Exercise 4-1: Spherical Mirror

Derive the expression for the focal length of a spherical mirror in the paraxial limit and also for finite height $y$ of a collimated incoming beam (parallel to $z$ axis). Calculate the formula of the axial spherical aberration for this collimated incoming beam through third order in $y$!

Hint: use the reflection law to eliminate the appearing angles in favour of $y$ and the radius of curvature $r$!

Exercise 4-2: Spherical Aberration of a single lens

A thin plane-convex lens with focal length $f' = 100$ mm is used to focus a collimated beam with diameter $D = 10$ mm and wavelength $\lambda = 1.06$ µm.

Draw a sketch of the lens in the optimal setup and explain why this orientation is advantageous! Calculate the magnification parameter $M$ and the bending parameter $X$ of the lens. The Quantity $M$ is defined to be

$$ M := \frac{u + u'}{u - u'}, $$

where $u$ and $u'$ are the angles of the ray with respect to the optical axis. The surface contribution of the primary spherical wave aberration of a single lens can be expressed by the formula

$$ A_s = \frac{1}{32n(n-1)f^2} \left[ \frac{n^3}{n-1} + \frac{n+2}{n-1} \left\{ X - \frac{2(n^2-1)}{n+2} M \right\}^2 - \frac{n^2(n-1)}{n+2} M^2 \right] $$

Calculate this coefficient for the refractive indices $n = 1.4$ and $n = 2.0$! Which choice of refractive index is more advantageous? Also, calculate the transverse aberration $\Delta y'$ in the image plane in the one-dimensional case with the pupil coordinate $y_p$ by using this formula in the form $W_{SPH}(y_p) = A_s \cdot y_p^4$ for both indices.

Compare the geometrical spot size and the diffraction Airy diameter!

Is the system diffraction limited?
Exercise 4-3: Thin Lens Formula for Spherical Aberration

The equation for the contribution of the spherical aberration of third order of a single dielectric surface reads

\[ A_s = -\frac{n'(n''-n')}{8n^2} \left( \frac{1}{R} - \frac{1}{s'} \right)^2 \left( \frac{n'}{R} - \frac{n''-n}{s'} \right) \]

Use this formula to derive the corresponding formula for a single thin lens from Exercise 4-2:

\[ A_{s\text{ (lens)}} = \frac{1}{32n(n-1)f''} \left[ \frac{n^3}{n-1} + \frac{n+2}{n-1} \left( X - \frac{2(n^2-1)}{n+2} \cdot M \right)^2 - \frac{n^2(n-1)}{n+2} \cdot M^2 \right] \]

(Hint: elimination with the help of imaging and lens maker equation)

Exercise 3-5: Field flattening lens

Calculate a thick meniscus shaped lens with refractive index \( n = 1.5 \) with vanishing Petzval curvature contribution, a focal length of \( f = 100 \text{ mm} \) and a front radius of \( r_1 = +10 \text{ mm} \). The back radius and the thickness are to be determined.