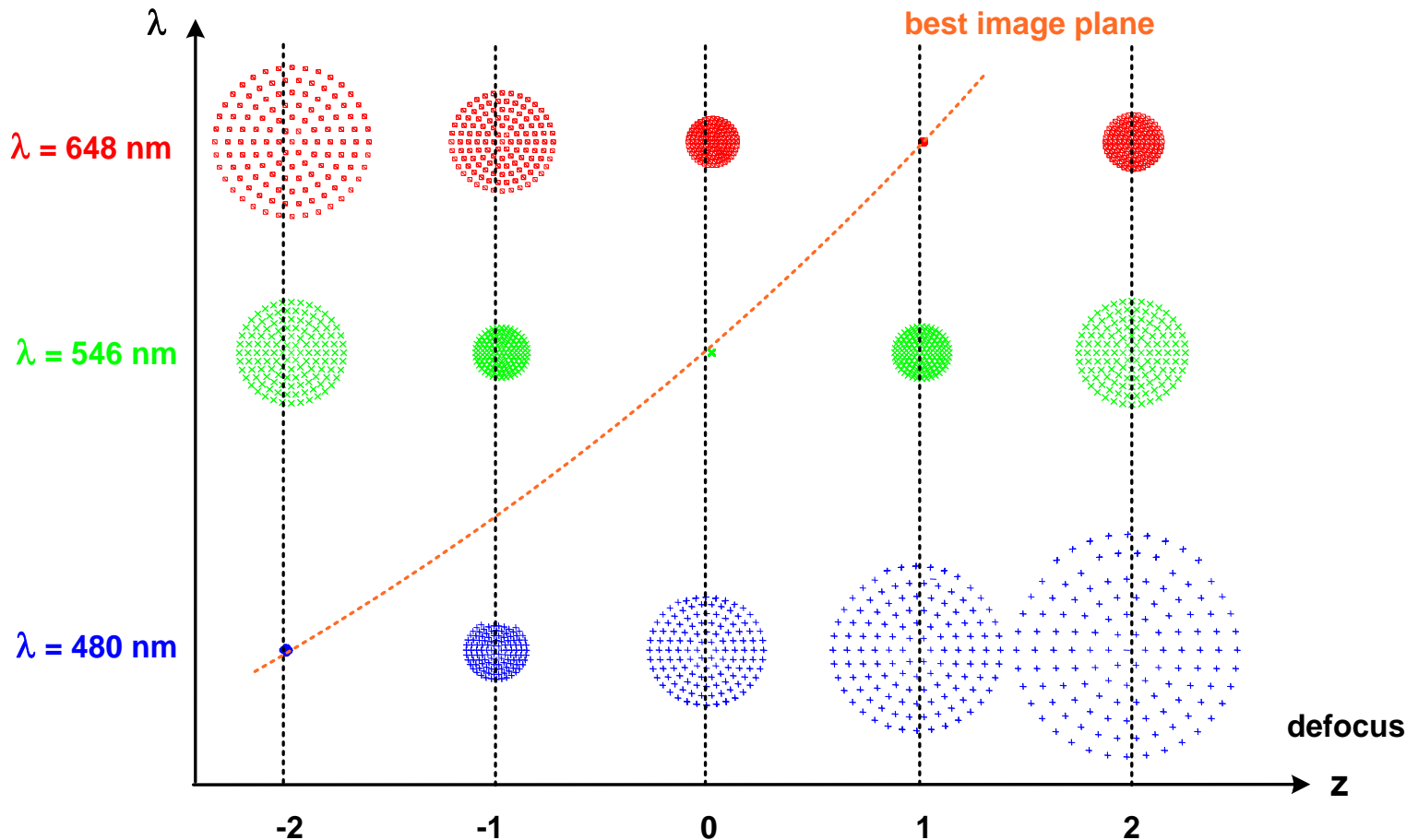
- Achromat:
 - coincidence of the image location for at least 3 wavelengths
 - three glasses necessary, only with anomalous partial dispersion (exceptions possible)



4 Aberrations I

Axial Chromatical Aberration

- Longitudinal chromatical aberration for a single lens
- Best image plane changes with wavelength



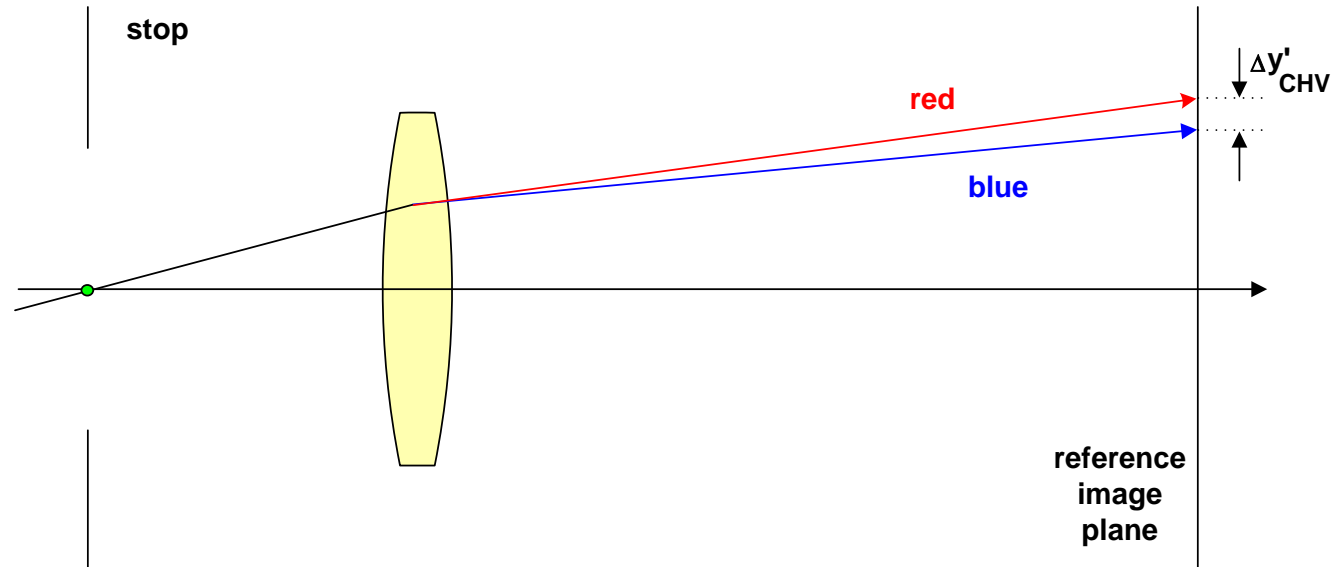
4 Aberrations I

Chromatic Variation of Magnification

- Lateral chromatical aberration:
Higher refractive index in the blue results in a stronger ray bending of the chief ray for a single lens
- The colored images have different size, the magnification is wavelength dependent
- Definition of the error: change in image height/magnification
- Correction needs several glasses with different dispersion
- The aberration strongly depends on the stop position

$$\Delta y'_{CHV} = y'_{F'} - y'_{C'}$$

$$\Delta \bar{y}'_{CHV} = \frac{y'_{F'} - y'_{C'}}{y'_e}$$



4 Aberrations I

Chromatic Variation of Magnification

- Impression of CHV in real images
- Typical colored fringes blue/red at edges visible
- Color sequence depends on sign of CHV

