Lens Design II

Lecture 1: Aberrations and optimization
2017-10-16

Herbert Gross
### Preliminary Schedule Lens Design II 2017

<table>
<thead>
<tr>
<th>Week</th>
<th>Date</th>
<th>Topic</th>
<th>Subtopics</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>16.10</td>
<td>Aberrations and optimization</td>
<td>Reiteration</td>
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<tr>
<td>2</td>
<td>23.10</td>
<td>Structural modifications</td>
<td>Zero operands, lens splitting, lens addition, lens removal, material selection</td>
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<td>3</td>
<td>30.10</td>
<td>Aspheres</td>
<td>Correction with aspheres, Forbes approach, optimal location of aspheres, several aspheres</td>
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<td>4</td>
<td>06.11</td>
<td>Freeforms</td>
<td>Freeform surfaces, general aspects, surface description, quality assessment, initial systems</td>
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<td>5</td>
<td>13.11</td>
<td>Field flattening</td>
<td>Astigmatism and field curvature, thick meniscus, plus-minus pairs, field lenses</td>
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<td>6</td>
<td>20.11</td>
<td>Chromatical correction I</td>
<td>Achromatization, axial versus transversal, glass selection rules, burried surfaces</td>
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<td>7</td>
<td>27.11</td>
<td>Chromatical correction II</td>
<td>Secondary spectrum, apochromatic correction, aplanatic achromates, spherochromatism</td>
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<tr>
<td>8</td>
<td>04.12</td>
<td>Special correction topics I</td>
<td>Symmetry, wide field systems, stop position, vignetting</td>
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<tr>
<td>9</td>
<td>11.12</td>
<td>Special correction topics II</td>
<td>Telecentricity, monocentric systems, anamorphic lenses, Scheimpflug systems</td>
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<tr>
<td>10</td>
<td>18.12</td>
<td>Higher order aberrations</td>
<td>High NA systems, broken achromates, induced aberrations</td>
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<tr>
<td>11</td>
<td>08.01</td>
<td>Further topics</td>
<td>Sensitivity, scan systems, eyepieces</td>
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<tr>
<td>12</td>
<td>15.01</td>
<td>Mirror systems</td>
<td>special aspects, double passes, catadioptric systems</td>
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<tr>
<td>13</td>
<td>22.01</td>
<td>Zoom systems</td>
<td>Mechanical compensation, optical compensation</td>
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<tr>
<td>14</td>
<td>30.01</td>
<td>Diffractive elements</td>
<td>Color correction, ray equivalent model, straylight, third order aberrations, manufacturing</td>
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<tr>
<td>15</td>
<td>05.02</td>
<td>Realization aspects</td>
<td>Tolerancing, adjustment</td>
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</table>
1. Geometrical aberrations
2. Wave aberrations and Zernikes
3. Point spread function
4. Modulation transfer function
5. Principles of optimization
Optical Image Formation

- Perfect optical image:
  All rays coming from one object point intersect in one image point
- Real system with aberrations:
  1. transverse aberrations in the image plane
  2. longitudinal aberrations from the image plane
  3. wave aberrations in the exit pupil
Notations for an Optical System

- \( x, y \): object coordinates, especially object height
- \( x', y' \): image coordinates, especially image height
- \( x_p, y_p \): coordinates of entrance pupil
- \( x'_p, y'_p \): coordinates of exit pupil
- \( s \): object distance from 1st surface
- \( s' \): image distance from last surface
- \( p \): entrance pupil distance from 1st surface
- \( p' \): exit pupil distance from last surface
- \( \Delta x' \): sagittal transverse aberration
- \( \Delta y' \): meridional transverse aberration
**Polynomial Expansion of Aberrations**

- Representation of 2-dimensional Taylor series vs field $y$ and aperture $r$
- Selection rules: checkerboard filling of the matrix
- Constant sum of exponents according to the order

<table>
<thead>
<tr>
<th>Aperture $r$</th>
<th>Field $y$</th>
<th>Spherical</th>
<th>Coma</th>
<th>Astigmatism</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r^0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$r^1$ Defocus</td>
<td>$y^0$</td>
<td>$y^1$</td>
<td>$y^2$</td>
<td>$y^3$</td>
</tr>
<tr>
<td>$r^2$</td>
<td>$r^1$</td>
<td>$y^2 r^1 \cos^2 \theta$</td>
<td>$y^2 r^1 \cos^2 \theta$</td>
<td>$y^3 r^2 \cos^3 \theta$</td>
</tr>
<tr>
<td>$r^3$ Spherical primary</td>
<td>$r^3$</td>
<td>$y r^2 c\cos \theta$</td>
<td>$y r^2 c\cos \theta$</td>
<td>$y^2 r^3 \cos^2 \theta$</td>
</tr>
<tr>
<td>$r^4$</td>
<td></td>
<td>$y^4$</td>
<td>$y^2 r^3$</td>
<td>$y^4 r^1 \cos^2 \theta$</td>
</tr>
<tr>
<td>$r^5$ Spherical secondary</td>
<td>$r^5$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Image location**

**Primary aberrations / Seidel**

**Secondary aberrations**
Pupil Sampling

- Ray plots
- Spot diagrams

- Sagittal ray fan
- Tangential ray fan
- Whole pupil area

\[
\begin{align*}
\Delta y & \quad \Delta x \\
\Delta y & \quad \Delta x
\end{align*}
\]
Spherical Aberration

- Typical chart of aberration representation
- Reference: at paraxial focus

**Primary spherical aberration at paraxial focus**

### Wave aberration

<table>
<thead>
<tr>
<th>Tangential</th>
<th>Sagittal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2\lambda$</td>
<td>$2\lambda$</td>
</tr>
</tbody>
</table>

**Transverse ray aberration**

<table>
<thead>
<tr>
<th>$\Delta y'$</th>
<th>$\Delta x'$</th>
<th>$\Delta y'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01 mm</td>
<td>0.01 mm</td>
<td>0.01 mm</td>
</tr>
</tbody>
</table>

**Pupil:**
- $y$-section
- $x$-section

**Modulation Transfer Function (MTF)**

- **MTF at paraxial focus**
- **MTF through focus for 100 cycles per mm**

**Geometrical spot through focus**

- $0.02 \text{ mm}$
- $-0.02 \text{ to } 0.02 \text{ mm}$

Ref: H. Zügge
Surface Contributions: Example

- Seidel aberrations: representation as sum of surface contributions possible
- Gives information on correction of a system
- Example: photographic lens

![Diagram of a photographic lens with labels for surfaces and aberrations]

<table>
<thead>
<tr>
<th>Surface</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>SI</td>
<td></td>
</tr>
<tr>
<td>SII</td>
<td></td>
</tr>
<tr>
<td>SIII</td>
<td></td>
</tr>
<tr>
<td>SIV</td>
<td></td>
</tr>
<tr>
<td>SVA</td>
<td></td>
</tr>
<tr>
<td>SVB</td>
<td></td>
</tr>
<tr>
<td>CVI</td>
<td></td>
</tr>
<tr>
<td>CVII</td>
<td></td>
</tr>
<tr>
<td>CIII</td>
<td></td>
</tr>
<tr>
<td>CIV</td>
<td></td>
</tr>
<tr>
<td>CVII</td>
<td></td>
</tr>
</tbody>
</table>

Aberration Types:
- Spherical Aberration (SI)
- Coma (SII)
- Astigmatism (SIII)
- Petzval field curvature (SIV)
- Distortion (SVA)
- Axial color (SVB)
- Lateral color (CVI)

Retrofocus F/2.8
Field: w=37°
Wave Aberration in Optical Systems

- Definition of optical path length in an optical system:
  Reference sphere around the ideal object point through the center of the pupil
- Chief ray serves as reference
  Difference of OPL : optical path difference OPD
- Practical calculation: discrete sampling of the pupil area,
  real wave surface represented as matrix
Pupil Sampling

- All rays start in one point in the object plane
- The entrance pupil is sampled equidistant
- In the exit pupil, the transferred grid may be distorted
- In the image plane a spreaded spot diagram is generated
Zernike Polynomials

- Expansion of wave aberration surface into elementary functions / shapes

\[ W(r, \varphi) = \sum_{n} \sum_{m=-n}^{n} c_{nm} Z_{n}^{m}(r, \varphi) \]

- Zernike functions are defined in circular coordinates \( r, \varphi \)

\[ Z_{n}^{m}(r, \varphi) = R_{n}^{m}(r) \cdot \begin{cases} 
\sin (m \varphi) & \text{for } m < 0 \\
\cos (m \varphi) & \text{for } m > 0 \\
1 & \text{for } m = 0 
\end{cases} \]

- Ordering of the Zernike polynomials by indices:
  - \( n \): radial
  - \( m \): azimuthal, sin/cos

- Mathematically orthonormal function on unit circle for a constant weighting function

- Direct relation to primary aberration types
Perfect Lateral Point Spread Function: Airy

Airy distribution:

- Gray scale picture
- Zeros non-equidistant
- Logarithmic scale
- Encircled energy
Circular homogeneous illuminated Aperture: intensity distribution

- transversal: Airy scale:
  \[ D_{Airy} = \frac{1.22 \cdot \lambda}{NA} \]

- axial: sinc scale
  \[ R_E = \frac{n \cdot \lambda}{NA^2} \]

- Resolution transversal better than axial: \( \Delta x < \Delta z \)

Scaled coordinates according to Wolf:
axial: \( u = 2 \pi \frac{z}{\lambda} \frac{n}{NA^2} \)
transversal: \( v = 2 \pi \frac{x}{\lambda} NA \)

Ref: M. Kempe
- Zernike coefficients $c$ in $\lambda$

- Spherical aberration, Circular symmetry
  - $c = 0.2$

- Astigmatism, Split of two azimuths
  - $c = 0.3$

- Coma, Asymmetric
  - $c = 0.5$
  - $c = 0.7$
  - $c = 1.0$
Optical Transfer Function: Definition

- Normalized optical transfer function (OTF) in frequency space

\[
H_{OTF}(v_x, v_y) = \frac{\int \int |g(x_p, y_p)|^2 \cdot e^{-2\pi i(x_p v_x + y_p v_y)} \, dx_p \, dy_p}{\int \int |g(x_p, y_p)|^2 \, dx_p \, dy_p}
\]

- Fourier transform of the Psf-intensity

\[
H_{OTF}(v_x, v_y) = \hat{F}[I_{PSF}(x, y)]
\]

- OTF: Autocorrelation of shifted pupil function, Duffieux-integral

\[
H_{OTF}(v_x, v_y) = \frac{\int \int P(x_p + \frac{\lambda f v_x}{2}, y_p + \frac{\lambda f v_y}{2}) \cdot P^*(x_p - \frac{\lambda f v_x}{2}, y_p - \frac{\lambda f v_y}{2}) \, dx_p \, dy_p}{\int \int |P(x_p, y_p)|^2 \, dx_p \, dy_p}
\]

- Absolute value of OTF: modulation transfer function (MTF)

- MTF is numerically identical to contrast of the image of a sine grating at the corresponding spatial frequency
Optical Transfer Function of a Perfect System

- Aberration free circular pupil:
  
  \[ v_o = \frac{a}{\lambda f} = \frac{\sin u'}{\lambda} \]

- Maximum cut-off frequency:
  
  \[ v_{\text{max}} = 2v_0 = \frac{2na}{\lambda f} = \frac{2n \sin u'}{\lambda} \]

- Analytical representation

  \[ H_{\text{MTF}}(v) = \frac{2}{\pi} \left[ \arccos \left( \frac{v}{2v_0} \right) - \left( \frac{v}{2v_0} \right) \sqrt{1 - \left( \frac{v}{2v_0} \right)^2} \right] \]

- Separation of the complex OTF function into:
  - absolute value: modulation transfer MTF
  - phase value: phase transfer function PTF

  \[ H_{\text{OTF}}(v_x, v_y) = H_{\text{MTF}}(v_x, v_y) \cdot e^{iH_{\text{PTF}}(v_x, v_y)} \]
Basic Idea of Optimization

- Topology of the merit function in 2 dimensions
- Iterative down climbing in the topology
Mathematical description of the problem:

- \( n \) variable parameters
- \( m \) target values
- Jacobi system matrix of derivatives, influence of a parameter change on the various target values, sensitivity function
- Scalar merit function
- Gradient vector of topology
- Hesse matrix of 2nd derivatives

Given:

- \( \vec{x} \)
- \( \vec{f}(\vec{x}) \)
- \( J_{ij} = \frac{\partial f_i}{\partial x_j} \)
- \( F(\vec{x}) = \sum_{i=1}^{m} w_i \cdot [y_i - f_i(\vec{x})]^2 \)
- \( g_j = \frac{\partial F}{\partial x_j} \)
- \( H_{jk} = \frac{\partial^2 F}{\partial x_j \partial x_k} \)
Optimization in Optical Design

- Merit function:
  Weighted sum of deviations from target values

- Formulation of target values:
  1. fixed numbers
  2. one-sided interval (e.g. maximum value)
  3. interval

- Problems:
  1. linear dependence of variables
  2. internal contradiction of requirements
  3. initial value far off from final solution

- Types of constraints:
  1. exact condition (hard requirements)
  2. soft constraints: weighted target

- Finding initial system setup:
  1. modification of similar known solution
  2. Literature and patents
  3. Intuition and experience

\[ \Phi = \sum_{j=1}^{m} g_j \cdot \left( f_{j}^{\text{ist}} - f_{j}^{\text{soll}} \right)^2 \]
Goal of optimization:
Find the system layout which meets the required performance targets according of the specification

Formulation of performance criteria must be done for:
- Apertur rays
- Field points
- Wavelengths
- Optional several zoom or scan positions

Selection of performance criteria depends on the application:
- Ray aberrations
- Spot diameter
- Wavefornt description by Zernike coefficients, rms value
- Strehl ratio, Point spread function
- Contrast values for selected spatial frequencies
- Uniformity of illumination

Usual scenario:
Number of requirements and targets quite larger than degrees od freedom,
Therefore only solution with compromise possible
Ray path at a lens of constant focal length and different bending

Quantitative parameter of description $X$:

The ray angle inside the lens changes

The ray incidence angles at the surfaces changes strongly

The principal planes move

For invariant location of $P$, $P'$ the position of the lens moves

\[ X = \frac{R_1 + R_2}{R_2 - R_1} \]
Correcting Spherical Aberration: Lens Splitting

- Correction of spherical aberration: Splitting of lenses

- Distribution of ray bending on several surfaces:
  - smaller incidence angles reduces the effect of nonlinearity
  - decreasing of contributions at every surface, but same sign

- Last example (e): one surface with compensating effect

Ref: H. Zügge