Institute of Applied Physics
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Optical Design with Zemax

Lecture 2: Properties of optical systems II

2012-10-30

Herbert Gross

2 Properties of Optical Systems II
Preliminary time schedule

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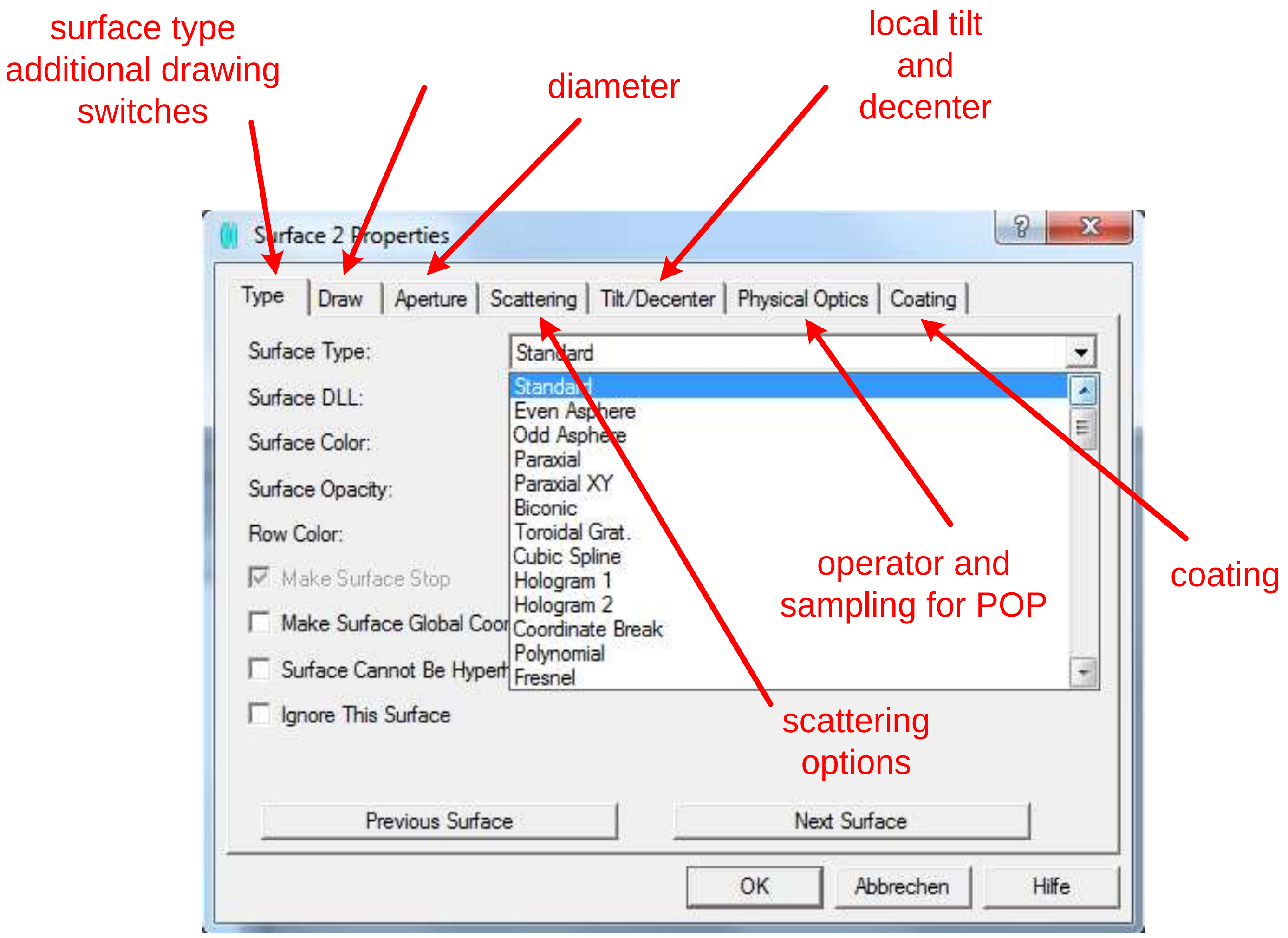
1	16.10.	Introduction	Introduction, Zemax interface, menues, file handling, preferences, Editors, updates, windows, Coordinate systems and notations, System description, Component reversal, system insertion, scaling, 3D geometry, aperture, field, wavelength
2	23.10.	Properties of optical systems I	Diameters, stop and pupil, vignetting, Layouts, Materials, Glass catalogs, Raytrace, Ray fans and sampling, Footprints
3	30.10.	Properties of optical systems II	Types of surfaces, Aspheres, Gratings and diffractive surfaces, Gradient media, Cardinal elements, Lens properties, Imaging, magnification, paraxial approximation and modelling
4	06.11.	Aberrations I	Representation of geometrical aberrations, Spot diagram, Transverse aberration diagrams, Aberration expansions, Primary aberrations,
5	13.11.	Aberrations II	Wave aberrations, Zernike polynomials, Point spread function, Optical transfer function
6	20.11.	Optimization I	Principles of nonlinear optimization, Optimization in optical design, Global optimization methods, Solves and pickups, variables, Sensitivity of variables in optical systems
7	27.11.	Optimization II	Systematic methods and optimization process, Starting points, Optimization in Zemax
8	04.12	Imaging	Fundamentals of Fourier optics, Physical optical image formation, Imaging in Zemax
9	11.12.	Illumination	Introduction in illumination, Simple photometry of optical systems, Non-sequential raytrace, Illumination in Zemax
10	18.12.	Advanced handling I	Telecentricity, infinity object distance and afocal image, Local/global coordinates, Add fold mirror, Scale system, Make double pass, Vignetting, Diameter types, Ray aiming, Material index fit
11	08.01.	Advanced handling II	Report graphics, Universal plot, Slider, Visual optimization, IO of data, Multiconfiguration, Fiber coupling, Macro language, Lens catalogs
12	15.01.	Correction I	Symmetry principle, Lens bending, Correcting spherical aberration, Coma, stop position, Astigmatism, Field flattening, Chromatical correction, Retrofocus and telephoto setup, Design method
13	22.01.	Correction II	Field lenses, Stop position influence, Aspheres and higher orders, Principles of glass selection, Sensitivity of a system correction, Microscopic objective lens, Zoom system
14	29.01.	Physical optical modelling I	Gaussian beams, POP propagation, polarization raytrace, polarization transmission, polarization aberrations
15	05.02.	Physical optical modelling II	coatings, representations, transmission and phase effects, ghost imaging, general straylight with BRDF

Contents 3rd Lecture

1. Types of surfaces
2. Aspheres
3. Gratings and diffractive surfaces
4. Gradient media
5. Cardinal elements
6. Lens properties
7. Imaging
8. Magnification
9. Paraxial approximation and modelling

2 Properties of optical systems II
Surface properties and settings

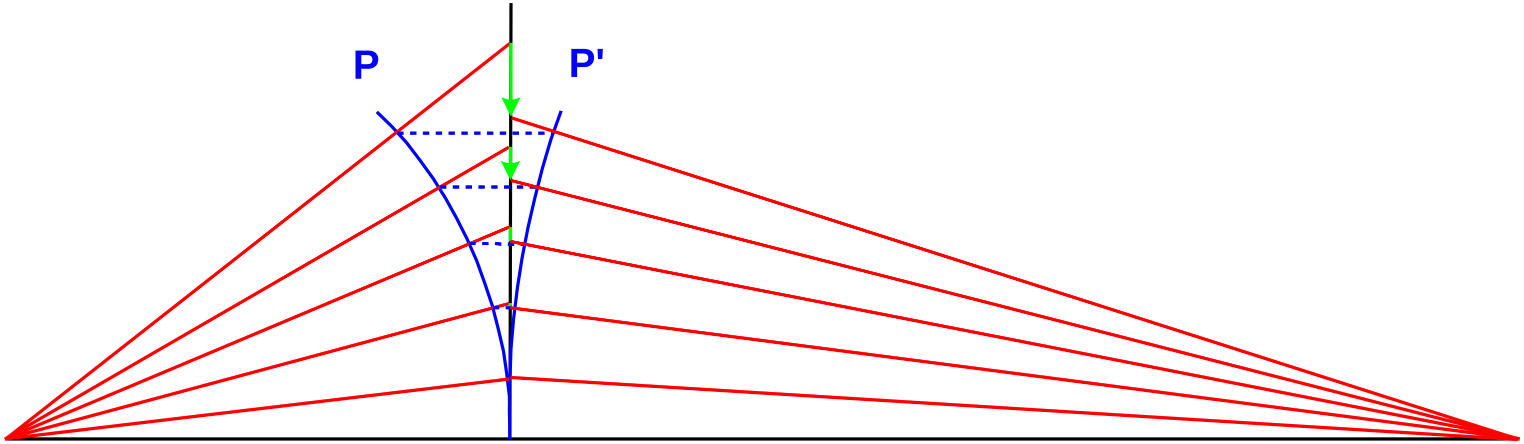
- Setting of surface properties



- Ideale lens
  Principal surfaces are spheres

- The marginal ray heights in the vortex plane are different for larger angles

- Inconsistencies in the layout drawings



- Conic section

  Special case spherical

$$z = \frac{c\left(x^2 + y^2\right)}{1 + \sqrt{1 - (1+\kappa)c^2\left(x^2 + y^2\right)}}$$

- Cone

$$z = \frac{\sqrt{x^2 + y^2}}{\theta}$$

- Toroidal surface with radii R_x and R_y in the two section planes

$$z = R_y - \sqrt{\left(R_y - R_x + \sqrt{R_x^2 - x^2}\right)^2 - y^2}$$

- Generalized onic section without circular symmetry

$$z = \frac{c_x x^2 + c_y y^2}{1 + \sqrt{1 - (1+\kappa_x)c_x^2 x^2 - (1+\kappa_y)c_y^2 y^2}}$$

- Roof surface

$$z = \left| y \right| \cdot \tan\theta$$

- Explicite surface equation, resolved to z

  Parameters: curvature c = 1 / R

  conic parameter κ

$$z = \frac{c\left(x^2+y^2\right)}{1+\sqrt{1-(1+\kappa)c^2\left(x^2+y^2\right)}}$$

- Influence of κ on the surface shape

Parameter	Surface shape
κ = - 1	paraboloid
κ < - 1	hyperboloid
κ = 0	sphere
κ > 0	oblate ellipsoid (disc)
0 > κ > - 1	prolate ellipsoid (cigar)

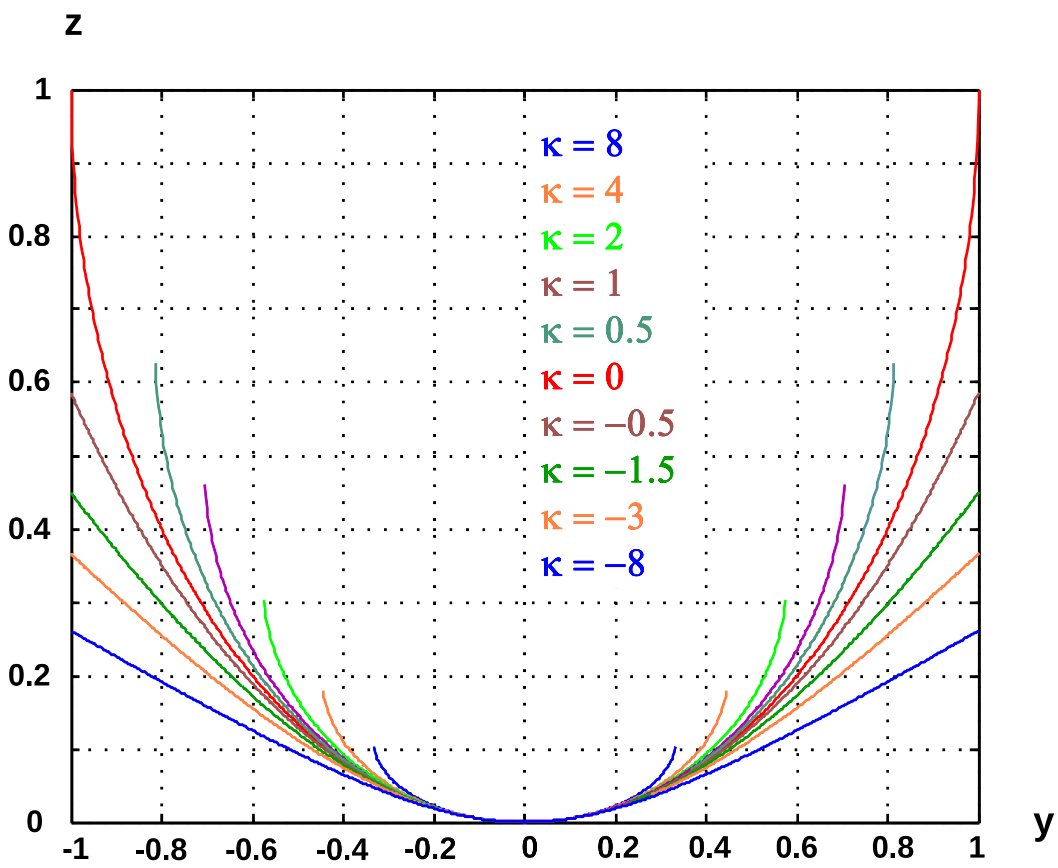
- Relations with axis lengths a,b of conic sections

$$\kappa = \left(\frac{a}{b}\right)^2 - 1 \qquad c = \frac{b}{a^2} \qquad b = \frac{1}{\left|c(1+\kappa)\right|} \qquad a = \frac{1}{\left|c\sqrt{\left|1+\kappa\right|}\right|}$$

Aspherical shape of conic sections

- Conic aspherical surface
- Variation of the conical parameter κ

$$z = \frac{cy^2}{1+\sqrt{1-(1+\kappa)\cdot c^2 y^2}}$$



Parabolic mirror

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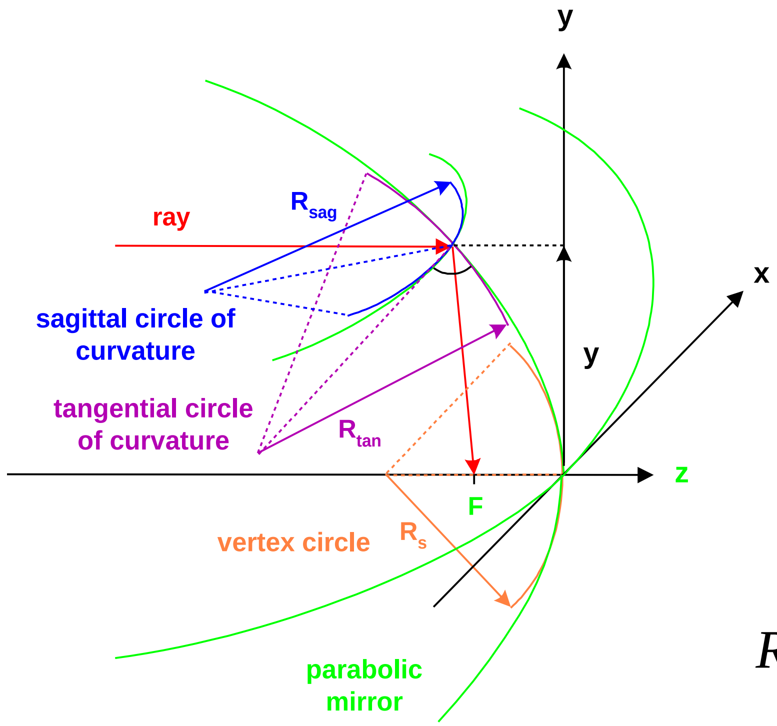
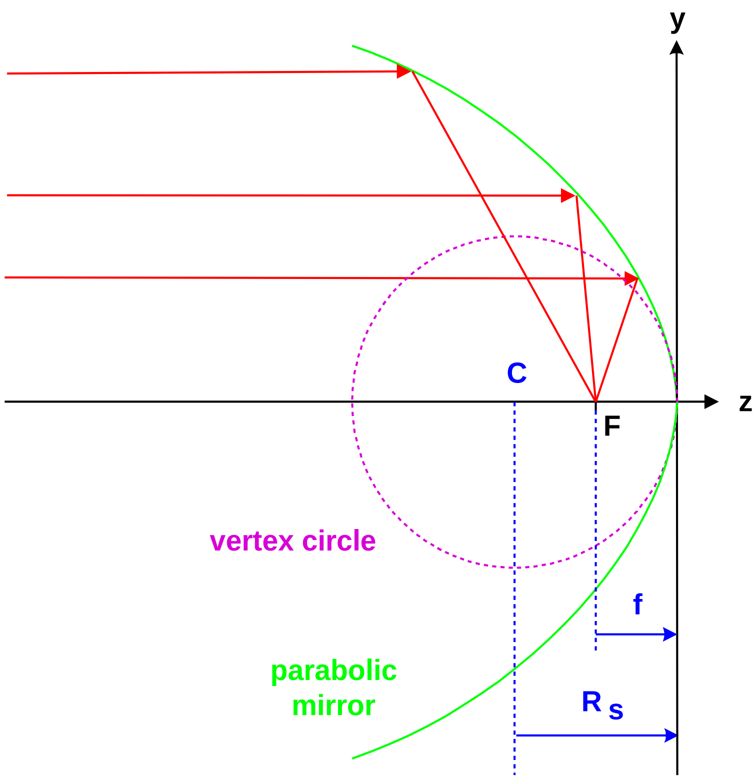
Equation

$$z = \frac{cy^2}{1+\sqrt{1-(1+\kappa)y^2c^2}}$$

c : curvature $1/R_s$

κ : eccentricity (= -1)





radii of curvature :

$$R_{\tan} = R_s \cdot \sqrt{1+\left(\frac{y}{R_s}\right)^2}$$

$$R_{\tan} = R_s \cdot \left[1+\left(\frac{y}{R_s}\right)^2\right]^{\frac{3}{2}}$$

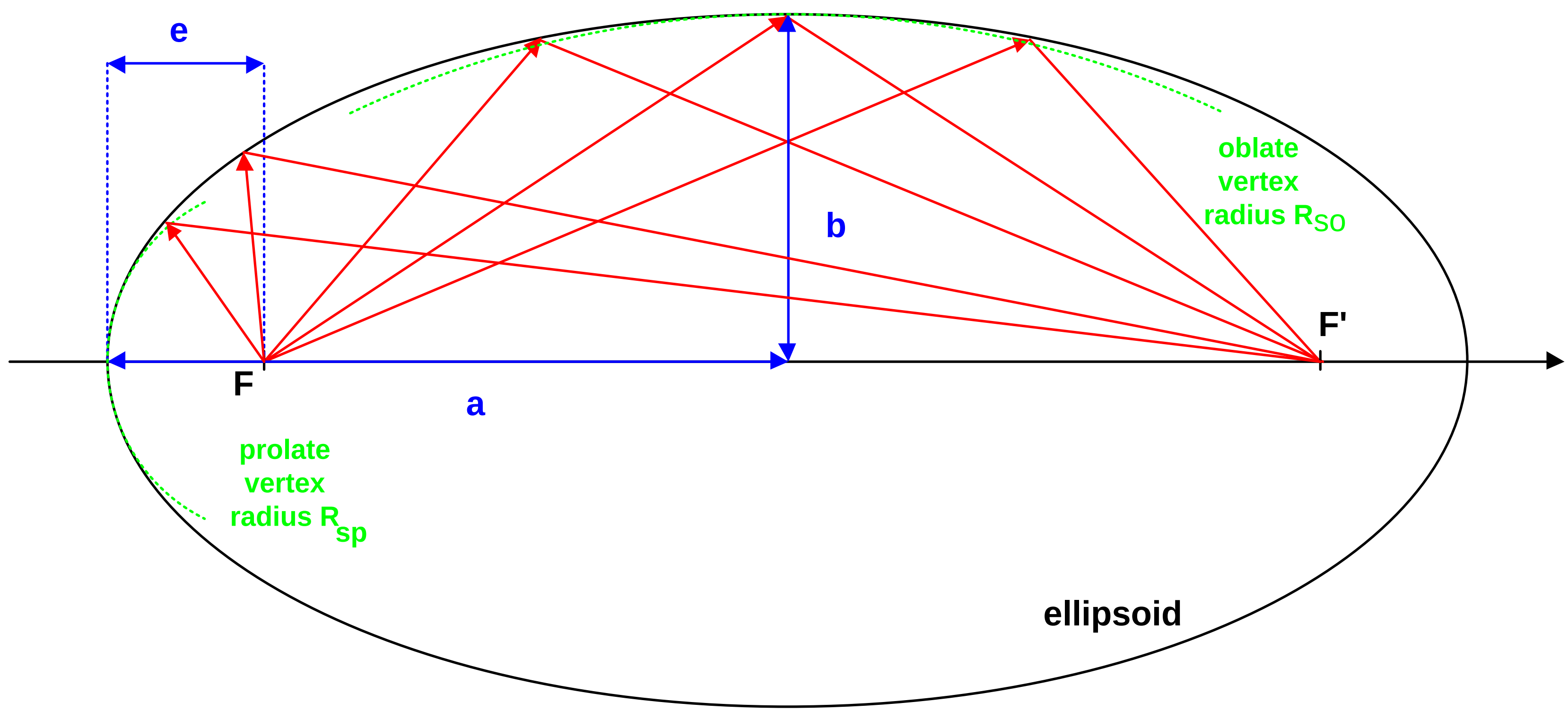
Ellipsoid mirror

Equation

$$z = \frac{cy^2}{1+\sqrt{1-(1+\kappa)y^2c^2}}$$

c: curvature 1/R

κ: Eccentricity



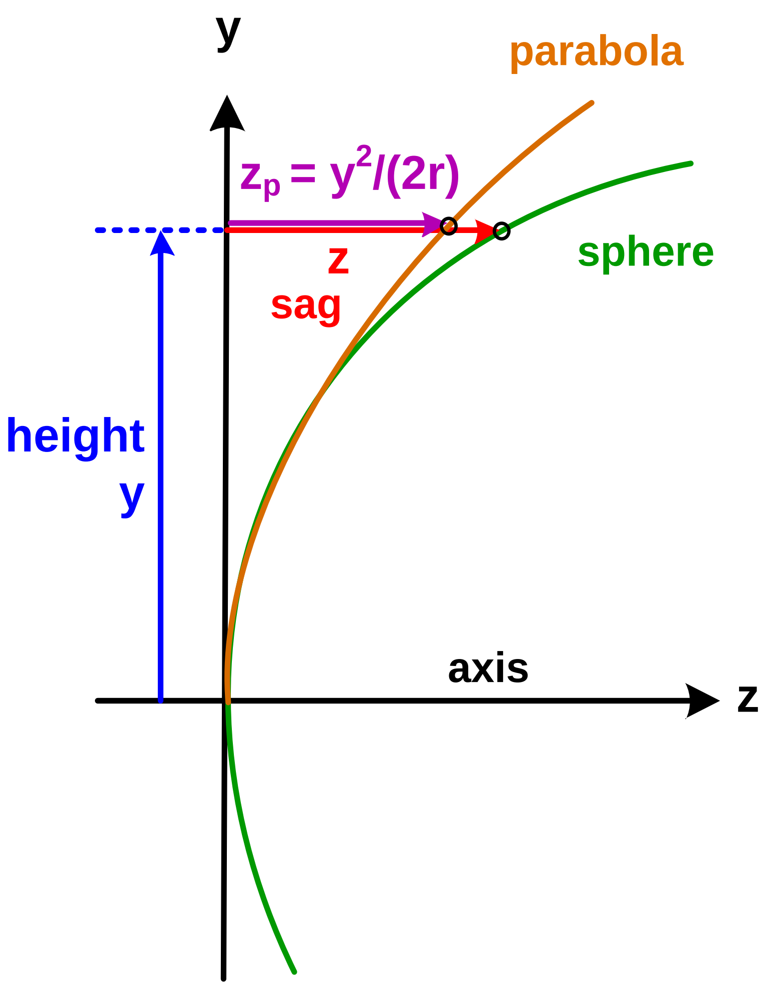
Sag of a surface

- Sag z at height y for a spherical surface:

$$z = r - \sqrt{r^2 - y^2}$$

- Paraxial approximation: quadratic term

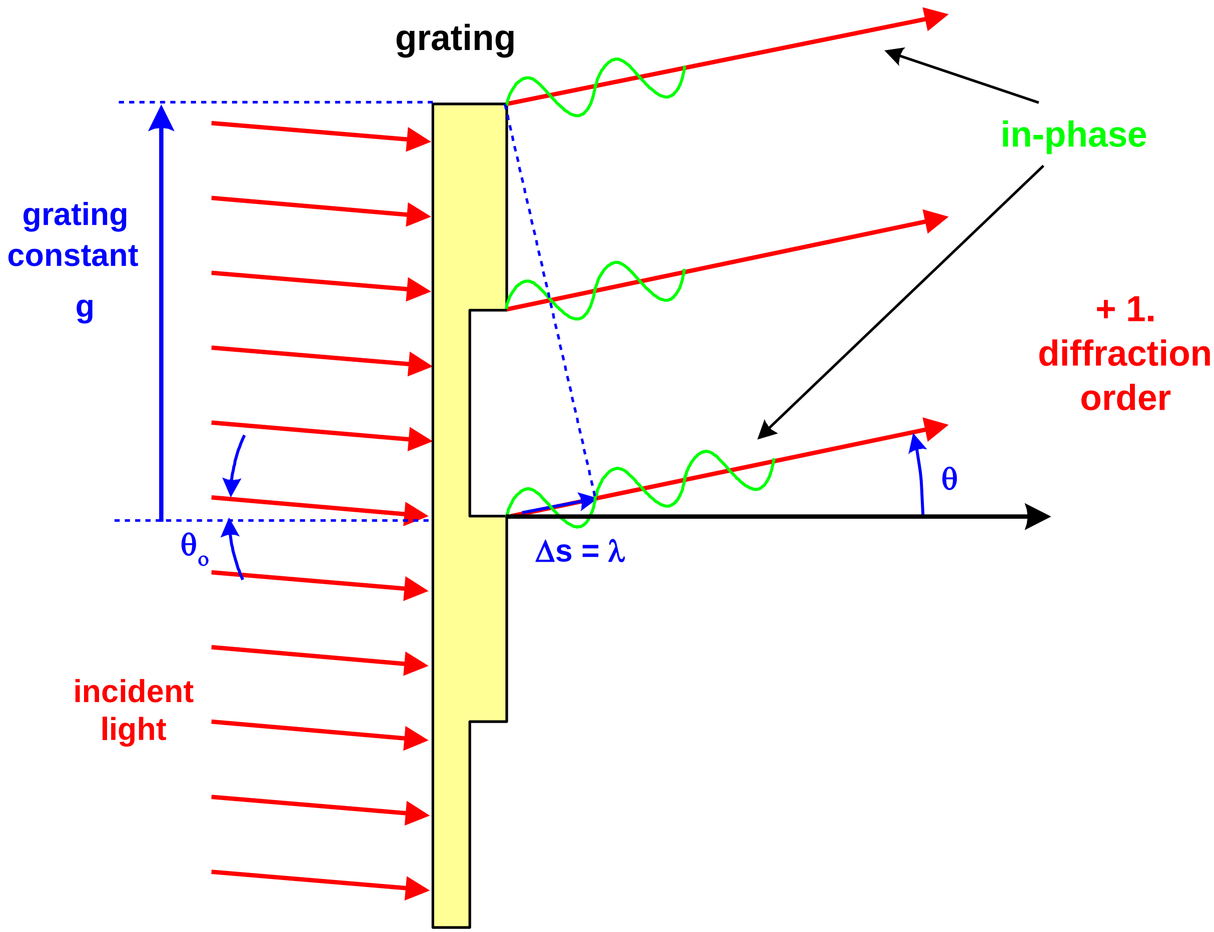
$$z_p \approx \frac{y^2}{2r}$$



- Maximum intensity:
  constructive interference of the contributions of all periods

- Grating equation

$$g \cdot (\sin\theta - \sin\theta_o) = m \cdot \lambda$$



Ideale diffraction grating

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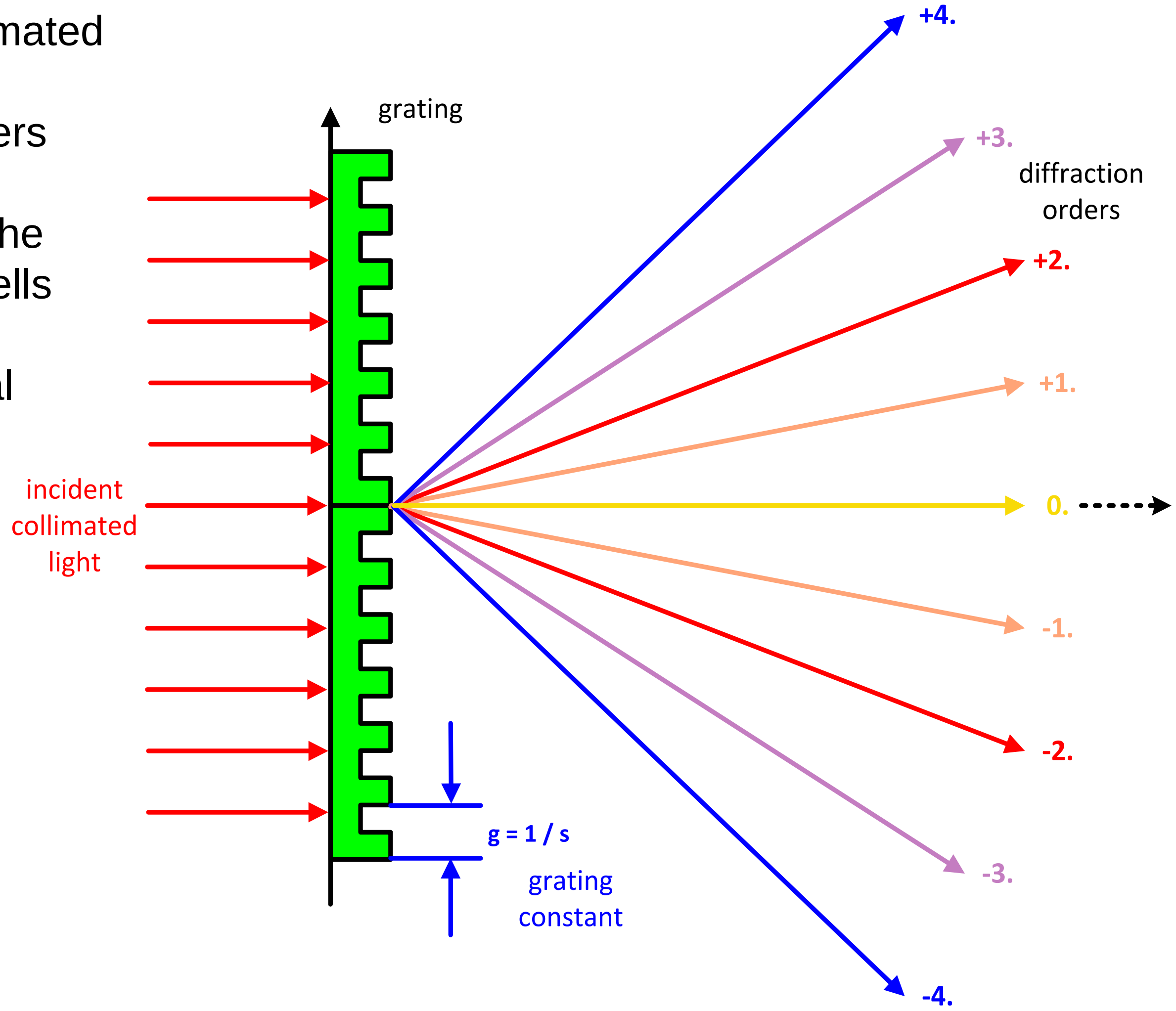
- Ideal diffraction grating: monochromatic incident collimated beam is decomposed into discrete sharp diffraction orders
- Constructive interference of the contributions of all periodic cells
- Only two orders for sinusoidal



2 Properties of optical systems II
Finite width of real grating orders

- Interference function of a finite number N of periods

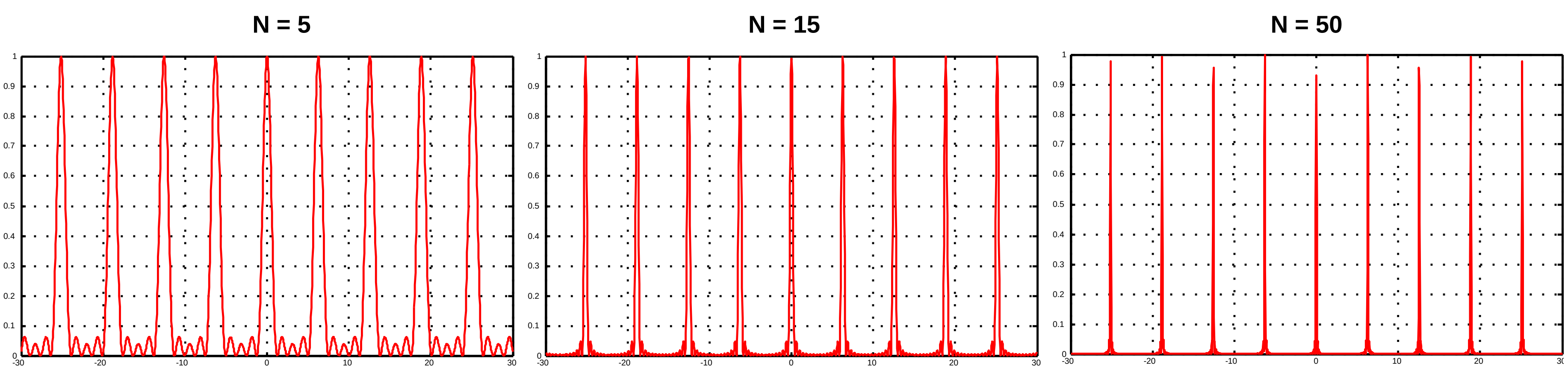
$$I = \frac{\sin^2\left(\dfrac{\pi \cdot g \cdot N \cdot \sin\theta}{\lambda}\right)}{\sin^2\left(\dfrac{\pi \cdot g \cdot \sin\theta}{\lambda}\right)}$$

- Finite width of every order depends on N

$$\sin\frac{\theta_{1/2}}{2} = \frac{\lambda}{4g \cdot N}$$

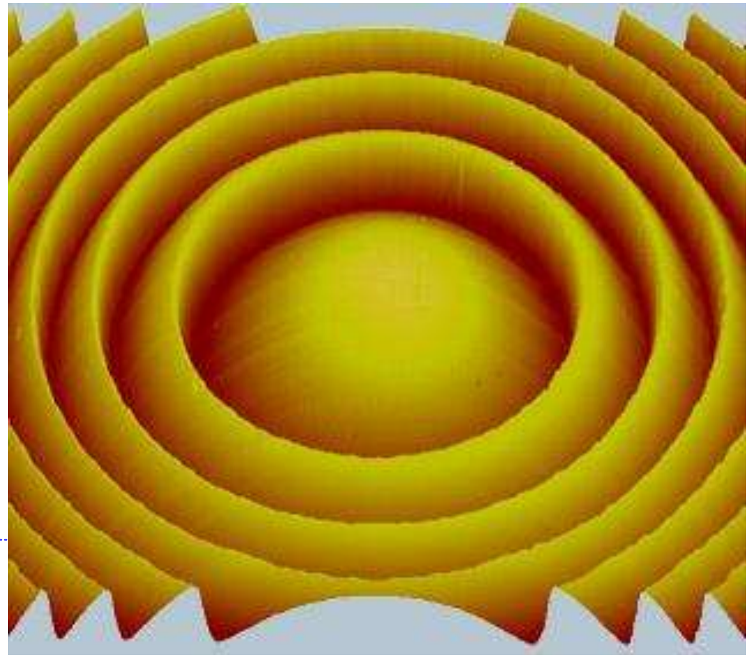
- Sharp order direction only in the limit of

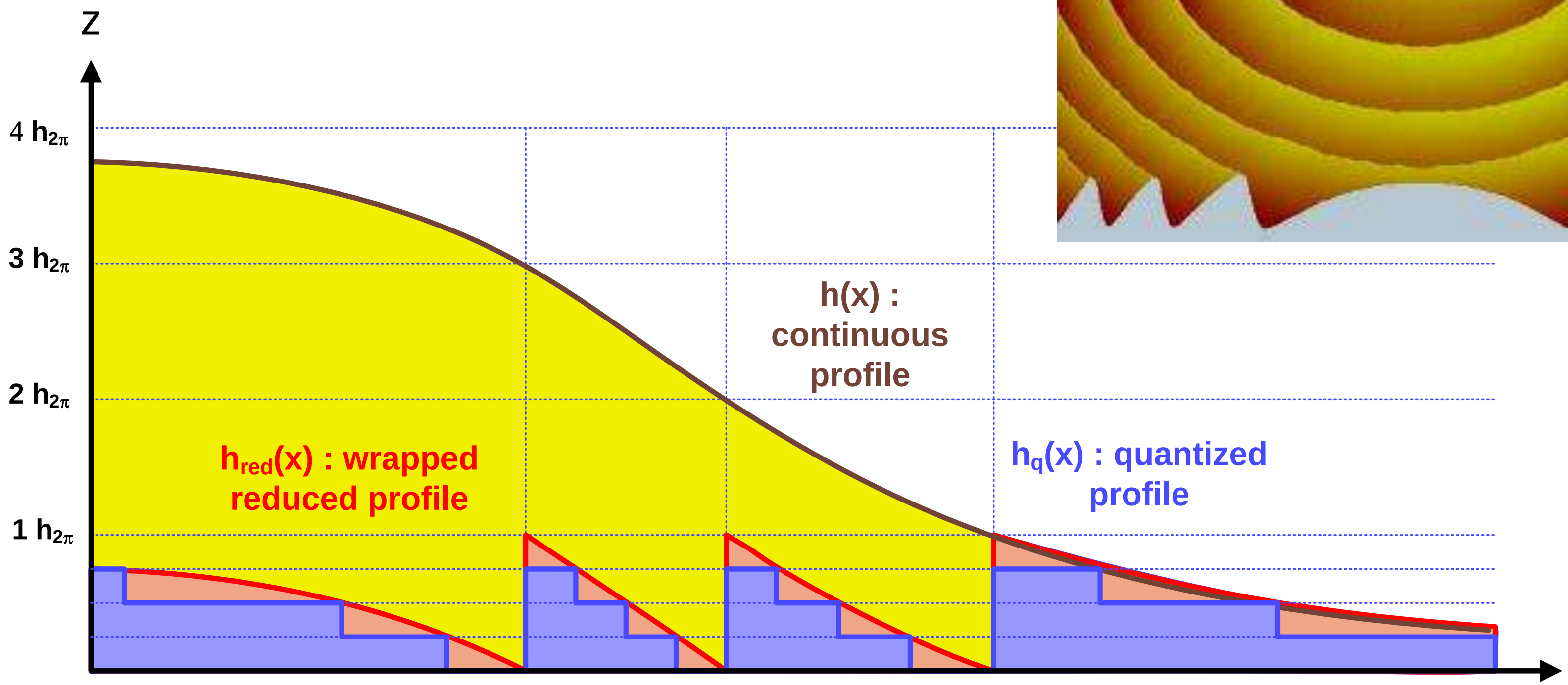
$$N \rightarrow \infty$$



2 Properties of optical systems II
Diffractive Elements

- Original lens height profile h(x)
- Wrapping of the lens profile: $h_{red}(x)$ reduction on maximal height $h_{2\pi}$
- Digitalization of the reduced profile: $h_q(x)$

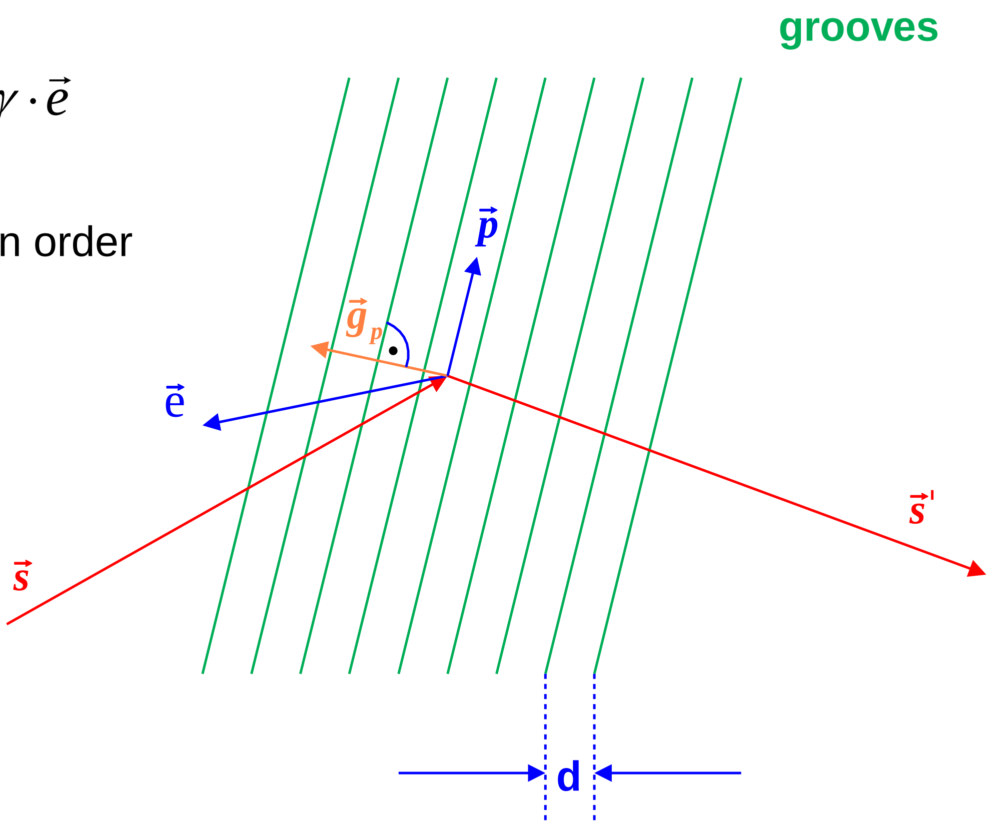




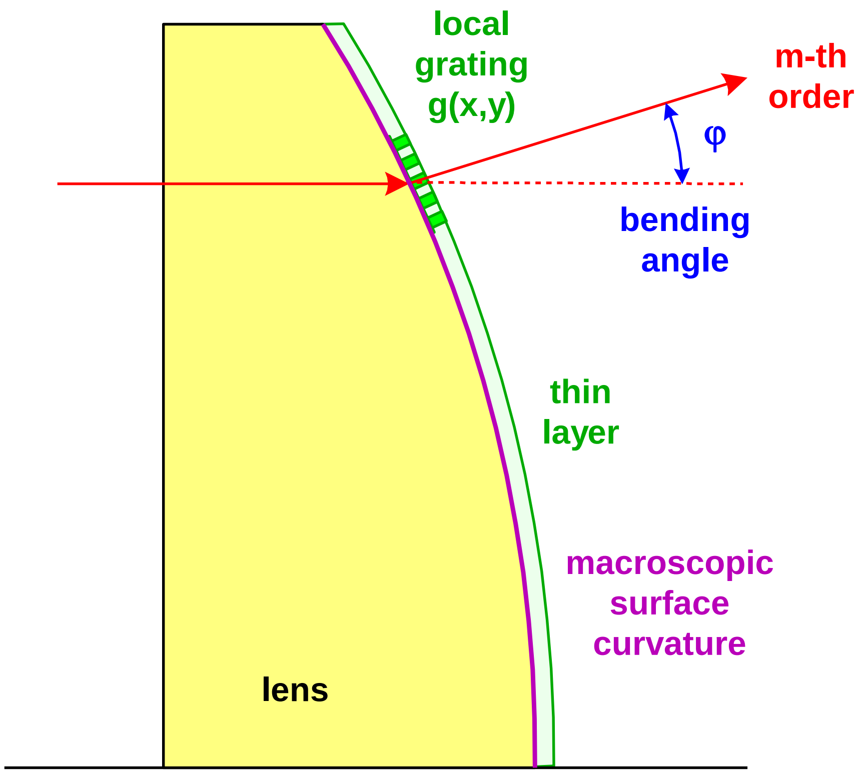
- Surface with grating structure:
  new ray direction follows the grating equation
- Local approximation in the case of space-varying grating width

$$\vec{s}\,' = \frac{n}{n'} \cdot \vec{s} + \frac{m\lambda g}{n' d} \cdot \hat{\vec{g}} + \gamma \cdot \vec{e}$$

- Raytrace only into one desired diffraction order
- Notations:
  g : unit vector perpendicular to grooves
  d : local grating width
  m : diffraction order
  e : unit normal vector of surface
- Applications:
  - diffractive elements
  - line gratings
  - holographic components



- Local micro-structured surface
- Location of ray bending :
  macroscopic carrier surface
- Direction of ray bending :
  local grating micro-structure
- Independent degrees of freedom:
  1. shape of substrate determines the point of the ray bending
  2. local grating constant determines the direction of the bended ray



2 Properties of optical systems II
Modellierung diffractive elements

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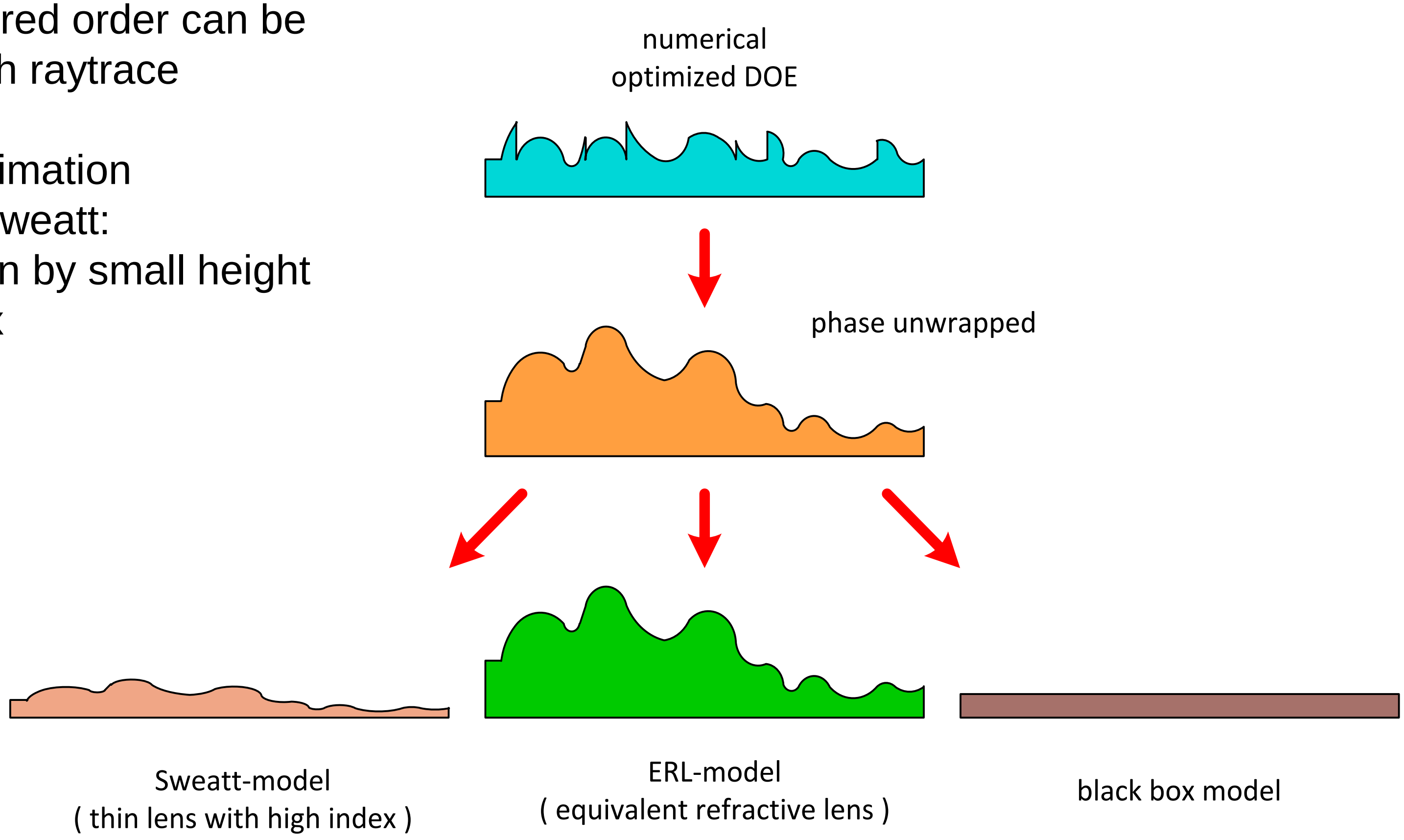
- Discrete topography on the surface
- Phase unwrapped to get a smooth surface
- Only one desired order can be calculated with raytrace
- Model approximation according to Sweatt: same refraction by small height and high index



Diffractive Optics: Sweatt Model

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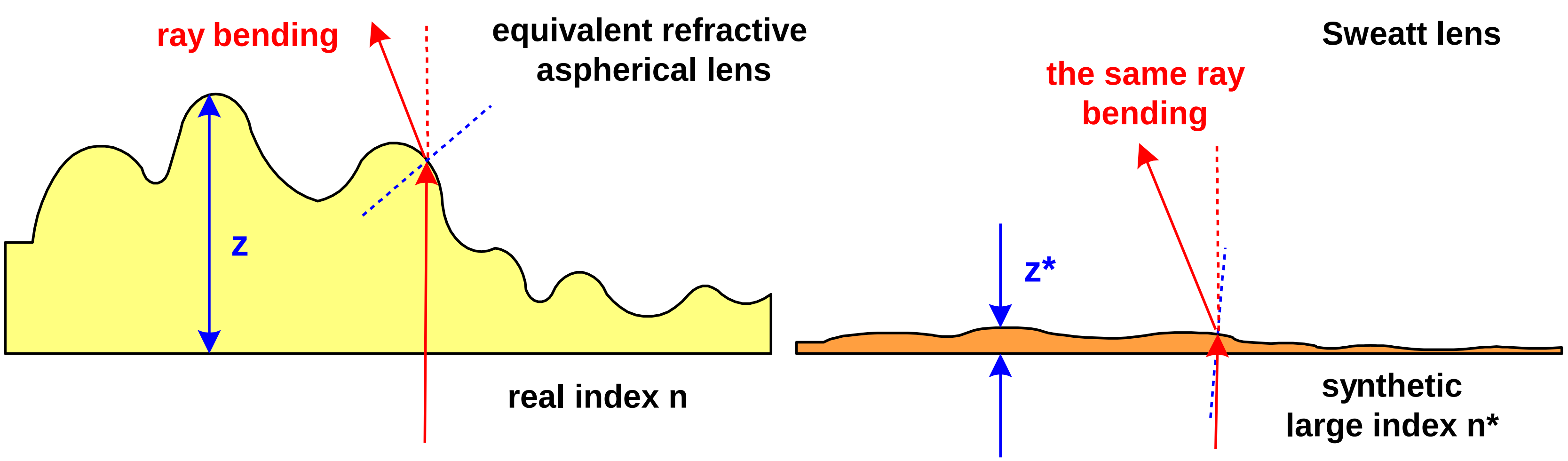
- Phase function redistributed:
  large index / small height
  typical : n = 10000

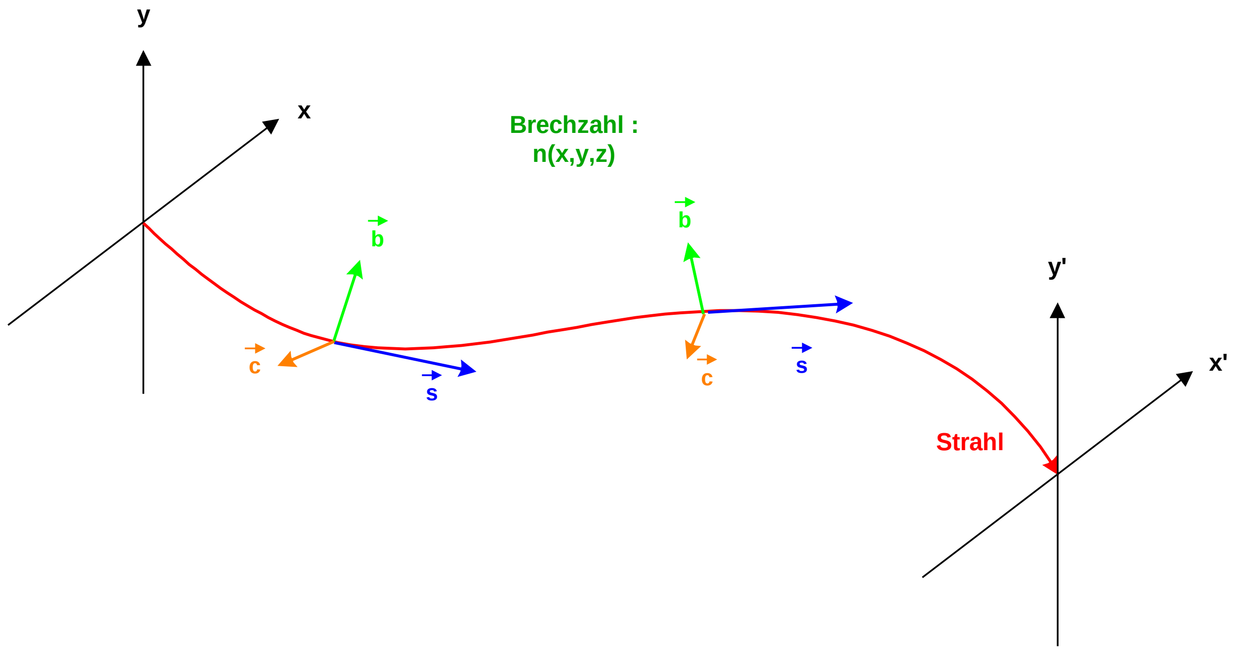
$$\varphi(x,y) = 2\pi \cdot n \cdot z(x,y) = 2\pi \cdot n^* \cdot z^*(x,y)$$

- Calculation in conventional software with raytrace possible



- Ray: in general curved line
- Numerical solution of Eikonal equation
- Step-based Runge-Kutta algorithm
  4th order expansion, adaptive step width
- Large computational times necessary for high accuracy

$$\frac{d^2\vec{r}}{dt^2} = n \cdot \nabla n = \vec{D} = \begin{pmatrix} n \dfrac{\partial n}{\partial x} \\ n \dfrac{\partial n}{\partial y} \\ n \dfrac{\partial n}{\partial z} \end{pmatrix}$$



- Analytical description of grin media by Taylor expansions of the function n(x,y,z)
- Separation of coordinates

$$n=n_{o,\lambda}+c_1h+c_2h^2+c_3h^4+c_4h^6+c_5h^8+c_6z+c_7z^2+c_8z^3+c_9z^4$$
$$+c_{10}x+c_{11}x^2+c_{12}x^3+c_{13}y+c_{14}y^2+c_{15}y^3$$

- Circular symmetry, nested expansion with mixed terms

$$n=n_{o,\lambda}+c_1h^2+c_2h^4+c_3h^6+c_4h^8+z\left(c_5+c_6h^2+c_7h^4+c_8h^6+c_9h^8\right)$$
$$+z^2\left(c_{10}+c_{11}h^2+c_{12}h^4+c_{13}h^6+c_{14}h^8\right)+z^3\left(c_{15}+c_{16}h^2+c_{17}h^4+c_{18}h^6+c_{19}h^8\right)$$

- Circular symmetry only radial

$$n=n_{o,\lambda}\sqrt{1+c_2(c_1h)^2+c_3(c_1h)^4+c_4(c_1h)^6+c_5(c_1h)^8+c_6(c_1h)^{10}}$$

- Only axial gradients

$$n=n_{o,\lambda}\sqrt{1+c_2(c_1z)^2+c_3(c_1z)^4+c_4(c_1z)^6+c_5(c_1z)^8}$$

- Circular symmetry, separated, wavelength dependent

$$n=n_{o,\lambda}+c_{1,\lambda}h^2+c_{2,\lambda}h^4+c_{3,\lambda}h^6+c_{4,\lambda}h^8+c_{5,\lambda}z+c_{6,\lambda}z^2+c_{7,\lambda}z^3$$

2 Properties of optical systems II
Gradient Lens Types

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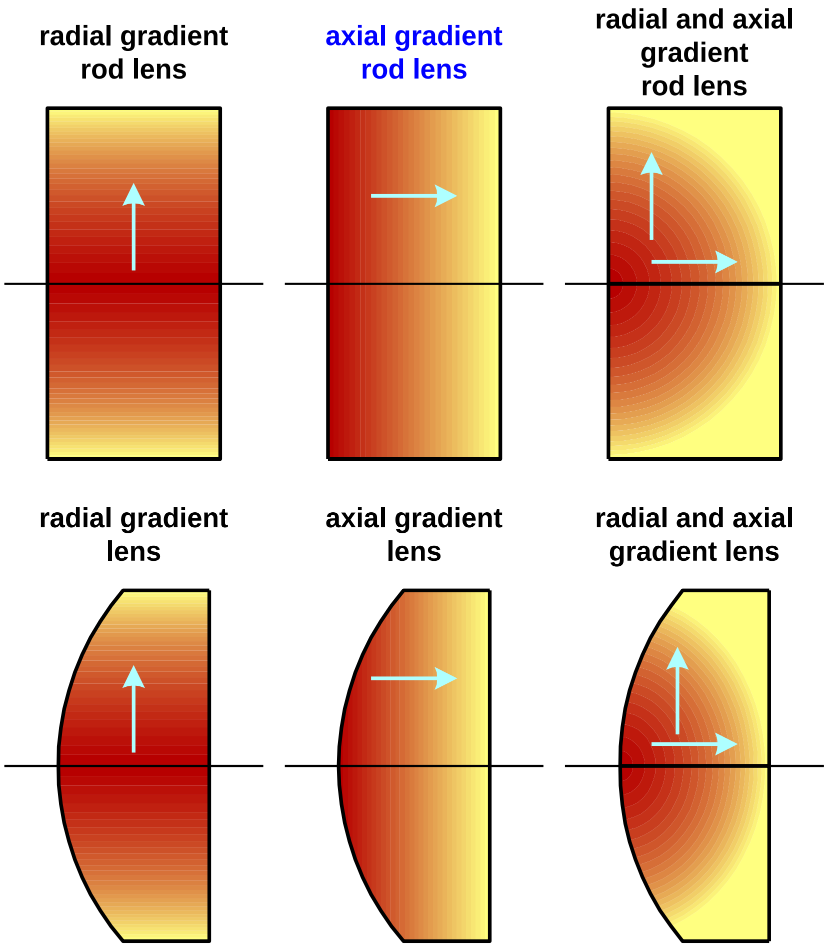
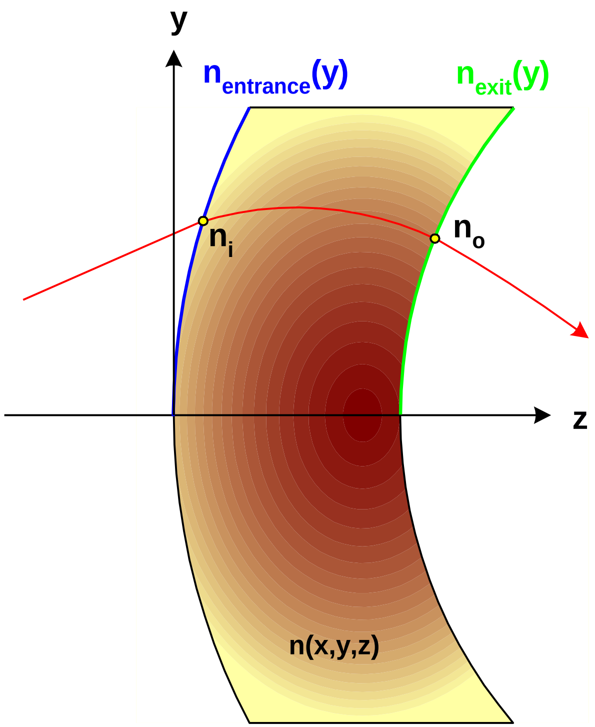
- Curved ray path in inhomogeneous media
- Different types of profiles

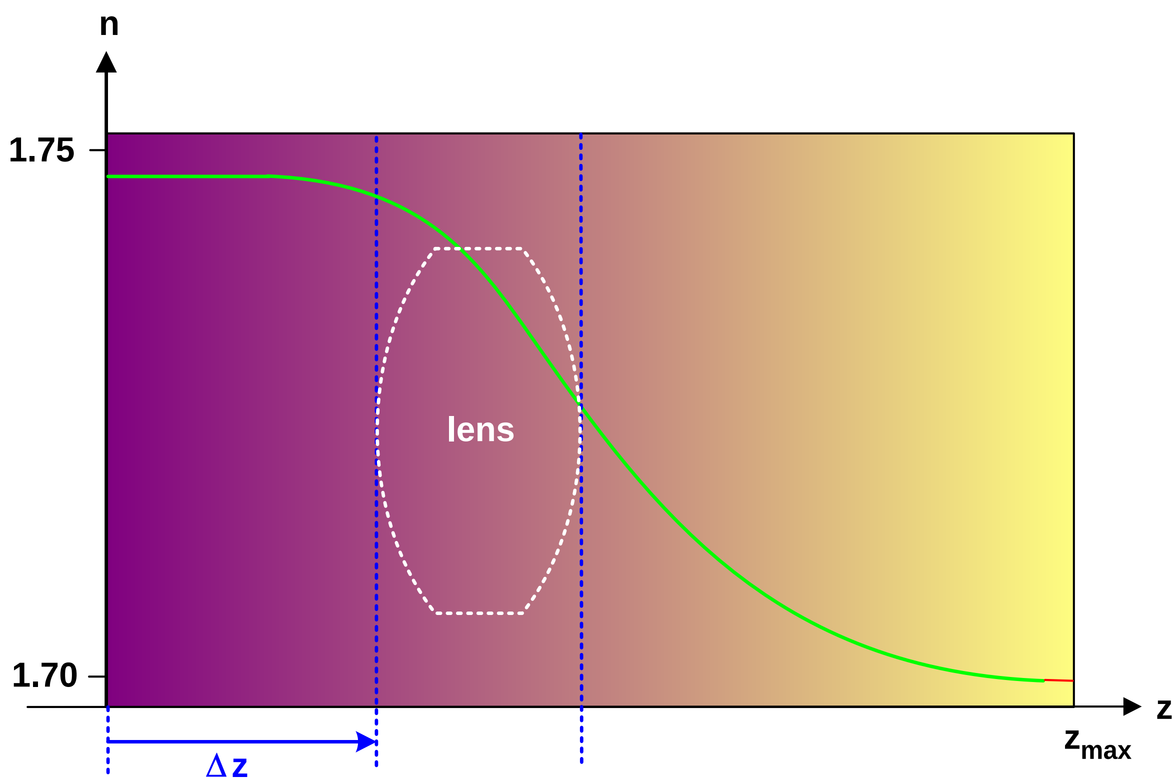


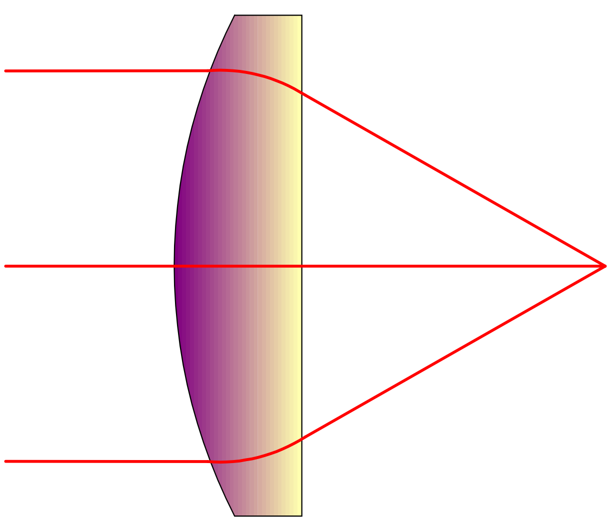


Gradium Lenses

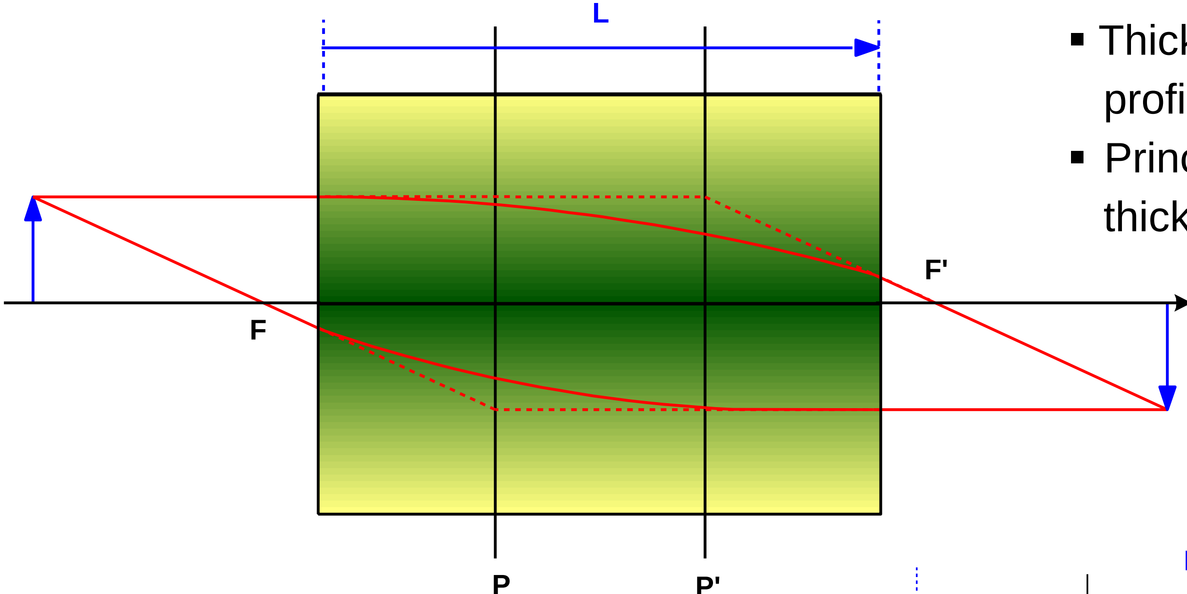
- Axial profile 'Gradium'
- Focussing effect only for oblique rays
- Combined effect of front surface curvature and index gradient
- Special cylindrical blanks with given profile allows choice of individual z-interval for the lens

$$n(z) = \sum_k n_k \cdot \left(\frac{z}{z_{max}} \right)^k$$





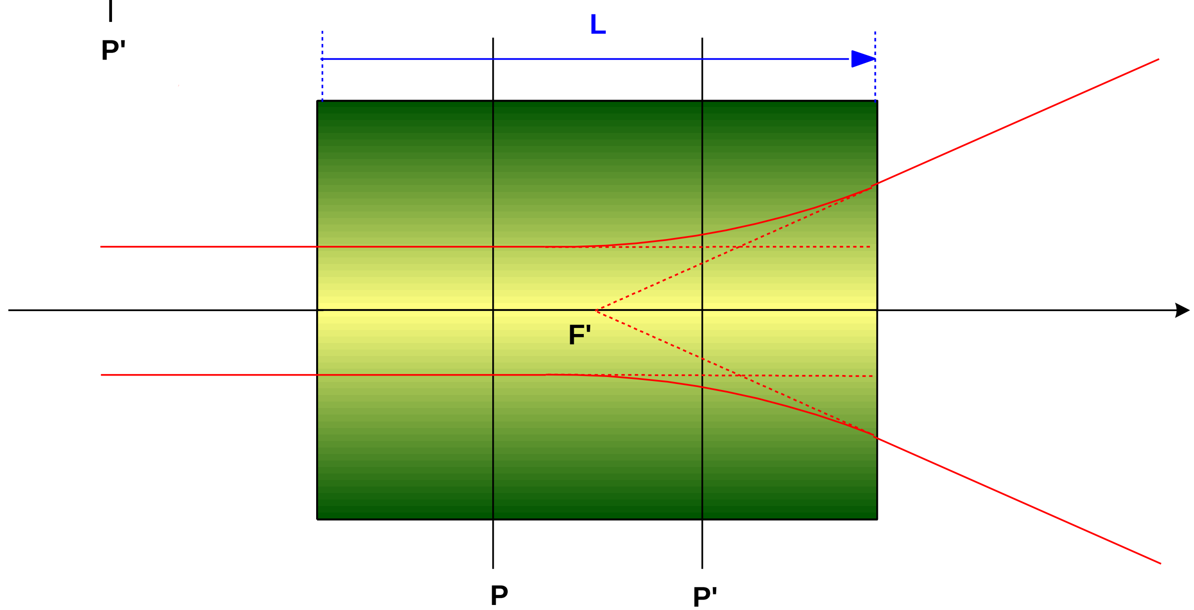
Collecting radial selfoc lens

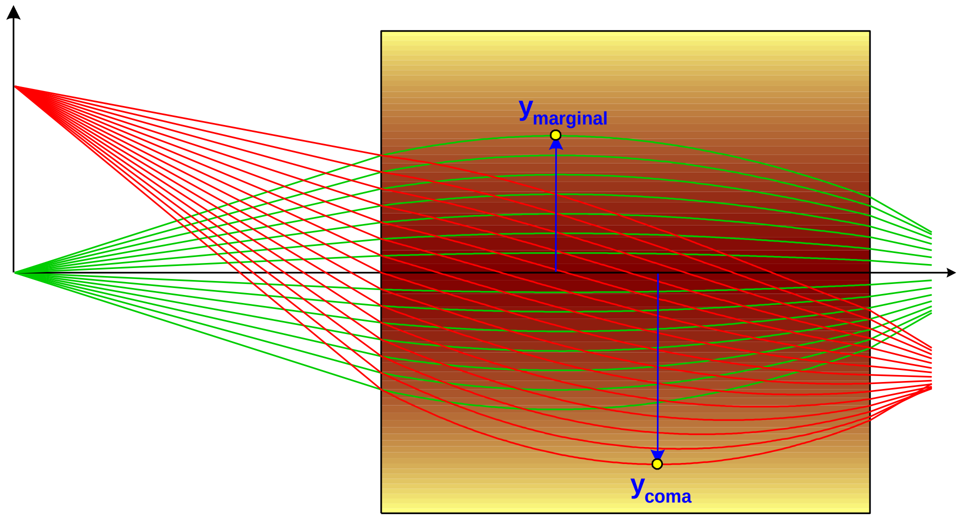


- Thick Wood lens with parabolic index profile
- Principal planes at 1/3 and 2/3 of thickness

$$n(r) = n_0 - n_2 \cdot r^2$$

- $n_2 > 0$: collecting lens
- $n_2 < 0$: negative lens



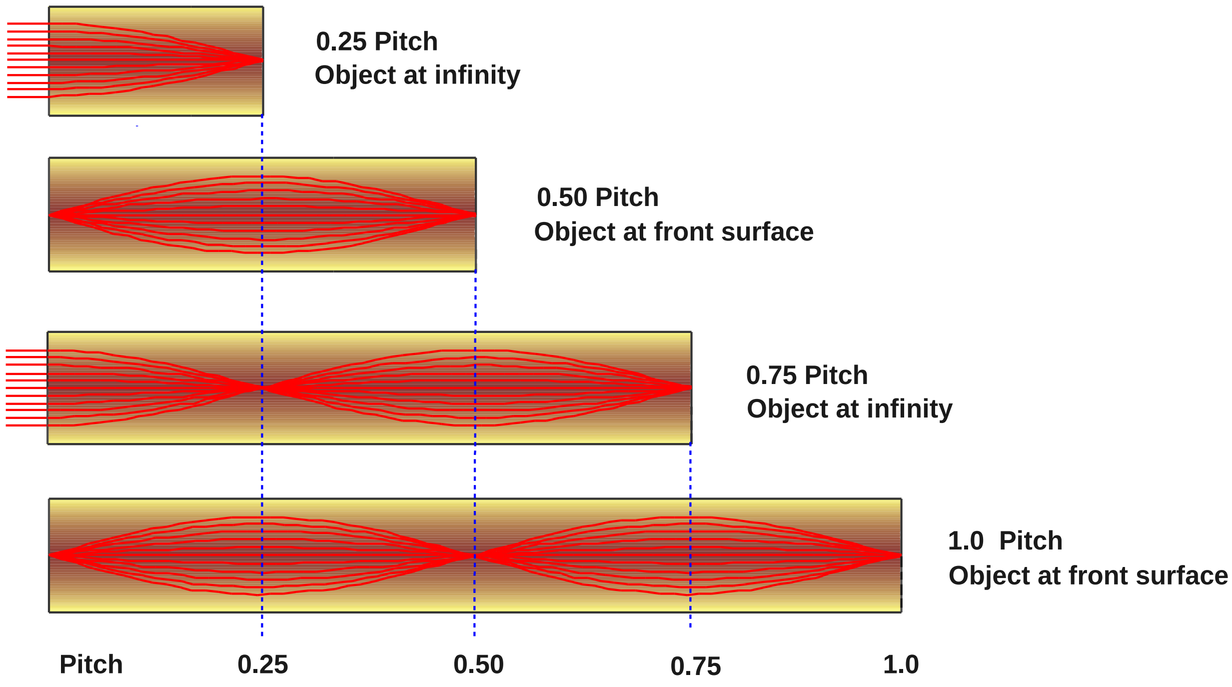


- Types of lenses with parabolic profile

$$n(r) = n_0 - n_2 \cdot r^2 = n_0 \cdot \left(1 - n_r \cdot r^2\right)$$
$$= n_0 \cdot \left(1 - \frac{1}{2} A \cdot r^2\right)$$

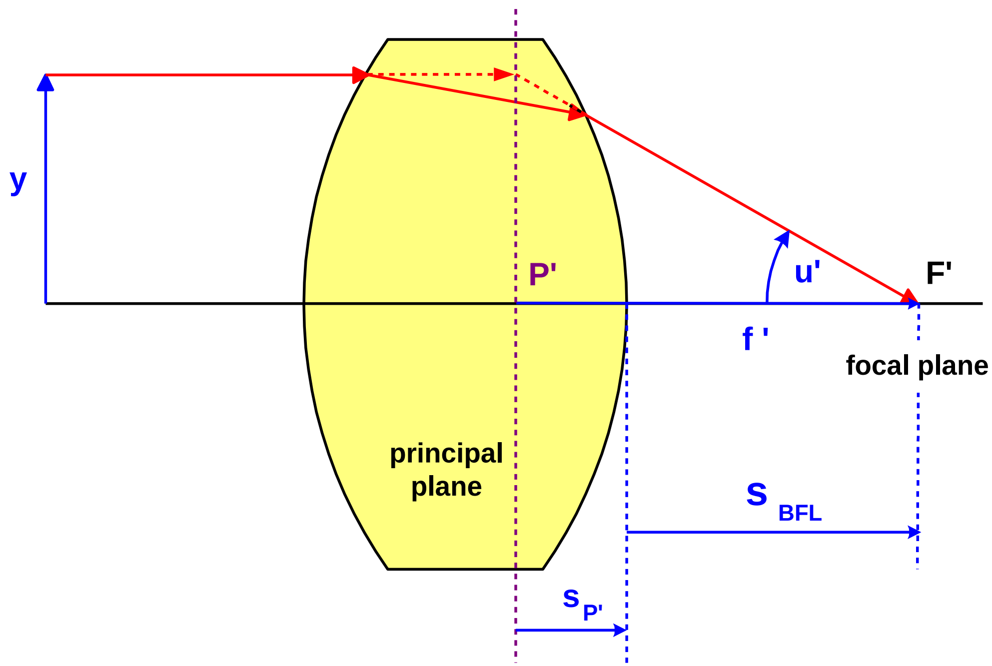
- Pitch length

$$p = 2\pi \cdot \sqrt{\frac{n_0}{2n_2}} = \frac{2\pi}{\sqrt{2n_r}}$$

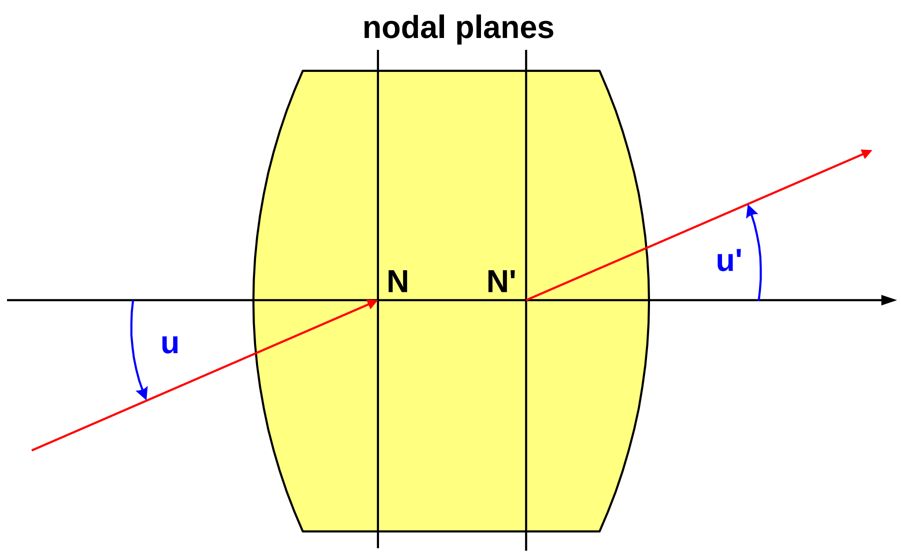


2 Properties of optical systems II
Cardinal elements of a lens

- Focal points:
  1. incoming parallel ray intersects the axis in F‘
  2. ray through F is leaves the lens parallel to the axis



- Principal plane P:
  location of apparent ray bending



- Nodal points:
  Ray through N goes through N‘ and preserves the direction

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P principal point

S vertex of the surface

F focal point

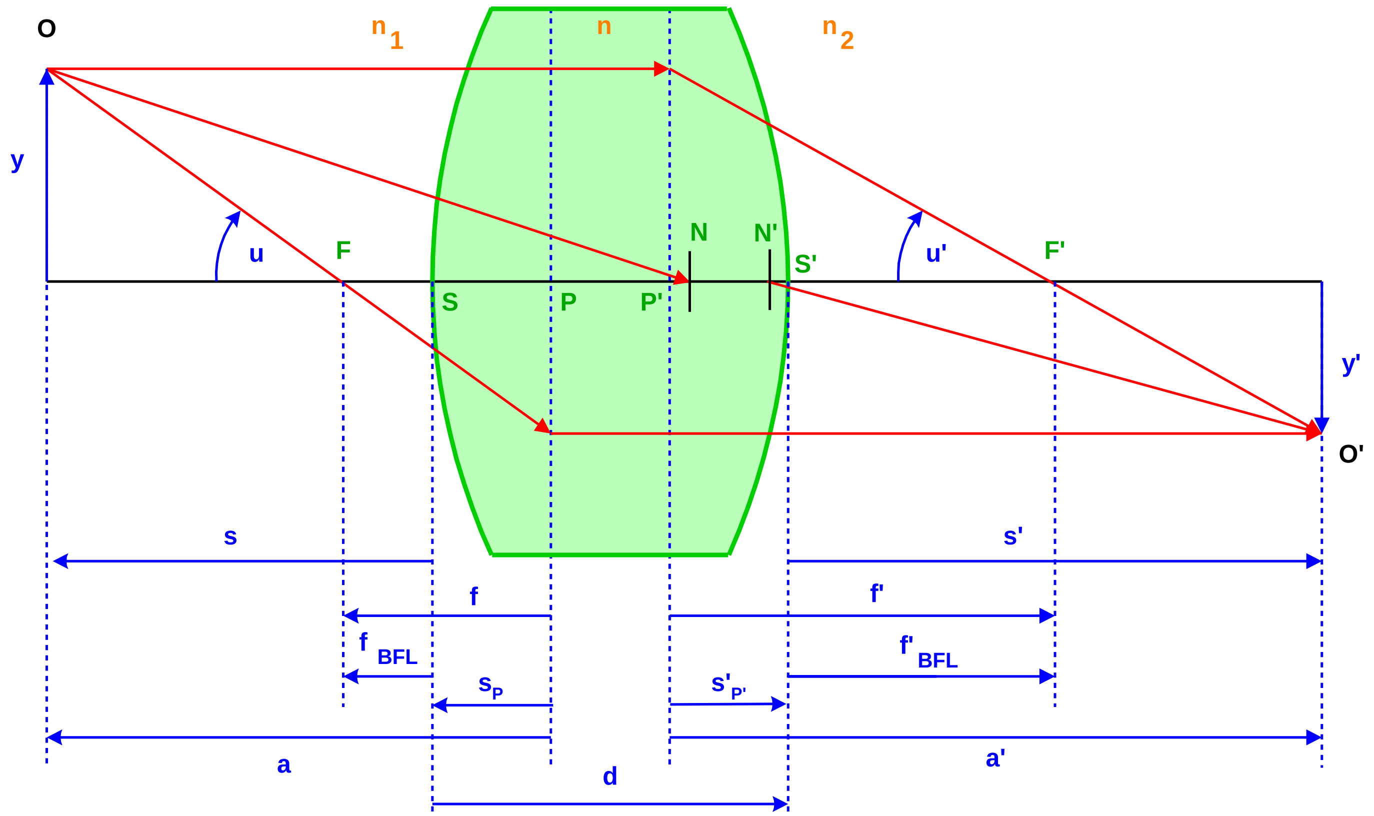
s intersection point of a ray with axis

f focal length PF

r radius of surface curvature

d thickness SS'

n refrative index



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- Main notations and properties of a lens:

  - radii of curvature r_1 , r_2
    curvatures c
    sign: r > 0 : center of curvature
    is located on the right side
  - thickness d along the axis
  - diameter D
  - index of refraction of lens material n

$$c_1 = \frac{1}{r_1} \qquad c_2 = \frac{1}{r_2}$$

- Focal length (paraxial)

$$f = \frac{y_F'}{\tan u} \quad , \quad f' = \frac{y}{\tan u'}$$

- Optical power

$$F = -\frac{n}{f} = \frac{n'}{f'}$$

- Back focal length
  intersection length,
  measured from the vertex point

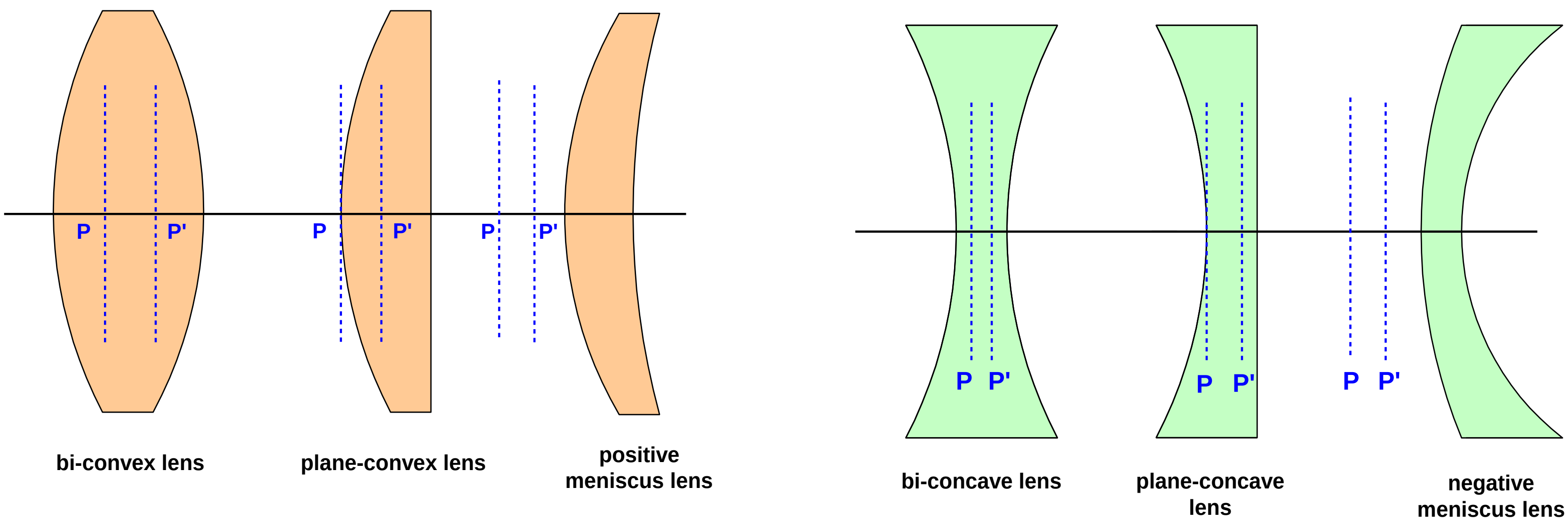
$$s_{F'} = f' + s_{H'}$$

2 Properties of optical systems II
Lens shape

- Different shapes of singlet lenses:
  1. bi-, symmetric
  2. plane convex / concave, one surface plane
  3. Meniscus, both surface radii with the same sign

- Convex: bending outside
  Concave: hollow surface

- Principal planes P, P‘: outside for mesicus shaped lenses



2 Properties of optical systems II
Lens bending und shift of principal plane

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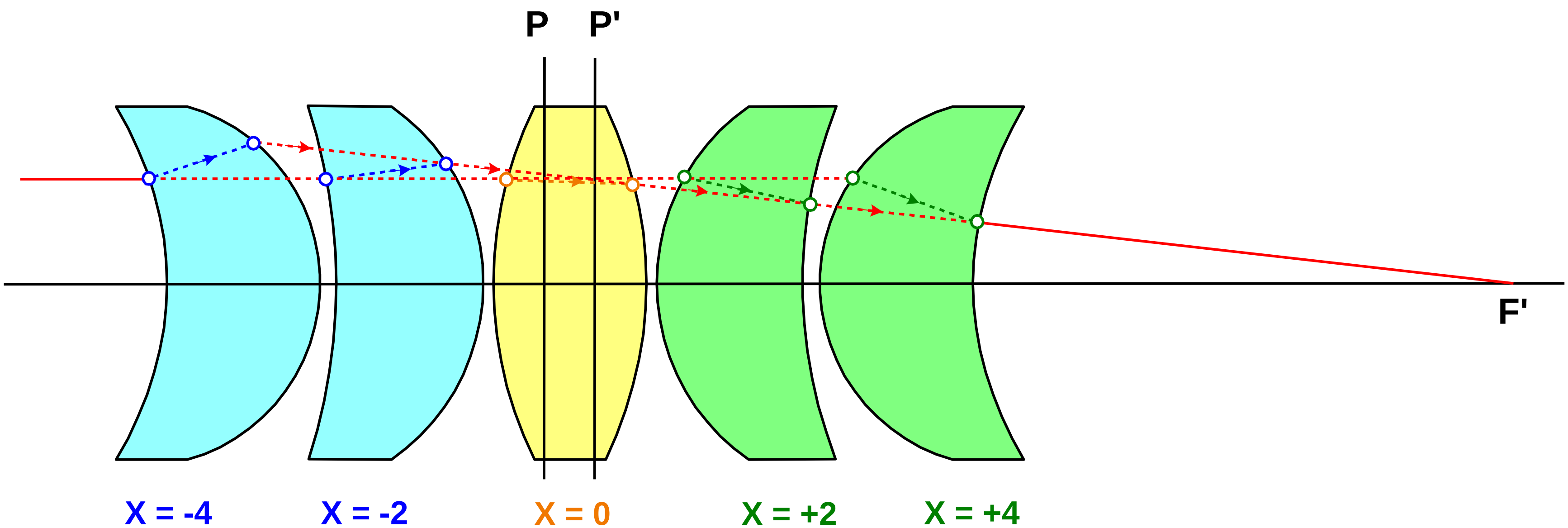
- Ray path at a lens of constant focal length and different bending
- The ray angle inside the lens changes
- The ray incidence angles at the surfaces changes strongly
- The principal planes move
  For invariant location of P, P‘ the position of the lens moves



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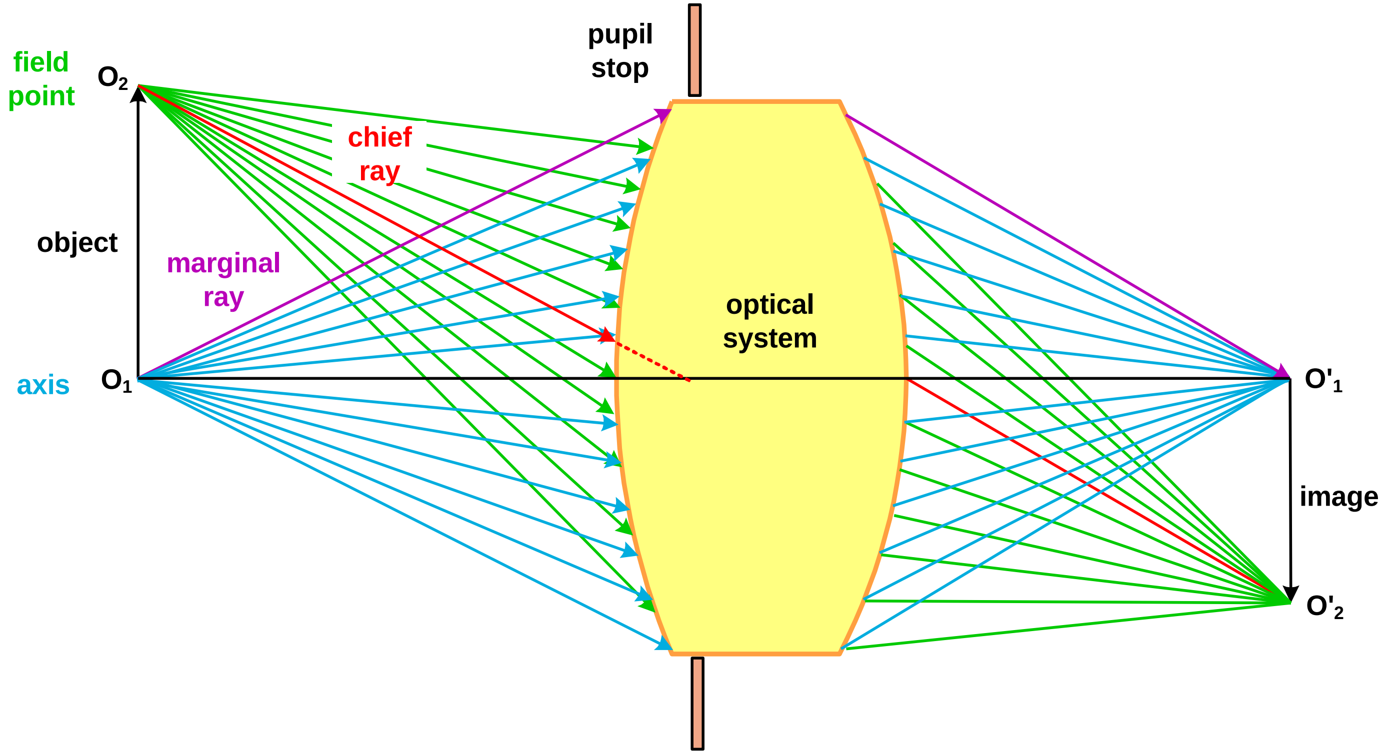
- Optical Image formation:
  All ray emerging from one object point meet in the perfect image point

- Region near axis:
  gaussian imaging
  ideal, paraxial

- Image field size:
  Chief ray

- Aperture/size of light cone:
  marginal ray
  defined by pupil stop



- Single surface imaging equation

$$\frac{n'}{s'} - \frac{n}{s} = \frac{n'-n}{r} = \frac{1}{f'}$$

- Thin lens in air focal length

$$\frac{1}{f'} = (n-1) \cdot \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

- Thin lens in air with one plane surface, focal length

$$f' = \frac{r}{n-1}$$

- Thin symmetrical bi-lens

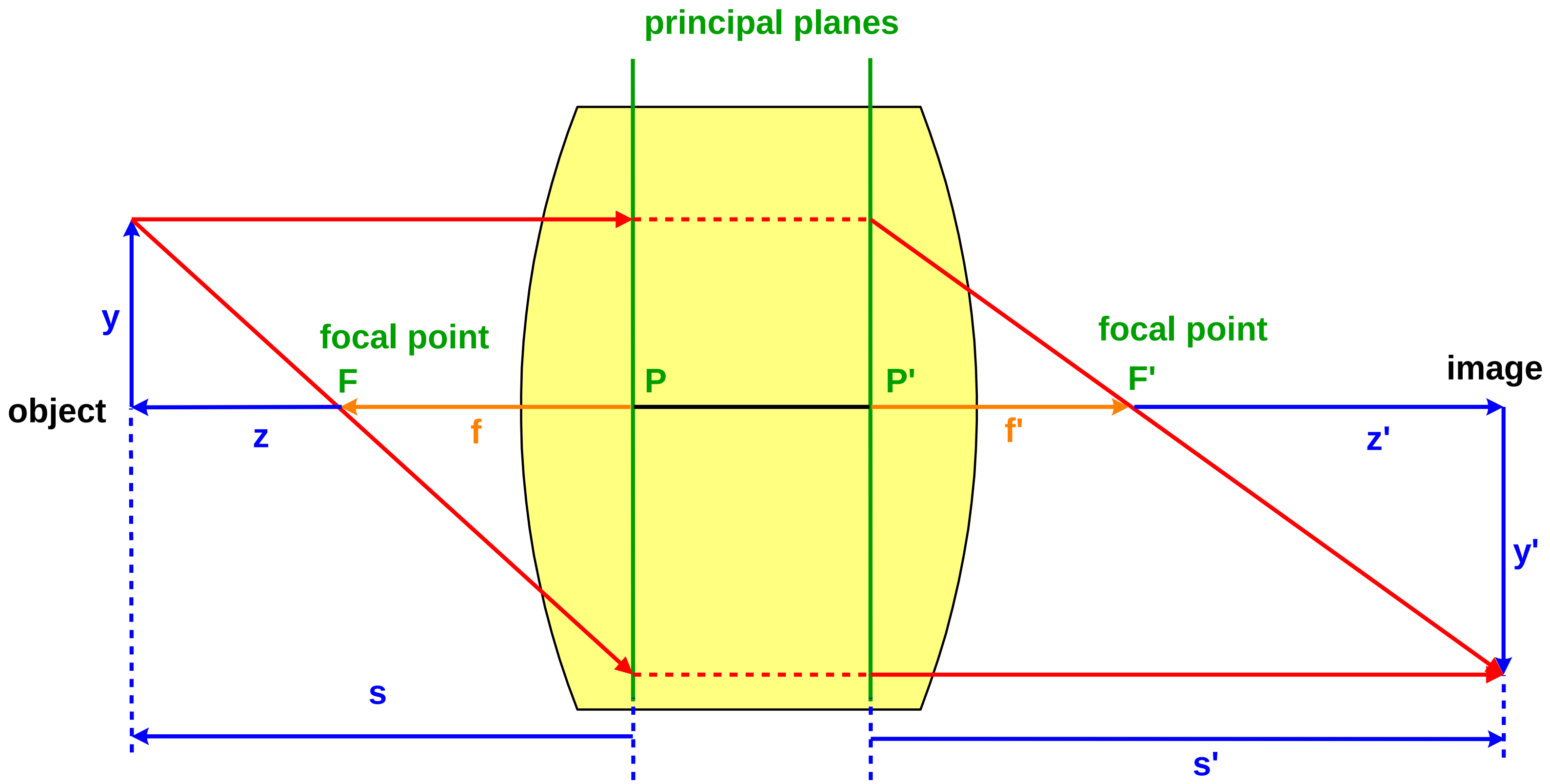
$$f' = \frac{r}{2 \cdot (n-1)}$$

- Thick lens in air focal length

$$\frac{1}{f'} = (n-1) \cdot \left(\frac{1}{r_1} - \frac{1}{r_2} \right) + \frac{(n-1)^2 d}{n \cdot r_1 r_2}$$

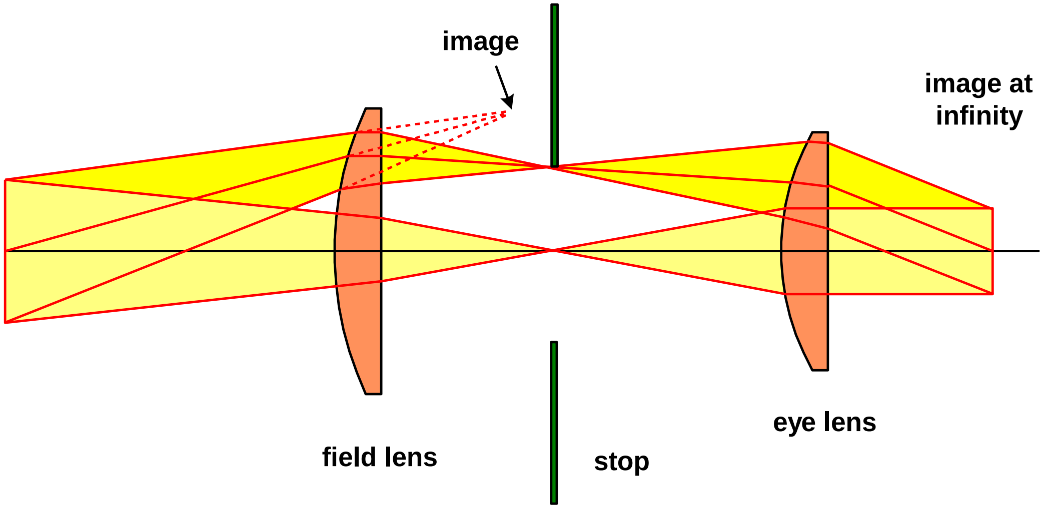
- Lateral magnification for finite imaging
- Scaling of image size

$$m = \frac{y'}{y} = -\frac{f \cdot \tan u}{f' \cdot \tan u'}$$

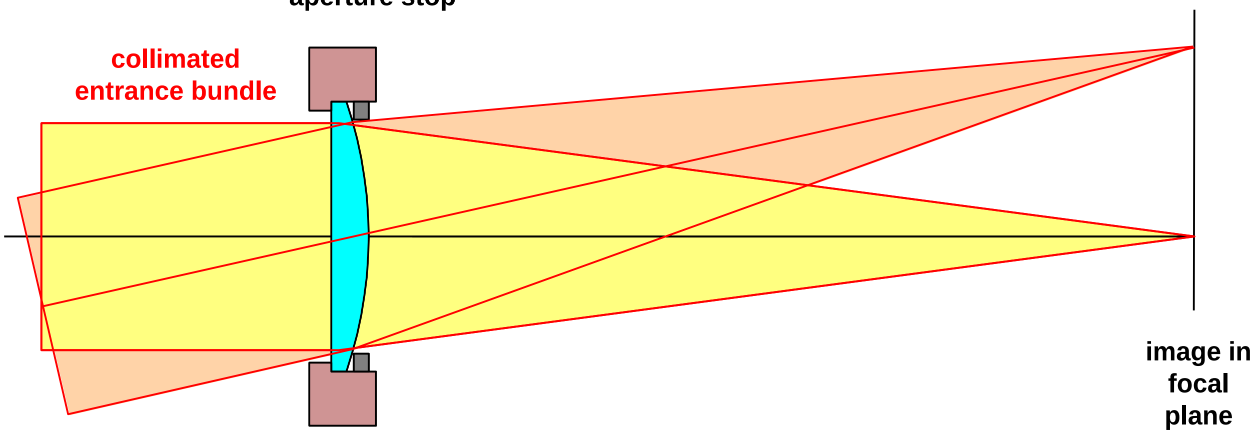


Object or field at infinity

- Image in infinity:
  - collimated exit ray bundle
  - realized in binoculars



- Object in infinity
  - input ray bundle collimated
  - realized in telescopes
  - aperture defined by diameter not by angle



Imaging equation

- Imaging by a lens in air:
  lens makers formula

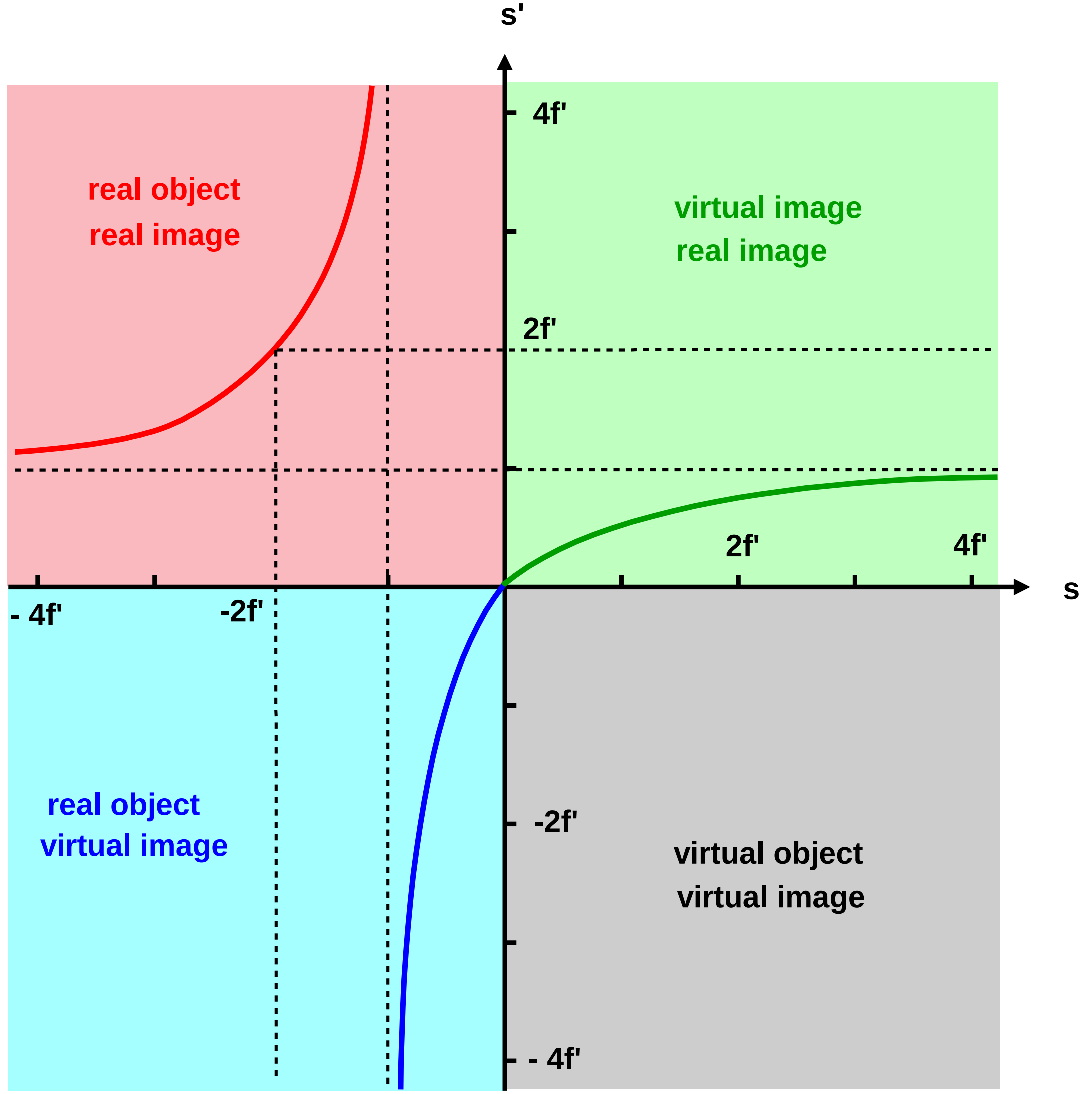
$$\frac{1}{s'} - \frac{1}{s} = \frac{1}{f}$$

- Magnification

$$m = \frac{s'}{s}$$

- Real imaging:
  s < 0 , s' > 0

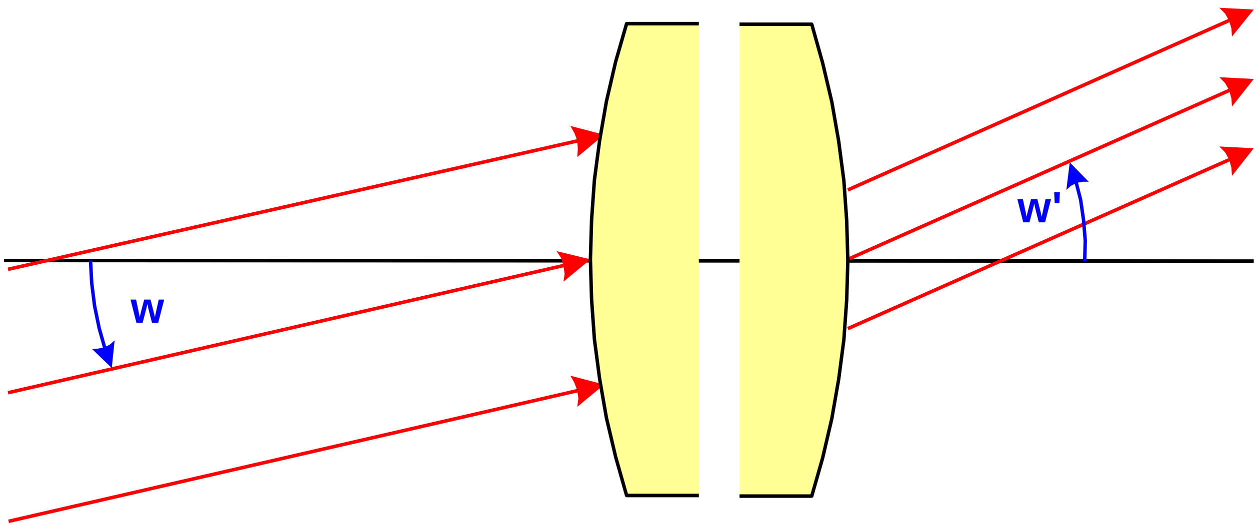
- Intersection lengths s, s' measured with respective to the principal planes P, P'



Angle Magnification

- Afocal systems with object/image in infinity
- Definition with field angle w
  angular magnification

$$\Gamma = \frac{\tan w'}{\tan w} = \frac{nh}{n'h'}$$

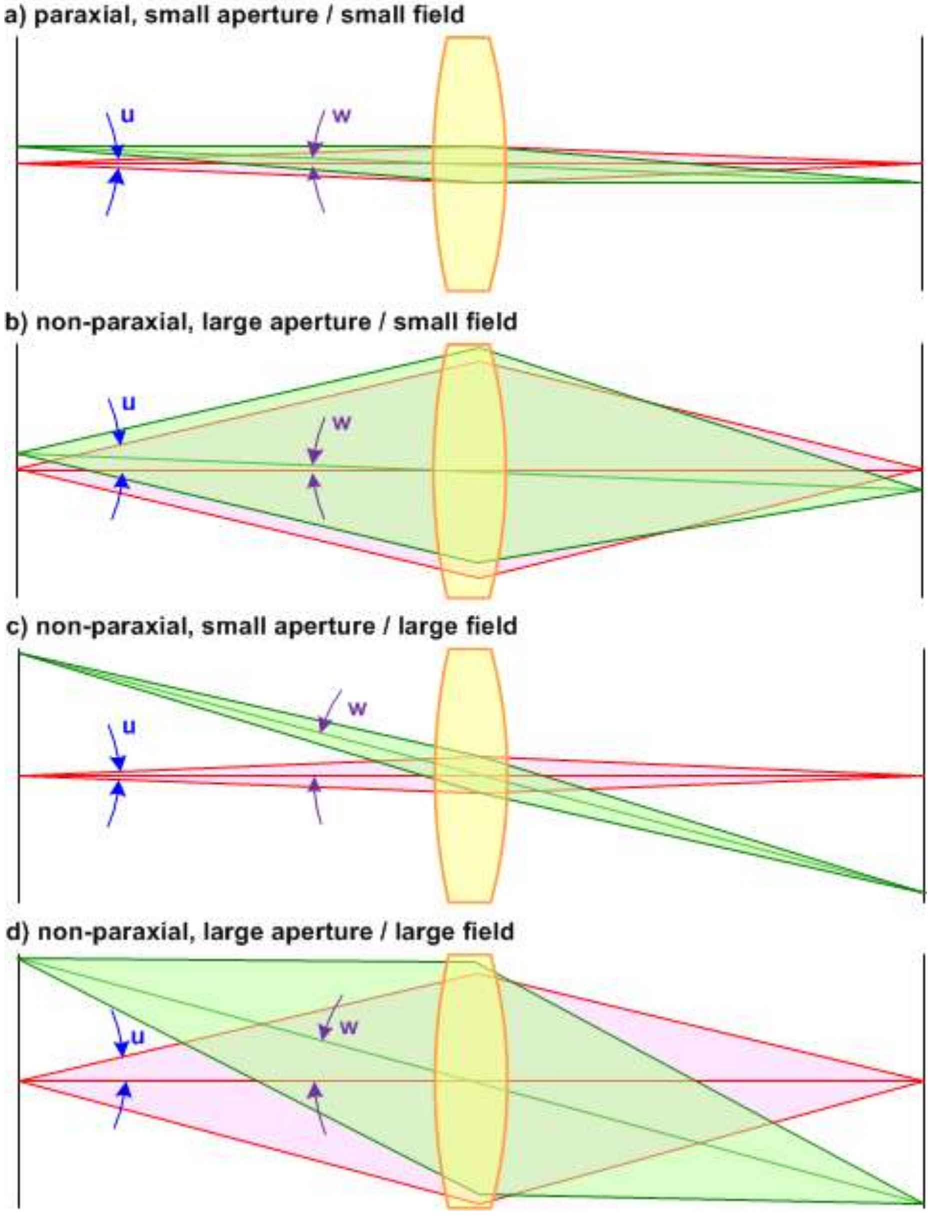


- Relation with finite-distance magnification

$$m \cdot \Gamma = -\frac{f}{f'}$$

Paraxial Approximation

- Paraxiality is given for small angles relative to the optical axis for all rays
- Large numerical aperture angle u violates the paraxiality, spherical aberration occurs
- Large field angles w violates the paraxiality, coma, astigmatism, distortion, field curvature occurs



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Paraxial approximation:

$$n \cdot i = n' \cdot i'$$

- Small angles of rays at every surface
- Small incidence angles allows for a linearization of the law of refraction
- All optical imaging conditions become linear (Gaussian optics), calculation with ABCD matrix calculus is possible
- No aberrations occur in optical systems
- There are no truncation effects due to transverse finite sized components
- Serves as a reference for ideal system conditions
- Is the fundament for many system properties (focal length, principal plane, magnification,...)
- The sag of optical surfaces (difference in z between vertex plane and real surface intersection point) can be neglected
- All waves are plane of spherical (parabolic)
- The phase factor of spherical waves is quadratic

$$E(x) = E_0 \cdot e^{-\frac{i \pi x^2}{\lambda R}}$$

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- Law of refraction

$$n \cdot \sin I = n' \cdot \sin I'$$

- Taylor expansion

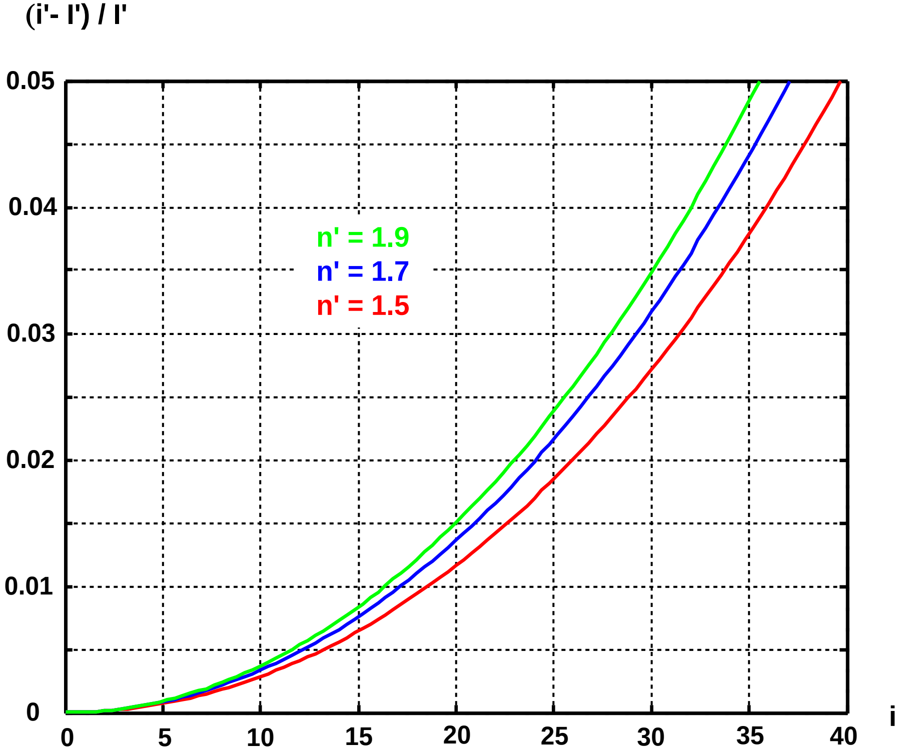
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$$

- Linear formulation of the law of refraction

$$n \cdot i = n' \cdot i'$$

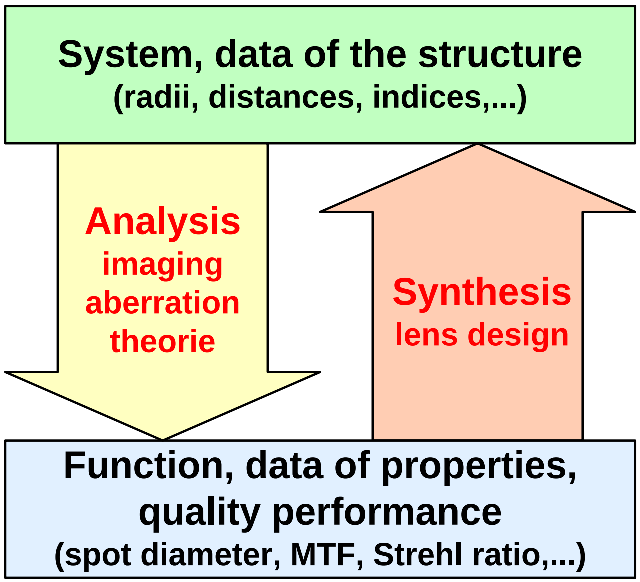
- Error of the paraxial approximation

$$\varepsilon = \frac{i' - I'}{I'} = \frac{\frac{n \cdot i}{n'}}{\arcsin\left(\frac{n \cdot \sin i}{n'}\right)} - 1$$

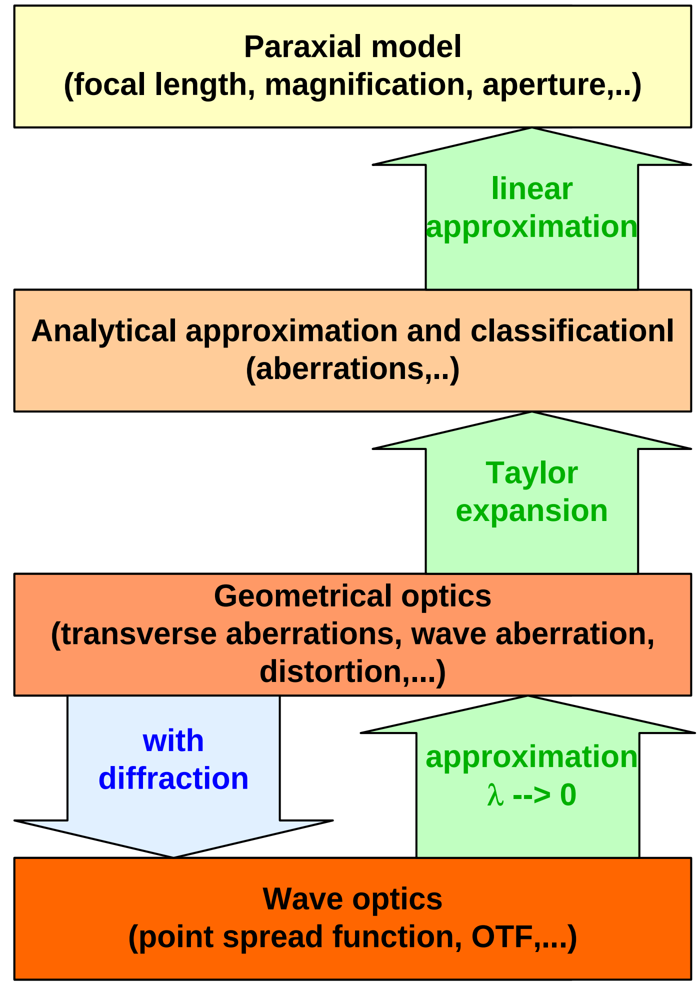


2 Properties of optical systems II
Modelling of Optical Systems

- Principal purpose of calculations:



- Imaging model with levels of refinement



Ref: W. Richter

2 Properties of optical systems II
Modelling of Optical Systems

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- Five levels of modelling:

1. Geometrical raytrace with analysis
2. Equivalent geometrical quantities, classification
3. Physical model:
   complex pupil function
4. Primary physical quantities
5. Secondary physical quantities

- Blue arrows: conversion of quantities